

# Chapter 10: Measures of Dispersions and Probability

**Ques ► 1** Scenario-1: One dice and two coins are thrown simultaneously.

Scenario-2: A frequency distribution table :

Age (year)	20-30	30-40	40-50	50-60	60-70
Number of labours	25	40	20	10	5

[D.B., Dj.B., S.B., J.B.-18]

- If  $P(A) = \frac{1}{3}$  and  $P(A \cap B) = \frac{1}{5}$ , find out  $P(B/A)$ ? 2
- Find the probability of getting odd numbers in dice constructing the sample space using scenario-1. 4
- Find the standard deviation from scenario-2. 4

**Solution to the question no. 1**

**a** Given that,  $P(A) = \frac{1}{3}$  and  $P(A \cap B) = \frac{1}{5}$

$$\therefore P(B | A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{5}}{\frac{1}{3}} = \frac{1}{5} \times 3 = \frac{3}{5} \text{ (Ans.)}$$

**b** The sample space of throwing one dice and two coins simultaneously can be determined from the following table:

The outcomes of throwing two coins	The outcomes of throwing a dice					
	1	2	3	4	5	6
HH	(HH1)	(HH2)	(HH3)	(HH4)	(HH5)	(HH6)
HT	(HT1)	(HT2)	(HT3)	(HT4)	(HT5)	(HT6)
TH	(TH1)	(TH2)	(TH3)	(TH4)	(TH5)	(TH6)
TT	(TT1)	(TT2)	(TT3)	(TT4)	(TT5)	(TT6)

$\therefore$  The required sample space,  
 $S = \{HH1, HH2, \dots, HH6, HT1, HT2, \dots, HT6, TH1, TH2, \dots, TH6, TT1, TT2, \dots, TT6\}$

Here, the number of sample points,  $n(S) = 24$   
 Again, the favourable outcomes in the sample space are  
 $= \{HH1, HT1, TH1, TT1, HH3, HT3, TH3, TT3, HH5, HT5, TH5, TT5\}$

$\therefore$  The number of favourable outcomes of getting odd numbers in dice = 12

$\therefore$  The Probability of getting odd numbers in dice =  $\frac{12}{24} = \frac{1}{2}$  (Ans.)

**c** We use the following table to calculate the standard deviation:

Age (Year)	Number of Labours $f_i$	Midpoint $(x_i)$	$u_i = \frac{x_i - 45}{10}$	$f_i u_i$	$f_i u_i^2$
20-30	25	25	-2	-50	100
30-40	40	35	-1	-40	40
40-50	20	45 = a	0	0	0
50-60	10	55	1	10	10
60-70	5	65	2	10	20
	$\Sigma f_i = N = 100$			$\Sigma f_i u_i = -70$	$\Sigma f_i u_i^2 = 170$

The standard deviation,

$$\begin{aligned} \sigma_x &= C \sqrt{\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2} \\ &= 10 \times \sqrt{\frac{170}{100} - \left(\frac{-70}{100}\right)^2} \\ &= 10 \times \sqrt{\frac{170}{100} - \frac{49}{100}} \\ &= 10 \sqrt{\frac{170 - 49}{100}} = 10 \sqrt{\frac{121}{100}} \\ &= 10 \times \frac{11}{10} = 11 \text{ (Ans.)} \end{aligned}$$

**Ques ► 2** Scenario-1: Out of 250 candidates of an examination in Mathematics and Statistics 25 fail in Statistics, 45 fail in Mathematics and 15 fail in both the subjects. A candidate is selected at random.

Scenario-2: The frequency distribution of marks obtained by 50 students is given below :

Marks	40	50	60	70	80	90
No. of students	4	6	11	13	12	4

[R.B., C.B., Ctg.B., B.B.-18]

- If  $P(A) = 0.6$  and  $P(A \cap B) = 0.48$  then find the value of  $P(B)$  so that A and B are independent. 2
- From scenario-1, find the probability of the candidate passing in Statistics but failing in Mathematics. 4
- According to scenario-2, find the difference between variance and standard deviation. 4

**Solution to the question no. 2**

**a** Given that,  $P(A) = 0.6$  and  $P(A \cap B) = 0.48$   
 If A and B are two independent events, then

$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B) \\ \text{or, } P(B) &= \frac{P(A \cap B)}{P(A)} = \frac{0.48}{0.6} = \frac{4}{5} = 0.8 \text{ (Ans.)} \end{aligned}$$

**b** Let, A = The event of fail in Mathematics  
 B = The event of fail in Statistics

Here,  $P(A) = \frac{45}{250} = \frac{9}{50}$

$P(B) = \frac{25}{250} = \frac{1}{10}$

and  $P(A \cap B) = \frac{15}{250} = \frac{3}{50}$

$\therefore$  The required Probability,  

$$\begin{aligned} P(A \cap B^c) &= P(A - B) \\ &= P(A) - P(A \cap B) \\ &= \frac{9}{50} - \frac{3}{50} \\ &= \frac{9 - 3}{50} \\ &= \frac{6}{50} \\ &= \frac{3}{25} \\ &= 0.12 \text{ (Ans.)} \end{aligned}$$

**c** We use the following table to calculate the standard deviation:

Marks ( $x_i$ )	No. of Students ( $f_i$ )	$f_i x_i$	$f_i x_i^2$
40	4	160	6400
50	6	300	15000
60	11	660	39600
70	13	910	63700
80	12	960	76800
90	4	360	32400
Total	$N = 50$	$\Sigma f_i x_i = 3350$	$\Sigma f_i x_i^2 = 233900$

$$\begin{aligned} \therefore \text{The standard deviation } \sigma &= \sqrt{\frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2} \\ &= \sqrt{\frac{233900}{50} - \left(\frac{3350}{50}\right)^2} \\ &= \sqrt{4678 - 4489} \\ &= \sqrt{189} \\ &= 13.75 \end{aligned}$$

and the variance,  $\sigma^2 = (13.75)^2 = 189$

$\therefore$  The difference between variance and standard deviation =  $189 - 13.75 = 175.25$  (Approx.) (Ans.)

**Ques 3**  $S = \{1, 2, 3, \dots, 50\}$

[D.B.-17]

- Write down the sample space of three coins thrown simultaneously. 2
- Find the probability of any number of S is either multiple of 3 or 5. 4
- Find the variance of the even numbers of S. 4

**Solution to the question no. 3**

**a** We use the following table to determine the sample space of throwing three coins simultaneously:

		Two coins			
		HH	HT	TH	TT
One coin	H	HHH	HHT	HTH	HTT
	T	THH	THT	TTH	TTT

$\therefore$  The sample space of three coins thrown simultaneously,  $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$  (Ans.)

**b** Given that,  $S = \{1, 2, 3, \dots, 50\}$

Let,

A = The event of numbers in S which are multiple of 3.

B = The event of numbers in S which are multiple of 5.

Here, total number of numbers in S = 50

The number of numbers in favour of A =  $\frac{48-3}{3} + 1 = 16$

The number of numbers in favour of B =  $\frac{50-5}{5} + 1 = 10$

The number of numbers in favour of both A and B

$$= \frac{45-15}{15} + 1 = 3 \quad [\because \text{The LCM of 3 and 5 is 15}]$$

$$\therefore P(A) = \frac{16}{50}, P(B) = \frac{10}{50} \text{ and } P(A \text{ and } B) = \frac{3}{50}$$

Therefore,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$\begin{aligned} &= \frac{16}{50} + \frac{10}{50} - \frac{3}{50} \\ &= \frac{16+10-3}{50} \\ &= \frac{23}{50} \end{aligned}$$

That is, the probability of any number of S is either multiple of 3 or 5 is  $\frac{23}{50}$  (Ans.)

**c** Given that,  $S = \{1, 2, 3, \dots, 50\}$

The even number of S are,

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50

Here, total number of even numbers,  $n = 25$

The sum of the numbers,

$$\begin{aligned} \Sigma x_i &= 2 + 4 + 6 + \dots + 50 \\ &= 2(1 + 2 + 3 + \dots + 25) \\ &= 2 \cdot \frac{25(25+1)}{2} \\ &= 650 \end{aligned}$$

And, the sum of square of the numbers,

$$\begin{aligned} \Sigma x_i^2 &= 2^2 + 4^2 + 6^2 + \dots + 50^2 \\ &= 2^2(1^2 + 2^2 + 3^2 + \dots + 25^2) \\ &= 4 \cdot \frac{25(25+1)(2 \cdot 25 + 1)}{6} \\ &= \frac{4 \cdot 25 \cdot 26 \cdot 51}{6} \\ &= 22100 \end{aligned}$$

$$\begin{aligned} \therefore \text{The variance, } \sigma^2 &= \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 \\ &= \frac{22100}{25} - \left(\frac{650}{25}\right)^2 \\ &= 884 - 676 \\ &= 208 \text{ (Ans.)} \end{aligned}$$

**Ques 4 Scenario-1:** The probabilities of Poly and Tuly for solving a problem are  $\frac{1}{3}$  and  $\frac{1}{4}$ .

**Scenario-2:** Daily income of ten labours of a company are 210, 220, 225, 230, 235, 238, 240, 242, 245, 248 respectively.

[R.B.-17]

- A box contains 4 white and 5 black balls. Three balls are drawn at random. Find the probability of all the three balls to be black. 2
- From Scenario-1, find the probability of Poly and Tuly solving the problem together. 4
- From Scenario-2, find the standard deviation and variance. 4

**Solution to the question no. 4**

**a** There are (4 + 5) or 9 balls in the box in where 5 are black. If we draw three balls at random, then the probability of all

three balls will be  $= \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{5}{42}$  (Ans.)

**b** Let, A = The event of solving the problem by Poly.  
B = The event of solving the problem by Tuly.

Therefore,  $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$

But, the events A and B are independent,

$$\therefore P(A \text{ and } B) = P(A) \times P(B) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

Now,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$\begin{aligned} &= \frac{1}{3} + \frac{1}{4} - \frac{1}{12} \\ &= \frac{4+3-1}{12} \\ &= \frac{6}{12} = \frac{1}{2} \end{aligned}$$

That is, the probability of Poly and Tuly solving the problem together is  $\frac{1}{2}$ . (Ans.)

- c** We use the following table to calculate the standard deviation:

Income ( $x_i$ )	$x_i^2$
210	44100
220	48400
225	50625
230	52900
235	55225
238	56644
240	57600
242	58564
245	60025
248	61504
$\Sigma x_i = 2333$	$\Sigma x_i^2 = 545587$

Here, Total number of observations,  $n = 10$

$\therefore$  The standard deviation,

$$\begin{aligned}\sigma &= \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2 - \left(\frac{1}{n} \sum x_i\right)^2} \\ &= \sqrt{\frac{1}{10} \times 545587 - \left(\frac{2333}{10}\right)^2} \\ &= \sqrt{54558.7 - 54428.89} \\ &= \sqrt{129.81} \\ &= 11.39 \text{ (Approx.)}\end{aligned}$$

and the variance  $= \sigma^2 = (11.39)^2 = 129.81$  (Approx.)

**Ques 5**

Marks	51-60	61-70	71-80	81-90	91-100
Students	10	20	15	10	5

[Dj.B.-17]

- What do you mean by Range according to the stem? 2
- Determine Standard Deviation according the grouped data of the stem. 4
- In light of the stem find out Mean Deviation. 4

**Solution to the question no. 5**

**a** Here, the upper limit = 100, the lower limit = 51

$\therefore$  The range,  $R = 100 - 51 = 49$  (Ans.)

**b** Table of calculating standard deviation:

Class limit	Mid point $x_i$	Frequency $f_i$	$f_i x_i$	$d_i = \frac{x_i - a}{c}$ $a=75.5$ $c=10$	$f_i d_i$	$f_i d_i^2$	$ x_i - \bar{x} $	$f_i  x_i - \bar{x} $
51-60	55.5	10	555	-2	-20	40	16.66	166.7
61-70	65.5	20	1310	-1	-20	20	6.66	133.3
71-80	75.5	15	1132.5	0	0	0	3.33	49.95
81-90	85.5	10	855	1	10	10	13.33	133.4
91-100	95.5	5	477.5	2	10	20	23.33	116.65
Total		$N=60$	$\Sigma f_i x_i = 4330$		$\Sigma f_i d_i = -20$	$\Sigma f_i d_i^2 = 90$		$\Sigma f_i  x_i - \bar{x}  = 600$

The standard deviation  $\sigma = \sqrt{\left[\frac{\Sigma f_i d_i^2}{N} - \left\{\frac{\Sigma f_i d_i}{N}\right\}^2\right] \times c^2}$

$$\begin{aligned}&= \sqrt{\left[\frac{90}{60} - \left(\frac{-20}{60}\right)^2\right] \times 10^2} \\ &= \sqrt{138.89} = 11.785 \text{ (Ans.)}\end{aligned}$$

**c** Table of calculate mean deviation:

Class limit	Mid point $x_i$	Frequency $f_i$	$f_i x_i$	$d_i = \frac{x_i - a}{c}$ $a=75.5$ $c=10$	$f_i d_i$	$f_i d_i^2$	$ x_i - \bar{x} $	$f_i  x_i - \bar{x} $
51-60	55.5	10	555	-2	-20	40	16.66	166.7
61-70	65.5	20	1310	-1	-20	20	6.66	133.3
71-80	75.5	15	1132.5	0	0	0	3.33	49.95
81-90	85.5	10	855	1	10	10	13.33	133.4
91-100	95.5	5	477.5	2	10	20	23.33	116.65
Total		$N=60$	$\Sigma f_i x_i = 4330$		$\Sigma f_i d_i = -20$	$\Sigma f_i d_i^2 = 90$		$\Sigma f_i  x_i - \bar{x}  = 600$

$\therefore$  The mean deviation  $MD = \frac{\Sigma f_i |x_i - \bar{x}|}{N} = \frac{600}{60} = 10$

(Ans.)

**Ques 6** Data:

Name of Colours	Number of Balls
White	3
Black	6
Red	7
Green	5
Yellow	4
Violet	9
Blue	8

[C.B.-17]

- Establish the multiplication law of probability for two dependent events. 2
- If the balls in the stem are in a box and three balls are drawn randomly, find the probability of getting 3 red balls or 3 green balls. 4
- Find the variance of the seven numbers in the stem. 4

**Solution to the question no. 6**

**a** Let, S be the sample space of an experiment and A and B are two events related to the experiment.

If  $n(S)$ ,  $n(A)$ ,  $n(B)$  and  $n(A \cap B)$  are the number of elements of S, A, B and  $A \cap B$  respectively, then

$$P(A|B) = \frac{n(A \cap B)}{n(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{P(A \cap B)}{P(B)}$$

$\therefore P(A \cap B) = P(B) P(A|B)$

Similarly,  $P(A \cap B) = P(A) P(B|A)$

That is, the probability of the joint occurrence of events A and B is equal to the product of the probability of A and the conditional probability of B given A.

**b** Total number of balls = 3 + 6 + 7 + 5 + 4 + 9 + 8 = 42

If there balls are drawn randomly, the probability of getting 3 red balls or 3 green balls,

$$\begin{aligned}&= \frac{{}^7C_3}{{}^{42}C_3} + \frac{{}^5C_3}{{}^{42}C_3} \\ &= \frac{35}{11480} + \frac{10}{11480} \\ &= \frac{45}{11480} \\ &= \frac{9}{2296} \text{ (Ans.)}\end{aligned}$$



**c Table of Calculating Variance:**

No. of balls, x	$x^2$
3	9
6	36
7	49
5	25
4	16
9	81
8	64
$\Sigma x = 42$	$\Sigma x^2 = 280$

$\therefore$  The variance,  $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2 = \frac{280}{7} - \left(\frac{42}{7}\right)^2$   
 $= 40 - 6^2 = 40 - 36 = 4$  (Ans.)

**Ques 7 Scenario-1:** Marks obtained in mathematics of 55 students in class XII are given below:

Marks	51-60	61-70	71-80	81-90	91-100
No. of students	7	18	15	10	5

**Scenario-2:** A bag contains 9 red balls and 7 white balls 6 balls are drawn at random. [Ctg.B.-17]

- a.  $P(A) = \frac{1}{3}$ ,  $P(B) = \frac{3}{4}$ . If A and B are independent events, then find  $P(A \cap B)$ . 2
- b. Form Scenario-1, find the standard deviation. 4
- c. Form Scenario-2, find the probability of having 3 red and 3 white balls. 4

**Solution to the question no. 7**

a Given that,  $P(A) = \frac{1}{3}$  and  $P(B) = \frac{3}{4}$

If A and B are two independent events, then

$P(A \cap B) = P(A) \times P(B) = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$  (Ans.)

b **Table to calculate the standard deviation:**

Number	No. of students ( $f_i$ )	Midpoint ( $x_i$ )	$u_i = \frac{x_i - 75.5}{10}$	$f_i u_i$	$f_i u_i^2$
51-60	7	55.5	-2	-14	28
61-70	18	65.5	-1	-18	18
71-80	15	75.5 = a	0	0	0
81-90	10	85.5	1	10	10
91-100	5	95.5	2	10	20
	$\Sigma f_i = N = 55$			$\Sigma f_i u_i = -12$	$\Sigma f_i u_i^2 = 76$

$\therefore$  The standard deviation,  $\sigma = \sqrt{\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2}$   
 $= \sqrt{\frac{76}{55} - \left(\frac{-12}{55}\right)^2}$   
 $= \sqrt{1.3818 - 0.0476}$   
 $= \sqrt{1.3342}$   
 $= 1.155$   
 $= 11.55$  (Approx.) (Ans.)

c There are (9 + 7) or 16 balls in where 9 are red and 7 are white.

Now, 6 balls can be drawn from 16 balls in  ${}^{16}C_6$  ways.

3 red balls can be drawn from 9 balls in  ${}^9C_3$  ways.

3 white balls can be drawn from 7 balls

If 6 balls are drawn at random, is  ${}^7C_3$  ways.

Then  $P(3 \text{ red balls and } 3 \text{ white balls})$

$= \frac{{}^9C_3 \times {}^7C_3}{{}^{16}C_6} = \frac{84 \times 35}{8008} = \frac{105}{286}$  (Ans.)

**Ques 8**  $S_1 = \{1, 3, 4, 5, 7, 9, 20\}$

$S_2 = \{12, 13, 14, 15, 16, 17, 18\}$

[S.B.-17]

- a. If a dice is randomly thrown, what is the probability of getting the number to be divisible by 2 or 3? 2
- b. If a number is taken randomly from  $S_1$  and  $S_2$  each, what is the probability of getting the prime number from  $S_1$  and multiple of 3 from  $S_2$ ? 4
- c. Find the standard deviation of the elements of  $S_1$ . 4

**Solution to the question no. 8**

a If a dice is thrown randomly, the outcomes are 1, 2, 3, 4, 5 and 6.

Here, 2, 4 and 6 are divisible by 2; 3 and 6 are divisible by 3; and 6 is divisible by both 2 and 3.

$\therefore P(\text{the number to be divisible by 2 or 3})$

$= P(\text{the number to be divisible by 2}) + P(\text{the number to be divisible by 3}) - P(\text{the number to be divisible by both 2 and 3})$

$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{3+2-1}{6} = \frac{4}{6} = \frac{2}{3}$  (Ans.)

b Given that,  $S_1 = \{1, 3, 4, 5, 7, 9, 20\}$

$S_2 = \{12, 13, 14, 15, 16, 17, 18\}$

Let A = The event of getting a prime number from  $S_1$ .

B = The event of getting a multiple of 3 from  $S_2$ .

Here, total number of numbers in  $S_1$  is 7 and in  $S_2$  is 7.

Again the number of prime numbers in  $S_1$  is 3 and the number of numbers which are multiple of 3 in  $S_2$  is 3.

$\therefore P(A) = \frac{3}{7}$  and  $P(B) = \frac{3}{7}$

The probability of getting the prime number from  $S_1$  and multiple of 3 from  $S_2 = P(A \cap B)$

$= P(A) P(B)$  [ $\because$  A and B are independent]

$= \frac{3}{7} \times \frac{3}{7} = \frac{9}{49}$

c  $S_1 = \{1, 3, 4, 5, 7, 9, 20\}$

The arithmetic mean of the elements of  $S_1$ ,

$\bar{x} = \frac{1+3+4+5+7+9+20}{7} = \frac{49}{7} = 7$

$\therefore$  The standard deviation,  $\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$

$= \sqrt{\{(1-7)^2 + (3-7)^2 + (4-7)^2 + (5-7)^2 + (7-7)^2 + (9-7)^2 + (20-7)^2\}}$

$= \sqrt{\frac{1}{7}(36 + 16 + 9 + 4 + 0 + 4 + 169)}$

$= \sqrt{\frac{238}{7}} = \sqrt{34} = 5.831$  (Ans.)

**Ques 9 Scenario-1:**

Class interval	10-16	17-22	23-28	29-34	35-40	41-46	47-52
Frequency	5	4	10	12	8	4	7

**Scenario-2:** In a college out of 100 students of eleven class 30 played football, 40 played cricket and 20 students played both football and cricket. A student is chosen at random. [J.B.-17]

- a. Find the value of  $P(A \cup B)$  if  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{3}{5}$  and A, B independent. 2
- b. From scenario-1, determine quartile deviation. 4
- c. According to the scenario-2, if the boy played cricket then what is the probability of his playing football? 4

**Solution to the question no. 9**

**a** Given that, A and B are two independent events and  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{3}{5}$ .

$$\therefore P(A \cap B) = P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{3}{5} - \frac{3}{10} = \frac{5+6-3}{10} = \frac{8}{10} = \frac{4}{5} \text{ (Ans.)}$$

**b** Calculation table of quartile deviation :

Class interval	Frequency	Cumulative frequency
10-16	5	5
17-22	4	9
23-28	10	19
29-34	12	31
35-40	8	39
41-46	4	43
47-52	7	50

Here,  $Q_1 = \frac{1 \times 50}{4}$ th observation = 12.5<sup>th</sup> observation

$\therefore Q_1$  lies in the class 23 – 28.

$$\therefore Q_1 = 23 + \frac{6}{10} (12.5 - 9) = 25.1$$

Again,  $Q_3 = \frac{3 \times 50}{4}$ th observation = 37.5<sup>th</sup> observation

$\therefore Q_3$  lies in the class 35 – 40.

$$\therefore Q_3 = 35 + \frac{6}{8} (37.5 - 31) = 39.875$$

$$\therefore \text{The quartile deviation} = \frac{Q_3 - Q_1}{2} = \frac{39.875 - 25.1}{2}$$

$$= 7.3875 \text{ (Ans.)}$$

**c** Let, F = The event of playing football  
C = The event of playing cricket

Given that, out of 100 students, 30 played football, 40 played cricket and 20 played both football and cricket.

It a student is chosen at random,

the probability of playing football,  $P(F) = \frac{30}{100} = \frac{3}{10}$

the probability of playing cricket,  $P(C) = \frac{40}{100} = \frac{2}{5}$

and the probability of playing both,  $P(F \cap C) = \frac{20}{100} = \frac{1}{5}$

$\therefore$  If the boy played cricket, then the probability of his playing

$$\text{football, } P(F | C) = \frac{P(F \cap C)}{P(C)} = \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2} \text{ (Ans.)}$$

**Ques 10 Scenario-1:** A dice and two coins are thrown together.

**Scenario-2:** The following frequency distribution:

Class interval	10-14	15-19	20-24	25-29	30-34	35-39
Frequency	5	8	14	12	9	6

[B.B.-17]

- Define mutually exclusive events and not mutually exclusive events. 2
- Find the probability of two head and odd number may be occurred with the help of sample space. 4
- Find the standard deviation from the frequency distribution. 4

**Solution to the question no. 10**

**a Mutually Exclusive Events :** Two (or more) events are said to be mutually exclusive when both can not happen simultaneously in a single trial. As for example, if a single coin is tossed, either head can be up or tail can be up, but both can not be up at the same time.

**Mutually Inclusive Events :** Two (or more) events are said to be mutually inclusive when they can both occur simultaneously in a single trial. As for example, if a single dice is thrown, it is possible to get a number which is even and a multiple of 3 at the same time.

**b** The sample space of throwing together a dice and two coins can be determined from the following table:

Outcomes of two coins	Outcomes of a dice					
	1	2	3	4	5	6
HH	(HH1)	(HH2)	(HH3)	(HH4)	(HH5)	(HH6)
HT	(HT1)	(HT2)	(HT3)	(HT4)	(HT5)	(HT6)
TH	(TH1)	(TH2)	(TH3)	(TH4)	(TH5)	(TH6)
TT	(TT1)	(TT2)	(TT3)	(TT4)	(TT5)	(TT6)

Here, total sample points in the sample space,  $n(S) = 24$

Let, A = The event of getting two heads and one odd number.

Then, the sample space in favour of A = {HH1, HH3, HH5}

$\therefore$  The sample points in favour of A,  $n(A) = 3$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{24} = \frac{1}{8} \text{ (Ans.)}$$

**c** The calculating table of standard deviation is given below:

Class interval	Midpoint $x_i$	Frequency $f_i$	$u_i = \frac{x_i - a}{c}$ a=22, c=5	$f_i u_i$	$f_i u_i^2$
10-14	12	5	-2	-10	20
15-19	17	8	-1	-8	8
20-24	22	14	0	0	0
25-29	27	12	1	12	12
30-34	32	9	2	18	36
35-39	37	6	3	18	54
	Total	N = 54		$\Sigma f_i u_i = 30$	$\Sigma f_i u_i^2 = 130$

$\therefore$  The Standard deviation,

$$\sigma = \sqrt{\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2} \times c$$

$$= \sqrt{\frac{130}{54} - \left(\frac{30}{54}\right)^2} \times 5$$

$$= \sqrt{2.4074 - 0.3086} \times 5$$

$$= \sqrt{2.0988} \times 5 = 1.4487 \times 5$$

$$= 7.2435 \text{ (Approx.) (Ans.)}$$

**Ques 11 Scenery-I :** A, B are independent and  $P(A) = \frac{1}{3}$ ,  $P(B) = \frac{3}{4}$

$P(B) = \frac{3}{4}$

**Scenery-II :** Two dice are thrown simultaneously.

[Mirzapur Cadet College, Tangail]

- Define sample space. 2
- From scenery-I, find  $P(A \cup B)$  and  $P(A \cap B)$ . 4
- From scenery-II, write down the sample space and find the probability of appearing two sixes. 4

**Solution to the question no. 11**

**a** **Sample Space:** The set of all possible outcomes of a random experiment is called sample space and it is usually denoted by S. Each outcome of a sample space is called sample point. For example, if an unbiased coin is tossed then the possible outcomes are H, T. Therefore the sample space,  $S = \{H, T\}$ .

**b** See the Question No. 7(a)

**c** The sample space of throwing two dice simultaneously is given below :

$\{(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6) (4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6) (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)\}$

Here, the number of sample points,  $n(S) = 36$

The probability of getting two 6s = 1

$\therefore$  The probability of getting two 6s  $\frac{1}{36}$  (Ans.)

**Ques 12** The probabilities of X and Y for solving a sum are  $\frac{1}{3}$

and  $\frac{1}{4}$  respectively and the probability of their solving it

together is  $\frac{1}{2}$ . They, also, form a set S taking all the multiples of 4 not greater than 48.

[Mymensingh Girls' Cadet College, Mymensingh]

**a** If the mean of the marks obtained by each of the students A and B is 84 and their standard deviations of marks are respectively 12.12 and 4.04, whose marks are more spread out? 2

**b** Ascertain whether X and Y are independent. If possible, show that  $P(X/Y) = P(x)$ . 4

**c** Find the variance of the elements of S. 4

**Solution to the question no. 12**

**a** Given that, the average number of A and B is 84. And the standard deviation of their numbers respectively 12.12 and 4.4.

We know,

If the standard deviation is more then the information will more spread out.

Since the standard deviation of A is more then B.

$\therefore$  The obtained number of A will more spread out.

**b** Given that, the probability of their sloving together is  $P(X \cap Y) = \frac{1}{2}$ .

$$P(X) = \frac{1}{3}$$

$$P(Y) = \frac{1}{4}$$

Since, the events are independent,  $\therefore P(X \cap Y) = \frac{1}{3} \times \frac{1}{4} =$

$$\frac{1}{12}$$

We know,

$$P(X | Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{\frac{1}{12}}{\frac{1}{4}} = \frac{1}{12} \times \frac{4}{1} = \frac{1}{3} = P(X)$$

$$\therefore P(X | Y) = P(X) \text{ (Showed)}$$

**c** Their set  $S = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48\}$

Here, Total number = 12

$$\therefore \text{Average} = \frac{\sum x_i}{n}$$

$$= \frac{4 + 8 + 12 + 16 + 20 + 24 + 28 + 32 + 36 + 40 + 44 + 48}{12}$$

$$= \frac{312}{12} = 26$$

$$\text{And } \sum x_i^2 = (4)^2 + (8)^2 + (12)^2 + (16)^2 + (20)^2 + (24)^2 + (28)^2 + (32)^2 + (36)^2 + (40)^2 + (44)^2 + (48)^2$$

$$= 10400$$

$$\therefore \text{Variance} = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$= \frac{10400}{12} - (26)^2$$

$$= 190.67 \text{ (Approx) (Ans.)}$$

**Ques 13** There are 6 red and 10 white balls in a box. A boy picks up two balls randomly one after another.

[Rajshahi Cadet College, Rajshahi]

- Find the probability of getting two red balls. 2
- Find the probability of getting two different color balls. 4
- Find the probability of getting two same color balls. 4

**Solution to the question no. 13**

**a** Number of red balls = 6

and number of white balls = 10.

$$\text{Probability of getting one red ball} = \frac{6}{16}$$

$$\text{Probability of getting another red ball} = \frac{5}{15}$$

$$\therefore \text{So probability of getting two red balls} = \frac{6}{16} \times \frac{5}{15} = \frac{1}{8} \text{ (Ans.)}$$

**b** Number of red balls = 6.

Number of white balls = 10.

$$\text{Probability of getting first ball red and second ball white} = \frac{6}{16} \times \frac{10}{15} = \frac{1}{4}$$

$$\text{Probability of getting first ball white and second ball red} = \frac{10}{16} \times \frac{6}{15} = \frac{1}{4}$$

$$\therefore \text{Required probability} = \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2} \text{ (Ans.)}$$

**c** For first ball:

$$\text{Probability of getting a red ball} = \frac{6}{16}$$

$$\text{Probability of getting a white ball} = \frac{10}{16}$$



For second ball:

If we drew a red ball initially, 5 red balls are left out of 15 balls.

$$\text{So probability of getting the second ball red} = \frac{5}{15} = \frac{1}{3}$$

If we drew a white ball initially 9 white balls are left out of 15.

$$\text{So probability of getting second ball white} = \frac{9}{15}$$

Now,

$$\text{Probability of getting two red balls} = \frac{6}{16} \times \frac{5}{15} = \frac{1}{8}$$

$$\text{and probability of getting two white balls} = \frac{10}{16} \times \frac{9}{15} = \frac{3}{8}$$

∴ Probability of getting two same color balls

$$= \frac{1}{8} + \frac{3}{8}$$

$$= \frac{4}{8} = \frac{1}{2} \text{ (Ans.)}$$

**Ques ► 14 Stem-1 :** Height (in inches) of 5 students of class XII are : 50, 65, 60, 60 and 65. *[Joypurhat Girl's Cadet College]*

**Stem-2 :** Two unbiased dice is thrown.

- What is the range of stem-1? 2
- Find mean deviation of stem-1. 4
- Write down the sample space and find the probability of getting same outcome of both the dices of stem-2. 4

**Solution to the question no. 14**

**a** Here, the highest value = 65

and the lowest value = 50

$$\therefore \text{The range} = 65 - 50 = 15 \text{ (Ans.)}$$

**b** Table of determining mean deviation:

Height $x_i$	No. of students $f_i$	$f_i x_i$	$ x_i - \bar{x} $	$f_i  x_i - \bar{x} $
50	1	50	10	10
60	2	120	0	0
65	2	130	5	10
	$\Sigma f_i = 5$	$\Sigma f_i x_i = 300$		$\Sigma f_i  x_i - \bar{x}  = 20$

$$\text{Here, } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{300}{5} = 60$$

$$\therefore \text{The mean deviation, M. D} = \frac{\Sigma f_i |x_i - \bar{x}|}{\Sigma f_i} = \frac{20}{5} = 4 \text{ (Ans.)}$$

**c** Sample space: The set of all possible outcomes of a random experiment is called sample space and it is usually denoted by S. Each outcome of a sample space is called sample point. For example if an unbiased coin is tossed, then the possible outcomes are H, T. Therefore, the sample space,  $S = \{H, T\}$

Here, total number of sample points = 36

The number of sample point in favour of getting same outcomes of both dices = 6 [(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)]

∴ The probability of getting same outcomes of both the dices

$$= \frac{6}{36} = \frac{1}{6} \text{ (Ans.)}$$

**Ques ► 15 Scenario-1:** A bag contains 5 red, 6 green and 4 white balls.

**Scenario-2:**

Class interval	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	3	7	15	20	12	9	4

*[Pabna Cadet College, Pabna]*

- Find the value of  $P(A \cup B)$  if  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{3}{5}$  and A, B are independent. 2
- If three balls are drawn random in the scenario-1, then find the probability of getting at best 2 red balls. 4
- From scenario-2, find the standard deviation and variance of the frequency table. 4

**Solution to the question no. 15**

**a** See the Question No. 9(a).

**b** Total number of balls in the bag = 5 red + 6 green + 4 white = 15 balls

If three balls are drawn randomly, the probability of getting at least 2 red balls,

$$\begin{aligned} &= \frac{{}^5C_0 \times {}^{10}C_3}{{}^{15}C_3} + \frac{{}^5C_1 \times {}^{10}C_2}{{}^{15}C_3} + \frac{{}^5C_2 \times {}^{10}C_1}{{}^{15}C_3} \\ &= \frac{1 \times 120}{455} + \frac{5 \times 45}{455} + \frac{10 \times 10}{455} \\ &= \frac{120 + 225 + 100}{455} \\ &= \frac{445}{455} = \frac{89}{91} \text{ (Ans.)} \end{aligned}$$

**c** The frequency table for standard deviation :

Class Interval	Mid-value $x_i$	frequency $f_i$	$d_i = \frac{x_i - A}{C}$ $A = 65.5, C = 10$	$f_i d_i$	$f_i d_i^2$
31 - 40	35.5	3	-3	-9	27
41 - 50	45.5	7	-2	-14	28
51 - 60	55.5	15	-1	-15	15
61 - 70	65.5	20	0	0	0
71 - 80	75.5	12	1	12	12
81 - 90	85.5	9	2	18	36
91 - 100	95.5	4	3	12	36
Total		$N = 70$		$\Sigma f_i d_i = 4$	$\Sigma f_i d_i^2 = 154$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2} \times C$$

$$= \sqrt{\frac{154}{70} - \left(\frac{4}{70}\right)^2} \times 10$$

$$= \sqrt{2.2 - (0.057)^2} \times 10$$

$$= \sqrt{2.2 - 0.00327} \times 10$$

$$= \sqrt{2.197} \times 10$$

$$= 1.48 \times 10 = 14.82 \text{ (Ans.)}$$

$$\text{Variance} = \sigma^2 = 14.82^2 = 219.63 \text{ (Ans.)}$$

**Ques ► 16** A frequency distribution table is given below :

Class interval	20 - 24	25 - 29	30 - 34	35 - 39	40 - 44	45 - 49
Frequency	7	10	15	13	9	6

*[Rangpur Cadet College, Rangpur]*

- Find the range from the given frequency distribution table. 2

- b. Find the variance from the given frequency distribution table. 4  
 c. Find the quartile deviation from the given frequency distribution table. 4

**Solution to the question no. 16**

a. From the given table,  $L_1 = 20$

$$L_n = 49$$

$$\therefore \text{Range, } R = L_n - L_1 = 49 - 20 = 29$$

b. The table to determination standard deviation and variance:

Class	Mid-value $x_i$	frequency $f_i$	$d_i = \frac{x_i - a}{c}$ $a=32,$ $c=5$	$f_i d_i$	$f_i d_i^2$
20-24	22	7	-2	-14	28
25-29	27	10	-1	-10	10
30-34	32	15	0	0	0
35-39	37	13	1	13	13
40-44	42	9	2	18	36
45-49	47	6	3	18	54
Total		$N = 60$		$\Sigma f_i d_i = 25$	$\Sigma f_i d_i^2 = 141$

We know,

$$\begin{aligned} \text{Standard deviation, } \sigma &= \sqrt{\left[ \frac{\Sigma f_i d_i^2}{N} - \left( \frac{\Sigma f_i d_i}{N} \right)^2 \right] \times c^2} \\ &= \sqrt{\left[ \frac{141}{60} - \left( \frac{25}{60} \right)^2 \right] \times c^2} \\ &= \sqrt{54.41} \\ &= 7.376 \text{ (Ans.)} \end{aligned}$$

$$\therefore \text{Variance, } \sigma^2 = (7.376)^2 = 54.41 \text{ (Ans.)}$$

c. The frequency table :

Class interval	Genuine class interval	frequency ( $f_i$ )	Cumulative frequency ( $F_i$ )
20-24	19.5-24.5	7	7
25-29	24.5-29.5	10	17
30-34	29.5-34.5	15	32
35-39	34.5-39.5	13	45
40-44	39.5-44.5	9	54
45-49	44.5-49.5	6	60
		$N = 60$	

$$\text{Quartile} = \frac{Q_3 - Q_1}{2}$$

$$\text{Here, } Q_1 = L_1 + \frac{\frac{N}{4} - F_1}{f_1} \times C$$

$$\begin{aligned} &= 24.5 + \frac{\frac{60}{4} - 7}{10} \times 5 \text{ [since } \frac{1 \times 60}{4} = 15^{\text{th}} \text{ term exists in 2nd} \\ &\text{class interval]} \\ &= 28.5 \end{aligned}$$

$$Q_3 = L_3 + \frac{\frac{3 \times N}{4} - F_3}{f_3} \times C$$

$$\begin{aligned} &= 34.5 + \frac{\frac{3 \times 60}{4} - 32}{13} \times 5 \text{ [}\therefore \frac{3 \times 60}{4} = 45 \text{ term exists in} \\ &\text{(35 - 39) class interval]} \\ &= 39.5 \end{aligned}$$

$$\therefore \text{Quartile} = \frac{Q_3 - Q_1}{2} = \frac{39.5 - 28.5}{2} = 5.5 \text{ (Ans.)}$$

**Ques 17** Among three bags, first bag contains 5 white and 4 red balls; second bag contains 3 white and 7 red balls; third bag contains 6 white, 7 red and 5 black balls. [Cumilla Cadet College]

- a. Determine the variance of first n natural number. 2  
 b. If three balls are drawn at random from the third bag, then what is the probability that two of them are same colour. 4  
 c. A bag is selected in equiprobable way and two balls are drawn from the bag. Find the probability that one ball is red and another ball is white. 4

**Solution to the question no. 17**

a. We know,

First n numbers of normal number 1, 2, ... .., n

Now, the sum of the numbers,

$$\sum_{i=1}^n x_i = 1 + 2 + 3 + \dots \dots + n = \frac{n(n+1)}{2}$$

$$\text{We know, mathematical average, } \bar{x} = \frac{\sum_{i=1}^n x_i}{n} = \frac{n(n+1)}{2 \cdot n} =$$

$$\frac{n+1}{2}$$

$$\text{and variance } \sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$\text{Now, } \sum_{i=1}^n x_i^2 = x_1^2 + x_2^2 + \dots \dots + x_n^2$$

$$= 1^2 + 2^2 + \dots \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\therefore \text{Variance, } \sigma^2 = \frac{n(n+1)(2n+1)}{6 \cdot n} - \left( \frac{n+1}{2} \right)^2$$

$$= \frac{n+1}{2} \left[ \frac{2n+1}{3} - \frac{n+1}{2} \right]$$

$$= \frac{n+1}{2} \left[ \frac{4n+2-3n-3}{6} \right]$$

$$= \frac{n^2-1}{12} \text{ (Ans.)}$$

b. Total balls in three bags = 6 white + 7 red + 5 black = 18 balls.

If three balls taken at random, the probability of getting 2 same colour balls

$$= \frac{{}^6C_2 \times {}^{12}C_1}{{}^{18}C_3} + \frac{{}^7C_2 \times {}^{11}C_1}{{}^{18}C_3} + \frac{{}^5C_2 \times {}^{13}C_1}{{}^{18}C_3}$$

$$= \frac{15 \times 12}{816} + \frac{21 \times 11}{816} + \frac{10 \times 13}{816}$$

$$= \frac{180 + 231 + 130}{816}$$

$$= \frac{541}{816} \text{ (Ans.)}$$

c. The probability to select a bag from three bag of equal probability =  $\frac{1}{3}$

Selecting a bag from equal probability if 2 ball is picked at random, then the probability of 1 red and 1 white ball

$$= \frac{1}{3} \times \frac{{}^5C_1 \times {}^4C_1}{{}^9C_2} + \frac{1}{3} \times \frac{{}^3C_1 \times {}^7C_1}{{}^{10}C_2} + \frac{1}{3} \times \frac{{}^6C_1 \times {}^7C_1}{{}^{13}C_2}$$

$$= \frac{1}{3} \times \frac{5 \times 4}{36} + \frac{1}{3} \times \frac{3 \times 7}{45} + \frac{1}{3} \times \frac{6 \times 7}{153}$$



$$= \frac{20}{108} + \frac{7}{45} + \frac{14}{153}$$

$$= \frac{992}{2295} \text{ (Ans.)}$$

**Ques ► 18 Scenario-1:** A dice and 2 coins are thrown.

**Scenario-2:** A frequency distribution table is given below—

Class	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
frequency	5	8	11	15	19	16	12	9	7	4

[Faujdarhat Cadet College, Faujdarhat]

- a. 3 cards are taken from a packet. What is the probability of the 3 cards not to be ace? 2
- b. According to Scenario-1 Determine the probability of getting even numbers and two heads. 4
- c. According to Scenario-2 Determine the quartile deviation. 4

**Solution to the question no. 18**

- a. 3 cards can taken from 4 ace by  ${}^4C_3$  ways.  
3 cards can taken from 52 cards by  ${}^{52}C_3$  ways.  
 $\therefore$  The probability of the 3 cards not to be ace  $1 - \frac{{}^4C_3}{{}^{52}C_3} = \frac{5524}{5525}$

(Ans.)

- b. See the Question No. 10(b).  
Consider odd number as even number.

c

Class interval	Frequency	Cumulative frequency
5-10	5	5
10-15	8	13
15-20	11	24
20-25	15	39
25-30	19	58
30-35	16	74
35-40	12	86
40-45	9	95
45-50	7	102
50-55	4	106
Total	106	

Here,  $N = 106$

$\therefore \frac{N}{4} = 26.5$  which in forth class interval that means in (20-25)

We know,

$$1^{\text{st}} \text{ quartile, } Q_1 = L_1 + \frac{\left(\frac{N}{4} - F_c\right)}{f_m} \times h$$

$$= 20 + \frac{26.5 - 24}{15} \times 5$$

$$= 20.83$$

$F_c$  = Previous class cumulative frequency of 1st quartile

$h$  = class interval,  $f_m$  = Frequency of 1st quartile

Here,  $\frac{3N}{4} = \frac{3 \times 106}{4} = 79.5$  Which situated in class interval (35-40)

$$3^{\text{rd}} \text{ quartile, } Q_3 = L_3 + \frac{\frac{3N}{4} - F_c}{f_m} \times h$$

$$= 35 + \frac{79.5 - 74}{12} \times 5$$

$$= 37.29$$

$$\therefore \text{Quartile} = \frac{Q_3 - Q_1}{2}$$

$$= \frac{37.29 - 20.83}{2}$$

$$= 8.23 \text{ (Ans.)}$$

**Ques ► 19**  $S = \{2, 4, 6, \dots, 160\}$

[Sylhet Cadet College, Sylhet]

- a. Prove that,  $0 \leq P(A) \leq 1$ , where  $P(A)$  is Probability of an event A. 2
- b. Find the probability of any number of S is either multiple of 3 or 6. 4
- c. Find the variance of S. 4

**Solution to the question no. 19**

- a Let, S be a finite sample space of a random experiment and A be an event associated with sample space.

Let, the total number of sample points of the sample space  $S = n(S)$ .

The number of favorable sample points of the event  $A = n(A)$ .

$\therefore$  According to the mathematical definition of probability we get,  $P(A) = \frac{n(A)}{n(S)}$  --- (1).

It is clear that, the number of elements of event A will remain in  $n(S)$  from 0.

That is,  $0 \leq n(A) \leq n(S)$

$$\text{Or, } \frac{0}{n(S)} \leq \frac{n(A)}{n(S)} \leq \frac{n(S)}{n(S)}$$

Or,  $0 \leq P(A) \leq 1$ . [Proved]

- b Given that,  $S = \{2, 4, 6, \dots, 160\}$

Let, the probability of any number of S is either multiple of 3 or 6 =  $P(A)$ .

Here, total number = 80.

In set S, the numbers that are multiple of 3 = 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, 120, 126, 132, 138, 144, 150, 156.

Total numbers = 26.

In set S, the numbers that are multiple of 6 are same as the numbers multiple of 3.

So the total numbers that are multiple of 6 = 26.

Now, probability of any number of S is either multiple of 3 or 6,

$P(A) = P(\text{number that is multiple of 3}) + P(\text{number that is multiple of 6}) - P(\text{number that is multiple of 3 and 6})$

$$= \frac{26C_1}{80C_1} + \frac{26C_1}{80C_1} - \frac{26C_1}{80C_1}$$

$$= \frac{26C_1}{80C_1} = \frac{26}{80} = \frac{13}{40} \text{ (Ans.)}$$

- c Let, the variable of set S,  $x_i = 2, 4, 6, 8, 10, \dots, 160$  total member,  $n = 80$ .

$$\therefore \sum x_i = 2+4+6+8+\dots+160$$

$$= 2(1+2+3+4+\dots+80)$$

$$= 2 \cdot \frac{80(80+1)}{2} = 80 \times 81 = 6480$$

Again,  $\sum x_i^2 = 2^2 + 4^2 + 6^2 + \dots + 160^2$

$= 2^2(1 + 2^2 + 3^2 + \dots + 80^2)$

$= 4 \cdot \frac{80(80+1)(160+1)}{6}$

$= \frac{4 \times 80 \times 81 \times 161}{6}$

$= 695,520$

$\therefore$  Variance of S,  $\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$= \frac{695,520}{80} - \left(\frac{6480}{80}\right)^2$

$= 8694 - 6561$

$= 2133. \text{ (Ans.)}$

**Ques 20 Scenario-1:**

Class interval	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	3	7	15	20	12	9	4

Scenario-2: Out of 50 students 35 play football, 25 play cricket. Each student plays at least one game.

[Jhenidah Cadet College]

a. IF  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{3}{5}$  and A and B are independent then find  $P(A \cup B)$ . 2

b. Find the standard deviation from the frequency distribution table maintained in the scenario-1. 4

c. In the scenario-2. A student is selected at random, find the probability of the student of playing both the games. 4

**Solution to the question no. 20**

**a** Since, A and B are independent

$\therefore P(A \cap B) = P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{3}{5} - \frac{3}{10}$

$= \frac{5+6-3}{10}$

$= \frac{8}{10}$

$= \frac{4}{5} \text{ (Ans.)}$

**b** See the Question No. 15(c)

**c** If a student is selected randomly, the probability of the

student whose are not playing football,  $P(F) = \frac{35}{50} = \frac{7}{10}$

The probability whose are playing cricket,  $P(C) = \frac{25}{50} = \frac{1}{2}$

Since, each students plays at least one game

$P(F \cup C) = P(F) + P(C) - P(F \cap C)$

or,  $\frac{50}{50} = \frac{7}{10} + \frac{1}{2} - P(F \cap C)$

$\therefore P(F \cap C) = \frac{7}{10} + \frac{1}{2} - 1$

$= \frac{7+5-10}{10}$

$= \frac{2}{10} = \frac{1}{5}$

$\therefore$  The probability of playing both games  $= \frac{1}{5} \text{ (Ans.)}$

**Ques 21 Scenario-1 : A frequency distribution table :**

Class interval	20-24	25-29	30-34	35-39	40-44	45-49
Frequency	7	10	15	13	9	6

**Scenario-2 : A bag contains 5 white, 7 red and 8 black balls.**

[Barishal Cadet College, Barishal]

a. Find the range of the data set 12, 5, 15, 7, 2, 9, 4. 2

b. From Scenario-1, find the variance. 4

c. From Scenario-2, If 4 balls are drawn successively without replacement, what is the probability of all the balls to be white? 4

**Solution to the question no. 21**

**a** Here, the largest value 15

and the smallest value = 2

$\therefore$  The range =  $15 - 2 = 13 \text{ (Ans.)}$

**b** Table of determining standard deviation and variance :

Class interval	Mid-point $x_i$	Frequency $f_i$	$d_i = \frac{x_i - a}{c}$ $a = 32, c = 5$	$f_i d_i$	$f_i d_i^2$
20-24	22	7	-2	-14	28
25-29	27	10	-1	-10	10
30-34	32	15	0	0	0
35-39	37	13	1	13	13
40-44	42	9	2	18	36
45-49	47	6	3	18	54
	Total	N = 60		$\sum f_i d_i = 25$	$\sum f_i d_i^2 = 141$

$\therefore$  The standard deviation

$\sigma = \sqrt{\left[ \frac{\sum f_i d_i^2}{N} - \left( \frac{\sum f_i d_i}{N} \right)^2 \right] \times c^2}$

$= \sqrt{\left[ \frac{141}{60} - \left( \frac{25}{60} \right)^2 \right] \times c^2}$

$= \sqrt{54.41}$

$= 7.376$

$\therefore$  The variance  $\sigma^2 = (7.376)^2 = 54.41 \text{ (Ans.)}$

**c** Total number of balls in the bag =  $(5 + 7 + 8) = 20$

If 4 ball are taken successively without replacement, then the probability of getting 4 white balls

$= \frac{{}^5C_1}{{}^{20}C_1} \times \frac{{}^4C_1}{{}^{19}C_1} \times \frac{{}^3C_1}{{}^{18}C_1} \times \frac{{}^2C_1}{{}^{17}C_1}$

$= \frac{5}{20} \times \frac{4}{19} \times \frac{1}{6} \times \frac{2}{17}$

$= \frac{1}{4} \times \frac{4}{19} \times \frac{1}{6} \times \frac{2}{17}$

$= \frac{1}{969} \text{ (Ans.)}$

**Ques 22 Scenario-I :** A boat travels directly across a river in time  $t_1$  and it travels a distance equal to the breadth of river down the stream in time  $t_2$ .

Scenario-II : Numbers obtaining by the 60 students of Rajuk Uttara Model College in Mathematics are as follows :

Number	50-59	60-69	70-79	80-89	90-99
Frequency	10	20	15	10	5

[RAJUK Uttara Model College, Dhaka]

- a. Probability of a student passing in Bengali is  $\frac{4}{5}$ .  
Probability of passing in Bengali and English both are  $\frac{3}{4}$   
and probability of passing in any one subject is  $\frac{7}{8}$ , then  
find the probability of passing only in English? 2
- b. From scenario-I : Let  $u$  be the velocity of the boat in still water and  $v$  be the velocity of stream then, prove that,  $t_1 : t_2 = \sqrt{u+v} : \sqrt{u-v}$  4
- c. From Scenario-II : Find the variance and standard deviation. 4

**Solution to the question no. 22**

- a Let, Probability of passing in Bangali = A  
and, probability of passing in English = B

Therefore,  $P(A) = \frac{4}{5}$

$P(A \text{ or } B) = P(A \cup B) = \frac{7}{8}$

$P(A \text{ and } B) = P(A \cap B) = \frac{3}{4}$

Now,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

or,  $\frac{7}{8} = \frac{4}{5} + P(B) - \frac{3}{4}$

or,  $P(B) = \frac{7}{8} + \frac{3}{4} - \frac{4}{5} = \frac{35 + 30 - 32}{40}$

$\therefore P(B) = \frac{33}{40}$

Therefore, the probability of passing in English =  $\frac{33}{40}$

$\therefore$  The probability of passing only in English.  
= The probability of passing in English - The probability of passing both in Bangla and English.

or,  $P(B \cap A^c) = P(B) - P(B \cap A) = \frac{33}{40} - \frac{3}{4} = \frac{3}{40}$ . (Ans.)

- b Let,  $\alpha$  be the angle between two velocities  $u$  and  $v$  and  $w$  be the resultant of them.

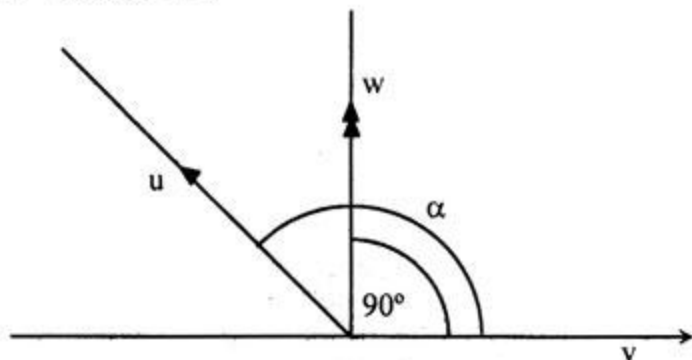
According to the parallelogram law,  
 $w^2 = u^2 + v^2 + 2uv \cos \alpha$  ... .. (i)

Again,  $\tan 90^\circ = \frac{u \sin \alpha}{v + u \cos \alpha}$

or,  $\cot 90^\circ = \frac{v + u \cos \alpha}{u \sin \alpha}$

or,  $0 = v + u \cos \alpha$

$\therefore u \cos \alpha = -v$



Using in (i)  $u \cos \alpha = -v$  we get,  
 $w^2 = u^2 + v^2 - 2v \cdot v = u^2 + v^2 - 2v^2 = u^2 - v^2$   
 $\therefore w = \sqrt{u^2 - v^2}$

If  $d$  be the width of the river then,  $d = wt_1$

$\therefore t_1 = \frac{d}{w} = \frac{d}{\sqrt{u^2 - v^2}}$

Again, the resultant velocity along the stream =  $u + v$

According to the stem,  $d = (u + v) t_2$

$\therefore t_2 = \frac{d}{u + v}$

$\therefore t_1 \text{ : } t_2 = \frac{d}{\sqrt{u^2 - v^2}} \text{ : } \frac{d}{u + v}$

or,  $t_1 \text{ : } t_2 = u + v \text{ : } \sqrt{u^2 - v^2}$

or,  $t_1 \text{ : } t_2 = (\sqrt{u + v})^2 \text{ : } (\sqrt{u + v})(\sqrt{u - v})$

$\therefore t_1 \text{ : } t_2 = \sqrt{u + v} \text{ : } \sqrt{u - v}$  (Proved)

- c See the Question No. 3(b)

The variance  $\sigma^2 = (11.785)^2 = 138.89$  (approx) (Ans.)

**Ques 23** The last Math quiz number of some students of Science-5 of Notre Dame College is as follows:

Number Interval	50-55	55-60	60-65	65-70	70-75	75-80
No. of Students	7	14	22	36	15	6

[Notre Dame College, Dhaka]

- a. Define Standard deviation and Variance. 2
- b. Using the information: find the Standard deviation and Variance. 4
- c. A box contains 5 red and 4 white balls. Another box contains 3 red and 6 white balls. One ball is drawn random from each box. Find the probability that at least one of the two balls to be red. 4

**Solution to the question no. 23**

**a Standard deviation :** The standard deviation is the positive square root of the sum of square deviations of the observations from their arithmetic mean divided by the total number of observations. It is denoted by  $\sigma$  or SD.

**Variance :** The sum of square deviations of the observations from their arithmetic mean divided by the total number of observations, is called variance. That is, variance is the square of standard deviation.

**b**

Class Interval	mid point $x_i$	Frequency $f_i$	$f_i x_i$	$f_i x_i^2$	$\bar{x}$	$ x_i - \bar{x} $	$f_i  x_i - \bar{x} $
50-55	52.5	7	367.5	19293.75		12.8	89.6
55-60	57.5	14	805	46287.5		7.8	109.2
60-65	62.5	22	1375	85937.5		2.8	61.6
65-70	67.5	36	2430	164025	65.3	2.2	79.2
70-75	72.5	15	1087.5	78843.75		7.2	108
75-80	77.5	6	465	36037.5		12.2	73.2
		$N = 100$	$\sum f_i x_i = 6530$	$\sum f_i x_i^2 = 430425$			$\sum f_i  x_i - \bar{x}  = 520.8$

Average Deviation (M.D) =  $\frac{\sum f_i |x_i - \bar{x}|}{N} = \frac{520.8}{100} = 5.208$

Standard Deviation  $\sigma = \sqrt{\frac{1}{N} (\sum f_i x_i^2) - \left(\frac{\sum f_i x_i}{N}\right)^2}$   
 $= \sqrt{\frac{430425}{100} - \left(\frac{6530}{100}\right)^2}$   
 $= 6.34$  (Ans.)



$$\begin{aligned} \text{Variance} &= \frac{\sigma}{\bar{x}} \times 100\% \\ &= \frac{6.34}{65.3} \times 100 \\ &= 9.709\% \text{ (Ans.)} \end{aligned}$$

**c** One box contains 5 red and 4 white balls whereas another box contains 3 red and 6 white balls. If two balls are drawn randomly one from each box, then the probability of getting at least one red ball = P (red ball from first box and white ball from second box) + P (red ball from second box white ball from first box) + P (red balls from both boxes)

$$\begin{aligned} &= \frac{{}^5C_1 \times {}^6C_1}{{}^9C_1} + \frac{{}^4C_1 \times {}^3C_1}{{}^9C_1} + \frac{{}^5C_1 \times {}^3C_1}{{}^9C_1} \\ &= \frac{5}{9} \times \frac{6}{9} + \frac{4}{9} \times \frac{3}{9} + \frac{5}{9} \times \frac{3}{9} \\ &= \frac{5}{9} \times \frac{2}{3} + \frac{4}{9} \times \frac{1}{3} + \frac{5}{9} \times \frac{1}{3} \\ &= \frac{10}{27} + \frac{4}{27} + \frac{5}{27} \\ &= \frac{19}{27} \text{ (Ans.)} \end{aligned}$$

**Ques ▶ 24** Scenerio- 1 : Out of 250 candidates of an examination in mathematics and statistics, 25 fail in statistics, 45 fail in mathematics and 15 fail in both the subjects. Among them a candidate is selected at random.

Scenerio-2 : A frequency distribution table of

Marks obtained	40	50	60	70	80	90
No. of students	4	6	11	13	12	4

[Ideal School & College, Motijheel, Dhaka]

- If  $P(A) = 0.6$  and  $P(A \cap B) = 0.48$ , then for what value of  $P(B)$ , the events A and B will be independent? 2
- From Scenerio-1, find the probability of the candidate's passing in statistics and failing in mathematics. 4
- Find the difference of the standard deviation and the variance from scenerio-2. 4

**Solution to the question no. 24**

See the Question No. 2

**Ques ▶ 25** Scenarion-1 : Frequency distribution table of 50 students is given below.

obtained marks	51-60	61-70	71-80	81-90	91-100
No. of students	6	10	16	14	4

Scenarion-2 In a box there are 8 green and 5 orange balls.

[Viqarunnisa Noon School & College, Dhaka]

- From a pack of 52 cards 1 card is drawn randomly. Find the probability of the card to be red color or ace. 2
- From Scenarion-2, if two balls are drawn at random, find the probability of getting two different color balls. 4
- From Scenarion-1 find the standard deviation and variance. 4

**Solution to the question no. 25**

- a** Let, the probability of the card will be red =  $P(A)$  and the probability of the card will be ace =  $P(B)$   
 $\therefore$  The probability of the card will be red or ace =  $P(A \cup B)$  and the probability of the card will be red and ace =  $P(A \cap B)$   
 We know,  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\begin{aligned} &= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} \\ &= \frac{7}{13} \text{ (Ans.)} \end{aligned}$$

- b** Total number of balls in the box =  $8 + 5 = 13$   
 Two ball can be drawn randomly  ${}^{13}C_2$  ways  
 $\therefore$  The probability of getting two different color balls = P(1 green ball and 1 orange ball)

$$\frac{{}^8C_1 \times {}^5C_1}{{}^{13}C_2} = \frac{20}{39} \text{ (Ans.)}$$

**c** Table of determing standard deviation and variance :

Obtained marks	Now of students ( $f_i$ )	Mid-value ( $x_i$ )	$u_i = \frac{x_i - 75.5}{10}$	$f_i u_i$	$f_i u_i^2$
51-60	6	55.5	-2	-12	24
61-70	10	65.5	-1	-10	10
71-80	16	75.5 = a	0	0	0
81-90	14	85.5	1	14	14
91-100	4	95.5	2	8	16
	$\Sigma f_i = N = 50$			$\Sigma f_i u_i = 0$	$\Sigma f_i u_i^2 = 64$

$$\begin{aligned} \therefore \text{The standard deviaten, } \sigma &= c \sqrt{\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2} \\ &= 10 \sqrt{\frac{64}{50} - \left(\frac{0}{50}\right)^2} \\ &= 10 \sqrt{1.28} \\ &= 10 \times 1.13 \\ &= 11.3 \text{ (Approx) (Ans.)} \end{aligned}$$

The variance,  $\sigma^2 = 11.3^2 = 127.69$  (Approx) (Ans.)

**Ques ▶ 26** Scenarion-1 : One dice and two coins are thrown simul-taneously.

Scenarion-2: A frequency distribution table is given below-

Age (years)	20-30	30-40	40-50	50-60	60-70
Frequency	25	40	20	10	5

[Dhaka Residential Model College, Dhaka]

- If  $P(A) = \frac{1}{3}$  and  $P(A \cap B) = \frac{1}{5}$  then find the value of  $P\left(\frac{B}{A}\right)$  2
- From scenarion-1: constructing the sample space, determine the probability of getting odd number. 4
- From scenarion-2; Find the standard deviation of the frequency distribution table. 4

**Solution to the question no. 26**

See the Question No. 1

**Ques ▶ 27** Scenarion-1 : Bag contains 3 white and 2 black balls. Two ball is drawn random from the bag.

Scenarion-2 :

Class interval	20-24	25-29	30-34	35-39	40-44	45-49
Frequency	7	10	15	12	10	6

[Birshreshtha Noor Mohammad Public College, Dhaka]

- A dice and two coins are thrown. Write the sample space. 2
- From scenarion-1, find the probability of getting two balls to be (i) same colour (ii) different colour. 4
- In scenarion-2, find standard deviation from given frequency distribution table. 4

**Solution to the question no. 27**

- a** See the Question No. 1(b)

- b** Total number of balls in the bag = 2 + 3 = 5

If two balls are drawn randomly, then

(i) The probability of getting same coloured two balls,

$$= \frac{{}^3C_2 + {}^2C_2}{{}^5C_2} = \frac{3}{10} + \frac{1}{10} = \frac{4}{10} = \frac{2}{5} \text{ (Ans.)}$$

(ii) The probability of getting different coloured two balls,

$$= \frac{{}^3C_1 \times {}^2C_1}{{}^5C_2} = \frac{3 \times 2}{10} = \frac{3}{5} \text{ (Ans.)}$$

- c** The table of determining standard deviation :

Class interval	Frequency (f <sub>i</sub> )	Mid-value (x <sub>i</sub> )	f <sub>i</sub> x <sub>i</sub>	f <sub>i</sub> x <sub>i</sub> <sup>2</sup>
20-24	7	22	154	3388
25-29	10	27	270	7290
30-34	15	32	480	15360
35-39	12	37	444	16428
40-44	10	42	420	17640
45-49	6	47	282	13254
মোট	60		2050	73360

$$\therefore \text{Arithmetic mean, } \bar{x} = \frac{\sum f_i x_i}{N} = \frac{2050}{60} = 34.17$$

$$\therefore \text{The standard deviation, } \sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$$

$$= \sqrt{\frac{73360}{60} - (34.17)^2}$$

$$= 7.42 \text{ (Ans.)}$$

**Ques ▶ 28** Stem-1: The probability of failing in Bengali in the final examination of a college of an examinee is  $\frac{1}{5}$  and that of passing both in Bengali and English is  $\frac{3}{4}$  and that of passing in any of these two subjects is  $\frac{7}{8}$ .

Stem-2 : Obtained marks in Mathematics of 100 students are given below :

Marks	20-30	30-40	40-50	50-60	60-70
No. of students	25	40	20	10	5

[Adamjee Cantonment College, Dhaka]

- a. If  $P(A) = \frac{1}{5}$ ,  $P(B) = \frac{1}{5}$  and  $P(B|A) = \frac{3}{8}$  then find the value of  $P(A \cup B)$ . 2
- b. Find the probability of passing only in English of an examinee in that final examination of the college. 4
- c. Find the variance of the frequency distribution. 4

**Solution to the question no. 28**

**a** Given that,  $P(A) = \frac{1}{5}$

$$P(B) = \frac{1}{5}$$

$$\text{and } P(B|A) = \frac{3}{8}$$

$$\therefore P(A \cap B) = P(B|A) \times P(A)$$

$$= \frac{3}{8} \times \frac{1}{5}$$

$$= \frac{3}{40}$$

For mutually inclusive events,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{5} + \frac{1}{5} - \frac{3}{40}$$

$$= \frac{40 + 16 - 15}{80}$$

$$= \frac{41}{80} \text{ (Ans.)}$$

- b** Let, the event of passing in Bengali = A and the event of passing in English = B

Then,  $P(A) = 1 - \frac{1}{5}$  [Complement rule]

$$= \frac{4}{5}$$

$$P(\text{Bengali or English}) = P(A \cup B) = \frac{7}{8}$$

$$P(\text{Bengali and English}) = P(A \cap B) = \frac{3}{4}$$

We know,  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\text{or, } \frac{7}{8} = \frac{4}{5} + P(B) - \frac{3}{4}$$

$$\text{or, } P(B) = \frac{7}{8} + \frac{3}{4} - \frac{4}{5} = \frac{35 + 30 - 32}{40}$$

$$\therefore P(B) = \frac{33}{40}$$

That is the probability of passing in English =  $\frac{33}{40}$

The probability of passing only in English =  $P(\text{English but not Bengali})$

$$= P(B \cap A^c) = P(B) - P(B \cap A)$$

$$= \frac{33}{40} - \frac{3}{4}$$

$$= \frac{3}{40} \text{ (Ans.)}$$

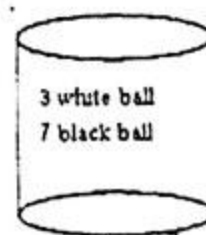
- c** See the Question No. 1(c)

Therefore, the variance,  $\sigma^2 = (11)^2 = 121$  (Ans.)

**Ques ▶ 29** Stem-1: The result of pre-test examination of 50 students are given as follows.

Obtained number	40	50	60	70	80	90
Number of students	4	6	11	13	12	4

Stem-2 :



first box



second box

A box is selected equally likely and a ball is drawn.

[BAF Shaheen College, Dhaka]

- a. One coin and two dices are thrown simultaneously, show its sample space. 2
- b. From stem 2 find the standard deviation of the result of mathematics. 4
- c. If the drawn ball is white then find the probability of that the ball is selected from the first box. 4

**Solution to the question no. 29**

- a** See the Question No. 2(c)

- b** See the Question No. 1(b)

- c** Let, A = The event of selection of first box  
B = The event of selection of second box  
W = The event of drawing a white ball

$$\therefore P(A) = P(B) = \frac{1}{2}$$

$$P(W/A) = \frac{3}{10}, P(W/B) = \frac{5}{13}$$

$\therefore$  The drawn ball is white then the probability of that the ball is selected from the first box,

$$P(A/W) = \frac{P(A) P(W/A)}{P(A) P(W/A) + P(B) P(W/B)}$$

$$= \frac{\frac{1}{2} \times \frac{3}{10}}{\frac{1}{2} \times \frac{3}{10} + \frac{1}{2} \times \frac{5}{13}}$$

$$= \frac{\frac{3}{20}}{\frac{3}{20} + \frac{5}{26}} = \frac{3}{\frac{3}{20} + \frac{5}{26}}$$

$$= \frac{3}{\frac{39}{260} + \frac{50}{260}} = \frac{3}{\frac{89}{260}} = \frac{3 \times 260}{89} = \frac{780}{89} \text{ (Ans.)}$$

**Ques 30** Scenario-1 : In a bag, there are 5 black and 7 red ball and in another bag, there are 6 black and 8 red balls.

Scenario-2 : The marks of a student of five subjects are 65, 85, 85, 90, 95.

[St. Joseph Higher Secondary School & College, Dhaka]

- a. If  $P(A) = \frac{1}{3}$ ,  $P(B) = \frac{3}{4}$ , then  $P(A \cup B) = ?$  where A, B are independent. 2
- b. From Scenario-1, if two balls (one from each bag) picked up, then find the probability of the balls to be at least one red. 4
- c. Find the standard deviation of the marks of the student. 4

**Solution to the question no. 30**

**a** Similar to the Question No. 6(a)

**b** Similar to Question No. 23(c)

**c** The obtained marks are 65, 85, 90, 95

The arithmetic mean of the marks,  $\bar{x} = \frac{65 + 85 + 85 + 90 + 95}{5}$

$$= \frac{420}{5} = 84$$

$\therefore$  The standard deviation of the marks,  $\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$

$$= \sqrt{\frac{(65 - 84)^2 + (85 - 84)^2 + (85 - 84)^2 + (90 - 84)^2 + (95 - 84)^2}{5}}$$

$$= \sqrt{\frac{361 + 1 + 1 + 36 + 12}{5}} = \sqrt{\frac{520}{5}} = \sqrt{104}$$

$$= 10.20 \text{ (Approx.) (Ans.)}$$

**Ques 31** Scenario-1 :

Marks obtained	50-60	60-70	70-80	80-90	90-100
Number of examinees	13	17	10	6	4

**Scenario-2 :**

White balls = 3  
Black balls = 2

White balls = 2  
Black balls = 5

[Chattogram Cant Public College, Chattogram]

- a. If  $P(A) = \frac{1}{5}$ ,  $P(B) = \frac{5}{7}$ , A & B are independent then find  $P(A \cup B)$ . 2
- b. From scenario-1 find the value of standard deviation. 4
- c. From scenario-2 one ball is drawn from each bag, find the probability of getting at least one ball to be white. 4

**Solution to the question no. 31**

**a** Given that,  $P(A) = \frac{1}{5}$  and  $P(B) = \frac{5}{7}$

Since A and B are independent events,

so  $P(A \cap B) = P(A) \cdot P(B)$

$$= \frac{1}{5} \cdot \frac{5}{7} = \frac{1}{7}$$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{5} + \frac{5}{7} - \frac{1}{7}$$

$$= \frac{7 + 25 - 1}{35}$$

$$= \frac{31}{35} \text{ (Ans.)}$$

**b** Table of determining S. D :

Class interval	Frequency $f_i$	Mid-value $x_i$	$u_i = \frac{x_i - a}{c}$ $a=75,$ $c=10$	$f_i u_i$	$f_i u_i^2$
50-60	13	55	-2	-26	676
60-70	17	65	-1	-17	289
70-80	10	75	0	0	0
80-90	6	85	1	6	36
90-100	4	95	2	8	64
	$\Sigma f_i = 50$			$\Sigma f_i u_i = -29$	$\Sigma f_i u_i^2 = 1065$

$\therefore$  The standard deviation  $\sigma = \sqrt{\frac{\sum f_i u_i^2}{N} - \left(\frac{\sum f_i u_i}{N}\right)^2} \times c$

$$= \sqrt{\frac{1065}{50} - \left(\frac{-29}{50}\right)^2} \times 10$$

$$= \sqrt{21.30 - 0.3364} \times 10$$

$$= \sqrt{20.9636} \times 10$$

$$= 45.79 \text{ (Approx.) (Ans.)}$$

**c** At least one white ball can be found if two balls are drawn from two different bags as the following ways:

Bag-1	Bag-2
White	Black
Black	White
White	White

$\therefore$  The Probability of getting at least one ball to be white

$$= {}^3C_1 \times {}^5C_1 + {}^2C_1 \times {}^2C_1 + {}^3C_1 \times {}^2C_1$$

$$= 25$$