Chapter-2: Vector

Question ▶ 1 $\vec{P} = 3\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{Q} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{R} = \hat{i} - \hat{j} + 2\hat{k}$. *[R.B.-17]*

a. Find the vector equation of a straight line passing through the point $\vec{P}$ and parallel to the vector $\vec{Q}$. 2

b. Show that, the vector $\vec{P} - \vec{Q}$ is perpendicular to the vector which is perpendicular to the plane formed with $\vec{P}$ and $\vec{Q}$. 4

c. According to the stem if A is a matrix formed with the co-efficients of $\hat{i}, \hat{j}, \hat{k}$ then find A^{-1}. 4

Solution to the question no. 1

a Given that, $\vec{P} = 3\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{Q} = 3\hat{i} - 2\hat{j} + 4\hat{k}$

∴ The vector equation of the line passing through the point $\vec{P}$ and parallel to the vector $\vec{Q}$ is , $\vec{r} = \vec{P} + t\vec{Q}$

$= 3\hat{i} - 3\hat{j} + 4\hat{k} + t(3\hat{i} - 2\hat{j} + 4\hat{k})$

$= (3 + 3t)\hat{i} - (3 + 2t)\hat{j} + (4 + 4t)\hat{k}$ **(Ans.)**

b Here, $\vec{P} - \vec{Q} = 3\hat{i} - 3\hat{j} + 4\hat{k} - 3\hat{i} + 2\hat{j} - 4\hat{k} = -\hat{j}$

∴ Again, the vector perpendicular to the plane consists with the vectors $\vec{P}$ and $\vec{Q}$ is

$$\vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -3 & 4 \\ 3 & -2 & 4 \end{vmatrix}$$

$= (-12 + 8)\hat{i} - (12 - 12)\hat{j} + (-6 + 9)\hat{k}$

$= -4\hat{i} + 3\hat{k}$

Now, $(\vec{P} - \vec{Q}).(\vec{P} \times \vec{Q}) = (-\hat{j}).(-4\hat{i} + 3\hat{k}) = 0$

∴ The vector $(\vec{P} - \vec{Q})$ is perpendicular to the vector that is perpendicular to the plane consists with the vectors $\vec{P}$ and $\vec{Q}$ **(Prove)**

c Given that, $\vec{P} = 3\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{Q} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{R} = \hat{i} - \hat{j} + 2\hat{k}$

The matrix conrists with the coefficients of $\hat{i}, \hat{j}, \hat{k}$ of the given vectors, $A = \begin{bmatrix} 3 & -3 & 4 \\ 3 & -2 & 4 \\ 1 & -1 & 2 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 3 & -3 & 4 \\ 3 & -2 & 4 \\ 1 & -1 & 2 \end{vmatrix}$$

$= 3(-4 + 4) - (-3)(6 - 4) + 4(-3 + 2)$

$= 0 + 6 - 4 = 2$

∴ A is invertible.

Now, $A_{11} = (-1)^{1+1} \begin{vmatrix} -2 & 4 \\ -1 & 2 \end{vmatrix} = -4 + 4 = 0$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = -(6 - 4) = -2$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & -2 \\ 1 & -1 \end{vmatrix} = -3 + 2 = -1$

$A_{21} = (-1)^{2+1} \begin{vmatrix} -3 & 4 \\ -1 & 2 \end{vmatrix} = -(-6 + 4) = 2$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = 6 - 4 = 2$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -3 \\ 1 & -1 \end{vmatrix} = -(-3 + 3) = 0$

$A_{31} = (-1)^{3+1} \begin{vmatrix} -3 & 4 \\ -2 & 4 \end{vmatrix} = -12 + 8 = -4$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 4 \\ 3 & 4 \end{vmatrix} = -(12 - 12) = 0$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -3 \\ 3 & -2 \end{vmatrix} = -6 + 9 = 3$

$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A$

$$= \frac{1}{2}\begin{bmatrix} 0 & -2 & -1 \\ 2 & 2 & 0 \\ -4 & 0 & 3 \end{bmatrix}^t$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 2 & -4 \\ -2 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 1 & 0 \\ \frac{-1}{2} & 0 & \frac{3}{2} \end{bmatrix}$$ **(Ans.)**

Question ▶ 2 $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{C} = \hat{i} + b\hat{j} + 3\hat{k}$. *[Dj.B.-17]*

a. What do you mean by position vector? 2

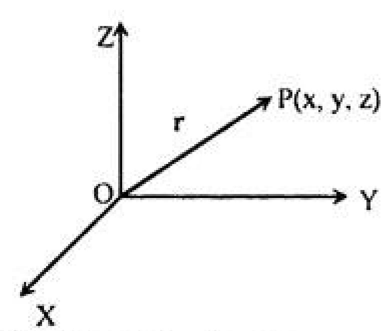
b. If the component of vector $\vec{B}$ along $\vec{A}$ is perpendicular to $\vec{C}$, then find the value of b. 4

c. Find the angle between the vectors $\vec{A} + \vec{B}$ and $\vec{A} \times \vec{B}$. 4

Solution to the question no. 2

a **Position vector:**

A vector which represents the position of a point in a space with respect to the origin, is called position vector of that point. It also represents the distance and direction of a point from the origin.



If P(x, y,z) is any point and O is the origin, then the position vector of P is $\overrightarrow{OP}$. If we denote $\overrightarrow{OP} = \vec{r}$, then $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, where $\hat{i}, \hat{j}, \hat{k}$ are unit vetors along with the direction of axes.

b Given that,

$\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{C} = \hat{i} + b\hat{j} + 3\hat{k}$

The component of the vector $\vec{B}$ in the direction of the vector $\vec{A} = \left(\frac{\vec{A}.\vec{B}}{|\vec{A}|}\right)\left(\frac{\vec{A}}{|\vec{A}|}\right)$

$$= \left(\frac{2 + 6 + 1}{\sqrt{2^2 + 3^2 + (-1)^2}}\right)\left(\frac{2\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{2^2 + 3^2 + (-1)^2}}\right)$$

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$= \frac{9}{14}(2\hat{i} + 3\hat{j} - \hat{k})$

If the component of the vector $\vec{B}$ in the direction of the vector $\vec{A}$ is perpendicular to the vector $\vec{C}$, then

$\frac{9}{14}(2\hat{i} + 3\hat{j} - \hat{k}) . (\hat{i} + b\hat{j} + 3\hat{k}) = 0$

or, $2 + 3b - 3 = 0$ or, $3b = 1$

$\therefore b = \frac{1}{3}$ **(Ans.)**

c Here, $\vec{A} + \vec{B} = 2\hat{i} + 3\hat{j} - \hat{k} + \hat{i} + 2\hat{j} - \hat{k} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

and $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & 2 & -1 \end{vmatrix}$

$= (-3 + 2)\hat{i} - (-2 + 1)\hat{j} + (4 - 3)\hat{k} = -\hat{i} + \hat{j} + \hat{k}$

and the angle between the vectors $\vec{A} + \vec{B}$ and $\vec{A} \times \vec{B}$ is,

$\theta = \cos^{-1}\left(\frac{(3\hat{i} + 5\hat{j} - 2\hat{k}). (-\hat{i} + \hat{j} + \hat{k})}{\sqrt{3^2 + 5^2 + (-2)^2}\sqrt{(-1)^2 + 1^2 + 1^2}}\right)$

$= \cos^{-1}\left(\frac{-3 + 5 - 2}{\sqrt{38} . \sqrt{3}}\right)$

$= \cos^{-1}(0) = 90°$ **(Ans.)**

Question ▶ 3 $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$; $\vec{B} = -\hat{i} - 4\hat{j} + 7\hat{k}$ and co-ordinates of three points are P(– 3, – 2, – 1); Q(4, 0, – 3) and S(6, – 7, 8). *[S.B.-17]*

a. Define Unit vector with an example. 2

b. According to stem find the component of $\vec{B}$ along the vector $\vec{A}$. 4

c. According to stem find the area of ΔPQS. 4

Solution to the question no. 3

a **Unit vector:** A vector that has a magnitude of one (1) is called unit vector. If $\overrightarrow{AB} = \vec{a}$ is any vector, then the unit vector in the direction of $\overrightarrow{AB}$ is $\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{\vec{a}}{|\vec{a}|}$. The unit vector $\frac{\vec{a}}{|\vec{a}|}$ is generally denoted by $\hat{a}$ (read as a hat).

Let $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$,

Then $|\vec{a}| = \sqrt{2^2 + (-3)^2 + 1^2} = \sqrt{14}$

$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{14}}$

b Given that, $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$, $\vec{B} = -\hat{i} - 4\hat{j} + 7\hat{k}$

$\therefore$ The component of $\vec{B}$ in the direction of $\vec{A}$

$= \left(\frac{\vec{A}.\vec{B}}{A}\right)\left(\frac{\vec{A}}{A}\right)$

$= \left(\frac{2(-1) + (-3)(-4) + (-1)7}{\sqrt{2^2 + (-3)^2 + (-1)^2}}\right)\left(\frac{2\hat{i} - 3\hat{j} - \hat{k}}{\sqrt{2^2 + (-3)^2 + (-1)^2}}\right)$

$= \left(\frac{-2 + 12 - 7}{\sqrt{14}}\right)\left(\frac{2\hat{i} - 3\hat{j} - \hat{k}}{\sqrt{14}}\right)$

$= \frac{3}{14}(2\hat{i} - 3\hat{j} - \hat{k})$ **(Ans.)**

c P(–3, –2, –1), Q(4, 0, –3) and S(6, –7, 8)

$\therefore \overrightarrow{PQ} = 4\hat{i} - 3\hat{k} - (-3\hat{i} - 2\hat{j} - \hat{k})$

$= 4\hat{i} - 3\hat{k} + 3\hat{i} + 2\hat{j} + \hat{k} = 7\hat{i} + 2\hat{j} - 2\hat{k}$

and $\overrightarrow{QS} = 6\hat{i} - 7\hat{j} + 8\hat{k} - 4\hat{i} + 3\hat{k} = 2\hat{i} - 7\hat{j} + 11\hat{k}$

$\therefore \overrightarrow{PQ} \times \overrightarrow{QS} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 2 & -2 \\ 2 & -7 & 11 \end{vmatrix}$

$= (22 - 14)\hat{i} - (77 + 4)\hat{j} + (-49 - 4)\hat{k}$

$= 8\hat{i} - 81\hat{j} - 53\hat{k}$

$\therefore$ The area of $\Delta PQS = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{QS}|$

$= \frac{1}{2}\sqrt{8^2 + (-81)^2 + (-53)^2}$

$= \frac{1}{2}\sqrt{9434}$

$= 48.564$ square units (Approx.) **(Ans.)**

Question ▶ 4 A (1, 2, 3), B (2, 3, 1) and C (3, 1, 2) as the position vectors of vertics of ΔABC.

[Mymensingh Girls' Cadet College, Mymensingh]

a. Show that, ΔABC is an equilateral triangle. 2

b. Determine the value of the angle A. 4

c. Determine the area of ΔABC. 4

Solution to the question no. 4

a Let, the position vecters of A, B and C with respect to the origin O are,

$\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\overrightarrow{OB} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\overrightarrow{OC} = 3\hat{i} + \hat{j} + 2\hat{k}$ respectively.

$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (2\hat{i} + 3\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= \hat{i} + \hat{j} - 2\hat{k}$

$\therefore |\overrightarrow{AB}| = \sqrt{1 + 1 + 4} = \sqrt{6}$

Again, $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= (3\hat{i} + \hat{j} + 2\hat{k}) - (2\hat{i} + 3\hat{j} + \hat{k})$

$= \hat{i} - 2\hat{j} + \hat{k}$

$\therefore |\overrightarrow{BC}| = \sqrt{1 + 4 + 1} = \sqrt{6}$

Again, $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= (3\hat{i} + \hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= 2\hat{i} - \hat{j} - \hat{k}$

$\therefore |\overrightarrow{AC}| = \sqrt{4 + 1 + 1} = \sqrt{6}$

$\therefore |\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{AC}|$

$\therefore$ ΔABC is an equiateral triangle. **(Shown)**

b From 'a', $\overrightarrow{AB} = \hat{i} + \hat{j} - 2\hat{k}$, $\overrightarrow{AC} = 2\hat{i} - \hat{j} - \hat{k}$

and $|\overrightarrow{AB}| = |\overrightarrow{AC}| = \sqrt{6}$

Since the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$, therefore,

$\cos A = \frac{\overrightarrow{AB} . \overrightarrow{AC}}{|\overrightarrow{AB}| |\overrightarrow{AC}|} = \frac{2 - 1 + 2}{6} = \frac{3}{6} = \frac{1}{2}$

$\therefore A = \cos^{-1}\left(\frac{1}{2}\right) = 60°$ **(Ans.)**

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c Here, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 2 & -1 & -1 \end{vmatrix}$

$= \hat{i}(-1-2) - \hat{j}(-1+4) + \hat{k}(-1-2)$

$= -3\hat{i} - 3\hat{j} - 3\hat{k}$

$\therefore \quad |\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-3)^2 + (-3)^2 + (-3)^2}$

$= \sqrt{27} = 3\sqrt{3}$

$\therefore$ The area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \frac{1}{2} \times 3\sqrt{3}$

$= \frac{3\sqrt{3}}{2}$ sq. units **(Ans.)**

Question ▶ 5 $\vec{P} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, $\vec{Q} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ are two vectors. *[Rajshahi Cadet College, Rajshahi]*

a. If A(2, –1, 3) and B(3, 2, – 4), then find $\overrightarrow{AB}$. 2

b. Find the included angle between $\vec{P}$ and $\vec{Q}$. 4

c. Find the unit vector perpendicular to vectors $\vec{P}$ and $\vec{Q}$. 4

Solution to the question no. 5

a The position vectors of A(2, –1, 3) and B(3, 2, – 4)

$\overrightarrow{OA} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\overrightarrow{OB} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$\therefore \overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (3\hat{i} + 2\hat{j} - 4\hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k})$

$= \hat{i} + 3\hat{j} - 7\hat{k}$

$\therefore |\overrightarrow{OP}| = \sqrt{1^2 + 3^2 + (-7)^2} = \sqrt{59}$ **(Ans.)**

b Given that,

$\vec{P} = 3\hat{i} - 6\hat{j} + 2\hat{k}$,

$\vec{Q} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

Let, the included angle between $\vec{P}$ and $\vec{Q}$ be θ.

$\therefore \vec{P}.\vec{Q} = PQ\cos\theta$. .... (i)

Now, $P = |\vec{P}| = \sqrt{(3)^2 + (-6)^2 + (2)^2} = \sqrt{49} = 7$.

$Q = |\vec{Q}| = \sqrt{(2)^2 + (3)^2 + (-4)^2} = \sqrt{29}$

$\vec{P}.\vec{Q} = (3\hat{i} - 6\hat{j} + 2\hat{k}).(2\hat{i} + 3\hat{j} - 4\hat{k})$

$= 6 - 18 - 8$

$= -20$

From equation (i) we get,

$-20 = 7.\sqrt{29}.\cos\theta$

or, $\cos\theta = \frac{-20}{7\sqrt{29}}$

or, $\theta = \cos^{-1}\left(-\frac{20}{7\sqrt{29}}\right)$.

$\therefore$ The required angle, $\cos^{-1}\left(-\frac{20}{7\sqrt{29}}\right)$ **(Ans.)**

c $P \times Q = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 2 & 3 & -4 \end{vmatrix}$

$= (24-6)\hat{i} - (-12-4)\hat{j} + (9+12)\hat{k}$

$= 18\hat{i} + 16\hat{j} + 21\hat{k}$

$\therefore \quad |P \times Q| = \sqrt{18^2 + 16^2 + 21^2} = \sqrt{324 + 256 + 441} = \sqrt{1021}$

Therefore, the required Unit vector Perpendicular to P and Q is $= \pm \frac{P \times Q}{|P \times Q|}$

$= \pm \frac{1}{\sqrt{1021}}(18\hat{i} + 16\hat{j} + 21\hat{k})$ **(Ans.)**

Question ▶ 6 $a = 3\hat{i} + 2\hat{j} - \hat{k}$, $b = 5\hat{i} - 4\hat{j} + \hat{k}$

[Pabna Cadet College, Pabna]

a. Find the magnitude of resultant of $\vec{a}$ and $\vec{b}$. 2

b. Find the projection of $\vec{b}$ along the vector $\vec{a}$. 4

c. Find the unit vector perpendicular to the vectors $\vec{a}$ and $\vec{b}$. 4

Solution to the question no. 6

a Given that, $a = 3\hat{i} + 2\hat{j} - \hat{k}$ and $b = 5\hat{i} - 4\hat{j} + \hat{k}$

$\therefore$ The resultant of a and b = a + b

$= (3\hat{i} + 2\hat{j} - \hat{k}) + (5\hat{i} - 4\hat{j} + \hat{k})$

$= 8\hat{i} - 2\hat{j}$

$\therefore$ The magnitude of the resultant

$\sqrt{8^2 + (-2)^2} = \sqrt{64 + 4} = \sqrt{68} = 2\sqrt{17}$ **(Ans.)**

b $|a| = \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$

$|b| = \sqrt{5^2 + (-4)^2 + 1^2} = \sqrt{25 + 16 + 1} = \sqrt{42}$

$a.b = (3\hat{i} + 2\hat{j} - \hat{k}).(5\hat{i} - 4\hat{j} + \hat{k}) = 15 - 8 - 1 = 6$

$\therefore$ The projection of $\vec{b}$ on $\vec{a} = |b|\cos\theta = \frac{a.b}{|a|}$

$= \frac{6}{\sqrt{14}} = \frac{6\sqrt{14}}{14} = \frac{3\sqrt{14}}{7}$ **(Ans.)**

c $a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -1 \\ 5 & -4 & 1 \end{vmatrix}$

$= (2-4)\hat{i} - (3+5)\hat{j} + (-12-10)\hat{k}$

$= -2\hat{i} - 8\hat{j} - 22\hat{k}$

$\therefore \quad |a \times b| = \sqrt{(-2)^2 + (-8)^2 + (-22)^2}$

$= \sqrt{4 + 64 + 484}$

$= \sqrt{552} = 2\sqrt{138}$

$\therefore$ The unit vector perpendicular to both $\vec{a}$ and $\vec{b} = \pm \frac{a \times b}{|a \times b|}$

$= \pm \frac{(-2\hat{i} - 8\hat{j} - 22\hat{k})}{2\sqrt{138}}$

$= \pm \frac{1}{\sqrt{138}}(\hat{i} + 4\hat{j} + 11\hat{k})$ **(Ans.)**

Question ▶ 7 $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$, $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$ and $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & p & p^2 \\ 1 & p^2 & p^4 \end{vmatrix}$

[Joypurhat Girls' Cadet College, Joypurhat]

a. Find the component of vector $\vec{B}$ along $\vec{A}$. 2

b. Prove that, $D = p(p-1)^2(p^2-1)$. 4

c. Determine the orthogonal unit vector on the plane that is formed by two vectors $\vec{A}$ and $\vec{B}$. 4

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Solution to the question no. 7

a The projection of B on A $= |B| \cos\theta = \frac{A.B}{|A|}$

$$= \frac{(2)(4) + (-6)(3) + (-3)(-1)}{\sqrt{2^2 + (-6)^2 + (-3)^2}}$$
$$= \frac{8 - 18 + 3}{\sqrt{49}}$$
$$= \frac{-7}{7} = -1 \text{ (Ans.)}$$

b $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & p & p^2 \\ 1 & p^2 & p^4 \end{vmatrix}$

$$= \begin{vmatrix} 0 & 0 & 1 \\ p-1 & p^2-p & p^2 \\ p^2-1 & p^4-p^2 & p^4 \end{vmatrix} \text{ [Applying } c'_1 = c_2 - c_1 \text{ and } c'_2 = c_3 - c_2]$$

$$= \begin{vmatrix} p-1 & p(p-1) \\ (p-1)(p+1) & p^2(p^2-1) \end{vmatrix}$$

$$= p(p-1)(p-1)\begin{vmatrix} 1 & 1 \\ p+1 & p(p+1) \end{vmatrix}$$

$$= p(p-1)^2 \{p(p+1) - (p+1)\}$$
$$= p(p-1)^2 (p+1)(p-1)$$
$$= p(p-1)^2 (p^2 - 1)$$

$\therefore D = p(p-1)^2(p^2 - 1)$ **(Proved)**

c Let, the orthogonal unit vector on the plane that is formed by two vectors A and B is $\hat{\eta}$

$\therefore \hat{\eta} = \pm\frac{A \times B}{|A \times B|}$

$\therefore A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$

$$= \hat{i}(6+9) - \hat{j}(-2+12) + \hat{k}(6+24)$$
$$= 15\hat{i} - 10\hat{j} + 30\hat{k}$$

and $|A \times B| = \sqrt{(15)^2 + (-10)^2 + 30^2}$

$$= \sqrt{225 + 100 + 900}$$
$$= \sqrt{1225} = 35$$

$\therefore$ The required orthogonal unit vector, $\hat{\eta} = \pm\frac{15\hat{i} - 10\hat{j} + 30\hat{k}}{35}$

$$= \pm\frac{5(3\hat{i} - 2\hat{j} + 6\hat{k})}{35}$$
$$= \pm\frac{1}{7}(3\hat{i} - 2\hat{j} + 6\hat{k}) \text{ (Ans.)}$$

Question ▶ 8 $\vec{P} = 3\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{Q} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{R} = \hat{i} - \hat{j} + 2\hat{k}$.

[Feni Girls' Cadet College, Feni]

a. Find the vector equation of the straight line passing through $\vec{P}$ and parallel to the vector $\vec{Q}$. 2

b. Show that, the vector, $\vec{P} - \vec{Q}$ is perpendicular to the vector which is perpendicular to the plane formed by the vectors $\vec{P}$ and $\vec{Q}$. 4

c. If A is the matrix formed by the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ vectors of the stem, then find A^{-1}. 4

Solution to the question no. 8

See the Question No.- 1

Question ▶ 9 $\vec{A} = -\hat{i} - \hat{j} + 2a\hat{k}$, $\vec{B} = \hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{C} = -4\hat{i} + \hat{j} + 3\hat{k}$

[Jhenidah Cadet College, Jhenidah]

a. Determine, $\vec{C} \times (-\hat{i} + \hat{j})$ 2

b. Find the component of $2\vec{B}$ along $\vec{C}$. 4

c. Considering the given vectors coplanar, determine the scalar product of $3\vec{A}$ and $4\vec{B}$. 4

Solution to the question no. 9

a Given that, $C = -4\hat{i} + \hat{j} + 3\hat{k}$

$\therefore C \times (-\hat{i} + \hat{j}) = (-4\hat{i} + \hat{j} + 3\hat{k}) \times (-\hat{i} + \hat{j})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 1 & 3 \\ -1 & 1 & 0 \end{vmatrix}$$
$$= \hat{i}(0-3) - \hat{j}(0+3) + \hat{k}(-4+1)$$
$$= -3\hat{i} - 3\hat{j} - 3\hat{k}$$
$$= -3(\hat{i} + \hat{j} + \hat{k}) \text{ (Ans.)}$$

b Given that, $B = \hat{i} - 3\hat{j} + 2\hat{k}$ and $C = -4\hat{i} + \hat{j} + 3\hat{k}$

$\therefore 2B = 2\hat{i} - 6\hat{j} + 4\hat{k}$

$\therefore$ The component of 2B along C $= \frac{(2B.C)}{|C|^2} C$

$$= \frac{(2\hat{i} - 6\hat{j} + 4\hat{k}).(-4\hat{i} + \hat{j} + 3\hat{k})}{(\sqrt{(-4)^2 + (1)^2 + (3)^2})^2}(-4\hat{i} + \hat{j} + 3\hat{k})$$
$$= \frac{-8 - 6 + 12}{16 + 1 + 9}(-4\hat{i} + \hat{j} + 3\hat{k})$$
$$= -\frac{1}{13}(-4\hat{i} + \hat{j} + 3\hat{k})$$
$$= \frac{1}{13}(4\hat{i} - \hat{j} - 3\hat{k}) \text{ (Ans.)}$$

c If the vectors are coplaner then,

$$\begin{vmatrix} -1 & -1 & 2a \\ 1 & -3 & 2 \\ -4 & 1 & 3 \end{vmatrix} = 0$$

or, $-(-9-2) + 1(3+8) + 2a(1-12) = 0$

or, $11 + 11 - 22a = 0$

or, $22a = 22 \quad \therefore a = 1$

$\therefore A = -\hat{i} - \hat{j} + 2\hat{k}$ and $B = \hat{i} - 3\hat{j} + 2\hat{k}$

$\therefore 3A = 3(-\hat{i} - \hat{j} + 2\hat{k}) = -3\hat{i} - 3\hat{j} + 6\hat{k}$

$4B = 4(\hat{i} - 3\hat{j} + 2\hat{k}) = 4\hat{i} - 12\hat{j} + 8\hat{k}$

$\therefore (3A).(4B) = (-3\hat{i} - 3\hat{j} + 6\hat{k}).(4\hat{i} - 12\hat{j} + 8\hat{k})$

$$= -12 + 36 + 48$$
$$= 72 \text{ (Ans.)}$$

Question ▶ 10 ▶ $\vec{a} = 2\hat{i} + \hat{j} + \mu\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$ are three vectors. *[Notre Dame College, Dhaka]*

a. If $\vec{a}$ and $\vec{b}$ are perpendicular then find the value of μ. 2

b. For what value of μ, vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar? 4

c. Determine the unit vector which is perpendicular to the plane containing $\vec{b}$ and $\vec{c}$. 4

Solution to the question no. 10

a Given that,

$$\vec{a} = 2\hat{i} + \hat{j} + \mu\hat{k}$$
$$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$$
$$\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$$

Since $\vec{a}$ and $\vec{b}$ are perpendicular.

$\therefore \vec{a}.\vec{b} = 0$

or, $(2\hat{i} + \hat{j} + \mu\hat{k}).(3\hat{i} - 2\hat{j} + \hat{k}) = 0$

or, $6 - 2 + \mu = 0$

or, $\mu = -4$.

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b Since vector $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar,

$$\vec{a}.(\vec{b}\times\vec{c}) = 0.$$

$$\text{or, } \begin{vmatrix} 2 & 1 & \mu \\ 3 & -2 & 1 \\ 1 & -3 & 5 \end{vmatrix} = 0$$

or, $2(-10+3) + 1(1-15) + \mu(-9+2) = 0$

or, $-14 - 14 + \mu(-7) = 0$

or, $-28 - 7\mu = 0$

or, $\mu = -4$. **(Ans.)**

c Let the unit vector perpendicular to the plane containing $\vec{b}$ and $\vec{c}$ is $\hat{\eta}$

$$\therefore \hat{\eta} = \pm\frac{\vec{b}\times\vec{c}}{|\vec{b}\times\vec{c}|}$$

$$\text{Now, } \vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 1 & -3 & 5 \end{vmatrix}$$

$$= \hat{i}(-10+3) + \hat{j}(1-15) + \hat{k}(-9+2)$$

$$= -7\hat{i} - 14\hat{j} - 7\hat{k}.$$

$$|\vec{b}\times\vec{c}| = \sqrt{(-7)^2 + (-14)^2 + (-7)^2}$$

$$= \sqrt{49 + 196 + 49}$$

$$= \sqrt{294}$$

$$= \sqrt{49 \times 6}$$

$$= 7\sqrt{6}$$

$$\therefore \hat{\eta} = \pm\frac{-7\hat{i} - 14\hat{j} - 7\hat{k}}{7\sqrt{6}}$$

$$= +\frac{(-\hat{i} - 2\hat{j} - \hat{k})}{\sqrt{6}} \textbf{ (Ans.)}$$

Question ▶ 11 $\vec{A} = \hat{i} + 2\hat{j}$, $\vec{B} = 2\hat{j} + 3\hat{k}$, $\vec{C} = \hat{i} + 3\hat{k}$

[Dhaka City College, Dhaka]

a. If $p\hat{i} - 2\hat{j} + \hat{k}$ and $p\hat{i} + p\hat{j} - \hat{k}$ are perpendicular then the value of p. 2

b. If M is matrix formed with the coefficients of $\hat{i}$, $\hat{j}$, $\hat{k}$ then find M^{-1}. 4

c. Find the component of the vector $(\vec{B} + \vec{C})$ along the vector $(\vec{A} - \vec{B})$. 4

Solution to the question no. 11

a Let, $\vec{M} = p\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{N} = p\hat{i} + p\hat{j} - \hat{k}$

The vectors M and N are perpendicular to each other if M.N = 0

i.e, $(p\hat{i} - 2\hat{j} + \hat{k}).(p\hat{i} + p\hat{j} - \hat{k}) = 0$

or, $p^2 - 2p - 1 = 0$

$$\therefore p = \frac{-(-2) \pm \sqrt{(-2)^2 - 4.1.(-1)}}{2.1}$$

$$= \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$$

$$= \frac{2(1 \pm \sqrt{2})}{2} = 1 \pm \sqrt{2} \textbf{ (Ans.)}$$

b Given that,

$\vec{A} = \hat{i} + 2\hat{j}$, $\vec{B} = 2\hat{j} + 3\hat{k}$, $\vec{C} = \hat{i} + 3\hat{k}$ and M is a matrix formed with the coefficients of $\hat{i}$, $\hat{j}$, $\hat{k}$ of $\vec{A}$, $\vec{B}$ and $\vec{C}$.

$$\therefore M = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\text{Here, } |M| = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 1 & 0 & 3 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & 3 \\ 0 & 3 \end{vmatrix} - 2\begin{vmatrix} 0 & 3 \\ 1 & 3 \end{vmatrix} + 0$$

$$= 1(6-0) - 2(0-3)$$

$$= 6 + 6 = 12 \neq 0$$

Now, $M_{11} = (-1)^{1+1}(6-0) = 6$

$M_{12} = (-1)^{1+2}(0-3) = 3$

$M_{13} = (-1)^{1+3}(0-2) = -2$

$M_{21} = (-1)^{2+1}(6-0) = -6$

$M_{22} = (-1)^{2+2}(3-0) = 3$

$M_{23} = (-1)^{2+3}(0-2) = 2$

$M_{31} = (-1)^{3+1}(6-0) = 6$

$M_{32} = (-1)^{3+2}(3-0) = -3$

$M_{33} = (-1)^{3+3}(2-0) = 2$

$$\therefore M^{-1} = \frac{1}{|M|}\text{adj } A = \frac{1}{12}\begin{bmatrix} 6 & -6 & 6 \\ 3 & 3 & -3 \\ -2 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{bmatrix}$$

(Ans.)

c Here, $\overline{B} + \overline{C} = (2\hat{j} + 3\hat{k}) + (\hat{i} + 3\hat{k})$

$= \hat{i} + 2\hat{j} + 6\hat{k}$

and $\overline{A} - \overline{B} = (\hat{i} + 2\hat{j}) - (2\hat{j} + 3\hat{k})$

$= \hat{i} - 3\hat{k}$

$\therefore$ The component of $(\overline{B} + \overline{C})$ along $(\overline{A} - \overline{B})$

$$= \frac{(\overline{A} - \overline{B}).(\overline{B} + \overline{C})}{(|\overline{A} - \overline{B}|)^2}.(\overline{A} - \overline{B})$$

$$= \frac{(\hat{i} - 3\hat{k}).(\hat{i} + 2\hat{j} + 6\hat{k})}{1^2 + (-3)^2}(\hat{i} - 3\hat{k})$$

$$= \frac{1 + 0 - 18}{10}(\hat{i} - 3\hat{k})$$

$$= -\frac{17}{10}(\hat{i} - 3\hat{k}) \textbf{ (Ans.)}$$

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