Chapter-12: Vectors in a Plane

Question ▶ 1 M and N are the mid-point of sides PQ and PR of a ΔPQR. [All Board-18]

a. Define the position vector of point with figure. 2

b. With the help of vector, prove that, $MN = \frac{1}{2} QR$. 4

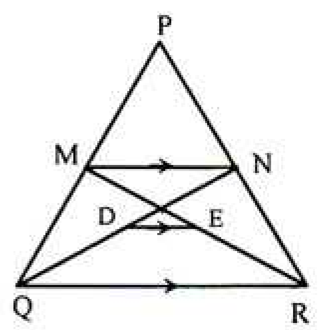
c. According to the information if D and E are mid-point of two diagonal of the trapezium QRNM, then with the help of vector prove that, $DE = \frac{1}{2}(QR - MN)$. 4

Solution to the question no. 1

a **Position vector:** With respect to a given point O in a plane, the position of any point P in the plane can be fixed by $\overrightarrow{OP}$. $\overrightarrow{OP}$ is called the position vector of P with respect to O and O is called the vector origin.

b Similar to example 3, chapter 12 of your textbook.

c



Given, in ΔPQR, M and N are the midpoints of PQ and PR respectively. The midpoints of the sides QN and RM of the trapezium QRNM are D and E respectively. Join E, D.

It is required to prove that, $DE = \frac{1}{2}(QR - MN)$

Proof: Let, the position vector of the points Q, R, N, M with respect to origin be $\underline{q}$, $\underline{r}$, $\underline{n}$, $\underline{m}$ respectively.

$\overrightarrow{QR} = \underline{r} - \underline{q}$

$\overrightarrow{MN} = \underline{n} - \underline{m}$

$\therefore$ Position vector of the point D $= \frac{1}{2}(\underline{q} + \underline{n})$ [$\because$ D is the midpoint of QN]

and position vector of the point E $= \frac{1}{2}(\underline{r} + \underline{m})$ [$\because$ E is the midpoint of RM]

$$\therefore \overrightarrow{DE} = \frac{1}{2}(\underline{r} + \underline{m}) - \frac{1}{2}(\underline{q} + \underline{n})$$
$$= \frac{1}{2}(\underline{r} + \underline{m} - \underline{q} - \underline{n})$$
$$= \frac{1}{2}\{(\underline{r} - \underline{q}) - (\underline{n} - \underline{m})\}$$
$$= \frac{1}{2}(\overrightarrow{QR} - \overrightarrow{MN})$$

Now, $|\overrightarrow{DE}| = \frac{1}{2}|(\overrightarrow{QR} - \overrightarrow{MN})|$

$\therefore DE = \frac{1}{2}(QR - MN)$ **(Proved)**

Question ▶ 2 The vertices of ΔABC are A (1, 3), B (–1, –1), C (3, –1) and the mid points of AB and AC are D and F. [D.B.17]

a. Determine the slope of AB. 2

b. Find the length of each side and the area of the triangle ABC. 4

c. Prove with the help of vectors that DF ∥ BC and $DF = \frac{1}{2}$ BC. 4

Solution to the question no. 2

a Given,
coordinates of the points A and B are (1, 3) and (– 1, –1) respectively.

$$\therefore \text{ Slope of the line AB} = \frac{-1-3}{-1-1} = \frac{-4}{-2} = 2 \textbf{ (Ans.)}$$

b Given, the vertices of ΔABC are A(1, 3), B(– 1, – 1) and C(3, –1) respectively.

Length of side AB $= \sqrt{(-1-1)^2 + (-1-3)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ unit

Length of side BC $= \sqrt{(3+1)^2 + (-1+1)^2} = \sqrt{16+0} = 4$ unit

Length of side AC $= \sqrt{(3-1)^2 + (-1-3)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ unit

$\therefore$ Half perimeter of ΔABC, $s = \frac{AB + BC + AC}{2} = \frac{2\sqrt{5} + 4 + 2\sqrt{5}}{2} = 2 + 2\sqrt{5}$ unit

$\therefore$ ΔABC area of $= \sqrt{s(s-AB)(s-BC)(s-AC)}$ sq. unit

$$= \sqrt{(2+2\sqrt{5})(2+2\sqrt{5}-2\sqrt{5})(2+2\sqrt{5}-4)(2+2\sqrt{5}-2\sqrt{5})}$$
$$= \sqrt{(2\sqrt{5}+2)(2\sqrt{5}-2) \times 2 \times 2}$$
$$= \sqrt{\{(2\sqrt{5})^2 - 2^2\} \times 4}$$
$$= \sqrt{(20-4) \times 4}$$
$$= \sqrt{64} = 8 \text{ sq. unit } \textbf{(Ans.)}$$

c See example-3 of chapter 12 from your textbook. Page-281. [**N.B.** replace E with F]

Question ▶ 3 A(p, 3p), B(p^2, 2p), C(p – 2, p) and D(1, 1) are the four different points. [R.B.17]

a. Find the value of p if the line joining the points B and C has slope $\frac{1}{2}$. 2

b. If the lines AB and CD are parallel, find the admissible value of p. 4

c. With negative value of p from (b) the straight line joining the middle points of the non-parallel side R and S of a trapezium ABCD. Prove with the help of vectors that

$RS \parallel AB \parallel CD$ and $RS = \frac{1}{2}(AB + CD)$. 4

Solution to the question no.3

a Coordinates of points B and C are $(p^2, 2p)$ and $(p - 2, p)$ respectively.

$\therefore$ Slope of the line BC $= \frac{p - 2p}{p - 2 - p^2} = \frac{-p}{p - 2 - p^2}$

According to question, $\frac{-p}{p - 2 - p^2} = \frac{1}{2}$

Or, $-2p = p - 2 - p^2$

Or, $p^2 - 3p + 2 = 0$

Or, $p^2 - 2p - p + 2 = 0$

Or, $p(p - 2) - 1(p - 2) = 0$

Or, $(p - 2)(p - 1) = 0$

$\therefore p = 1, 2$ **(Ans.)**

b See example-5 of exercise-11.3 from your textbook. Page-259

[N.B. t will be replaced with p]

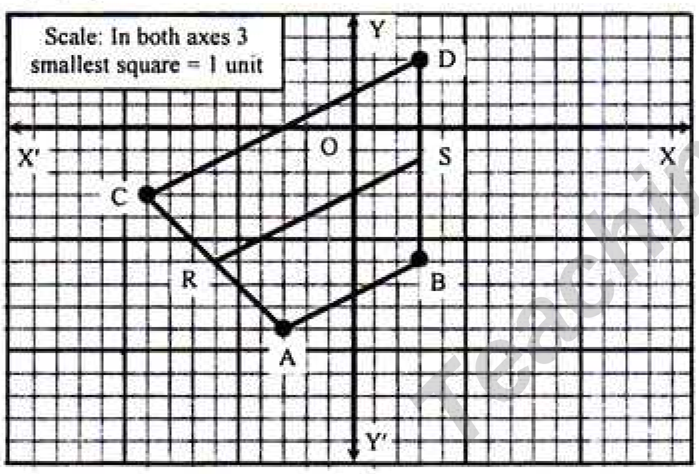
c From 'b' we get, $p = -1$

$\therefore A(p, 3p) \equiv (-1, -3)$

$B(p^2, 2p) \equiv (1, -2)$

$C(p - 2, p) \equiv (-3, -1)$

$D(1, 1)$



Let, ABCD be the trapezium whose sides AC and BD are non-parallel and the sides AB and CD are parallel. The midpoints of sides AC and BD are R and S respectively. R, S are joined. It is required to prove that, $RS \parallel AB \parallel CD$ and $RS = \frac{1}{2}(AB + CD)$.

Proof: Let, the position vectors of the points A, B, C and D with respect to origin be $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ respectively.

$\therefore \overrightarrow{AB} = \underline{b} - \underline{a}$, $\overrightarrow{CD} = \underline{d} - \underline{c}$

$\therefore$ The position vector of point R $= \frac{1}{2}(\underline{a} + \underline{c})$

and the position vector of point S $= \frac{1}{2}(\underline{b} + \underline{d})$

$\therefore \overrightarrow{RS} = \frac{1}{2}(\underline{b} + \underline{d}) - \frac{1}{2}(\underline{a} + \underline{c})$

$= \frac{1}{2}(\underline{b} + \underline{d} - \underline{a} - \underline{c})$

$= \frac{1}{2}\{(\underline{b} - \underline{a}) + (\underline{d} - \underline{c})\}$

$\therefore \overrightarrow{RS} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{CD})$

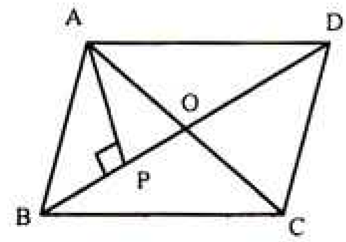
$\therefore |\overrightarrow{RS}| = \frac{1}{2}|(\overrightarrow{AB} + \overrightarrow{CD})|$

So, $RS = \frac{1}{2}(AB + CD)$

Since $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel so, $\overrightarrow{AB} + \overrightarrow{CD}$ will be parallel to $\overrightarrow{AB}$ and $\overrightarrow{CD}$. So the vector $\overrightarrow{RS}$ will be parallel to $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

Therefore, $RS \parallel AB \parallel CD$ and $RS = \frac{1}{2}(AB + CD)$ **(Proved)**

Question ▶4



In figure, ABCD is a parallelogram. [C.B.17]

a. Find the orthogonal projection of AB and AD. 2

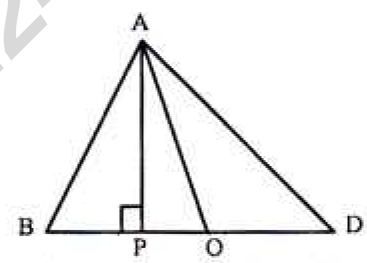
b. Prove that, $AB^2 + AD^2 = 2(AO^2 + BO^2)$. 4

c. Prove that, AO = OC and BO = OD with the help of vectors. 4

Solution to the question no. 4

a The projection of AB on BD is BP and projection of AD is DP.

b



Particular enunciation : Let, in ΔABD, AO is a median which bisects BD and $AP \perp BD$. It is required to prove that, $AB^2 + AD^2 = 2(AO^2 + BO^2)$

Proof: In ΔAOB, $\angle AOB$ is acute angle.

$\therefore$ As per the extension of the theorem of Pythagoras in the case of the acute angle we get,

$AB^2 = AO^2 + BO^2 - 2BO \cdot OP$ .... (i)

Again, in ΔAOD, $\angle AOD$ is obtuse angle.

$\therefore$ As per the extension of the theorem of Pythagoras in the case of the obtuse angle we get,

$AD^2 = AO^2 + OD^2 + 2OD.OP$ ... ... (ii)

Adding equations (i) and (ii) we get,

$AB^2 + AD^2 = AO^2 + BO^2 - 2BO \cdot OP + AO^2 + OD^2 + 2OD.OP$

$= 2AO^2 + BO^2 - 2BO.OP + BO^2 + 2BO.OP$ [$\because$ BO = OD]

$= 2AO^2 + 2BO^2$

$\therefore AB^2 + AD^2 = 2(AO^2 + BO)^2$ **(Proved)**

c See example-4 of exercise-12 from your textbook. Page-282.

Question ▶5 P (8, 3), Q (3, 8) and R (–2, 3) are the three vertices of a triangle 'S' and 'T' be the middle points of the sides PQ and PR respectively. [Ctg.B.17]

a. Find the slope of QR. 2

b. Show that, ΔPQR is an isosceles triangle and its area is 25 square units. 4

c. Prove with the help of vectors that, $ST \parallel QR$ and $ST = \frac{1}{2}QR$. 4

Solution to the question no. 5

a Given, coordinates of the point Q and R are respectively (3, 8) and (– 2, 3)

∴ Slope of the line QR $= \frac{3-8}{-2-3} = \frac{-5}{-5} = 1$ **(Ans.)**

b Given, the vertices of a triangle are P(8, 3), Q(3, 8) and R(– 2, 3)

Length of side PQ $= \sqrt{(3-8)^2 + (8-3)^2} = \sqrt{50}$ unit

Length of side QR $= \sqrt{(-2-3)^2 + (3-8)^2} = \sqrt{50}$ unit

Length of side PR $= \sqrt{(-2-8)^2 + (3-3)^2} = \sqrt{100}$ unit $= 10$ unit

since, in ΔPQR, PQ = QR $= \sqrt{50}$ unit

∴ ΔPQR is an isosceles triangle. **(Shown)**

Let, length of equal sides, a $= \sqrt{50}$ unit

and length of sides other, b = 10 unit

We know, the area of a isosceles triangle

$$= \frac{b}{4}\sqrt{4a^2 - b^2} \text{ sq. unit}$$

$$= \frac{10}{4}\sqrt{4(\sqrt{50})^2 - (10)^2}$$

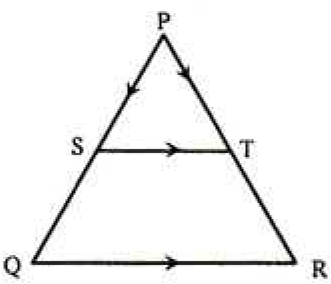
$$= \frac{5}{2}\sqrt{4 \times 50 - 100}$$

$$= \frac{5}{2}\sqrt{100}$$

$$= \frac{5}{2} \times 10 = 25$$

∴ Area of ΔPQR = 25 sq. unit. **(Ans.)**

c



Given, in ΔPQR, the midpoints of side PQ and PR are respectively S and T. Join S, T. It is required to prove that, ST ∥ QR and ST $= \frac{1}{2}$ QR

According to the triangle law of vector addition, $\overrightarrow{PS} + \overrightarrow{ST} = \overrightarrow{PT}$ ∴ $\overrightarrow{PT} - \overrightarrow{PS} = \overrightarrow{ST}$ .... (i)

and $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$ ∴ $\overrightarrow{PR} - \overrightarrow{PQ} = \overrightarrow{QR}$ .... (ii)

But $\overrightarrow{PR} = 2.\overrightarrow{PT}$ and $\overrightarrow{PQ} = 2.\overrightarrow{PS}$

[∵ S and T are the midpoints of PQ and PR respectively]

∴From (ii) we get, $2\overrightarrow{PT} - 2\overrightarrow{PS} = \overrightarrow{QR}$

Or, $2(\overrightarrow{PT} - \overrightarrow{PS}) = \overrightarrow{QR}$

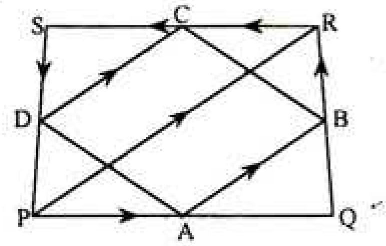
Or, $2\overrightarrow{ST} = \overrightarrow{QR}$ [from (i)]

∴ $\overrightarrow{ST} = \frac{1}{2}\overrightarrow{QR}$

Or, $|\overrightarrow{ST}| = \frac{1}{2}|\overrightarrow{QR}|$ or, ST $= \frac{1}{2}$ QR **(Proved)**

Again, the support line of $\overrightarrow{ST}$ and $\overrightarrow{QR}$ will be same or parallel. But here, support line is not same. So $\overrightarrow{ST}$ and $\overrightarrow{QR}$ are parallel that is ST ∥ QR. **(Proved)**

Question ▶ 6



In the figure, A, B, C and D are the middle points of quadrilateral PQRS. [J.B.17]

a. Express $\overrightarrow{AB}$ in terms of $\overrightarrow{PQ}$ and $\overrightarrow{QR}$. 2

b. Prove with the help of vectors that, the quadrilateral ABCD is a parallelogram. 4

c. Prove with the help of vectors that, AB ∥ PR and AB $= \frac{1}{2}$PR. 4

Solution to the question no. 6

a According to triangle law of vector addition, from ΔABQ,

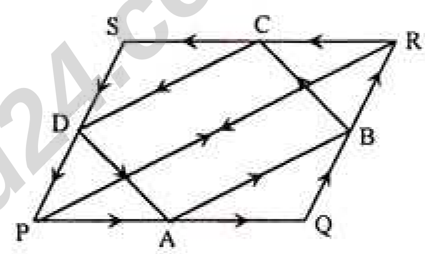
$\overrightarrow{AQ} + \overrightarrow{QB} = \overrightarrow{AB}$

Or, $\frac{1}{2}\overrightarrow{PQ} + \frac{1}{2}\overrightarrow{QR} = \overrightarrow{AB}$

[∵ A and B are the midpoints of PQ and QR respectively]

∴ $\overrightarrow{AB} = \frac{1}{2}(\overrightarrow{PQ} + \overrightarrow{QR})$ **(Ans.)**

b



Let, the midpoints of the sides of the quadrilateral PQRS be A, B, C and D respectively. It is required to prove that, ABCD is a parallelogram.

Proof: Let, $\overrightarrow{PQ} = \underline{p}$, $\overrightarrow{QR} = \underline{q}$, $\overrightarrow{RS} = \underline{r}$, $\overrightarrow{SP} = \underline{s}$

Thus,

$\overrightarrow{AB} = \overrightarrow{AQ} + \overrightarrow{QB} = \frac{1}{2}(\overrightarrow{PQ} + \overrightarrow{QR}) = \frac{1}{2}(\underline{p} + \underline{q})$

Similarly, $\overrightarrow{BC} = \frac{1}{2}(\underline{q} + \underline{r})$, $\overrightarrow{CD} = \frac{1}{2}(\underline{r} + \underline{s})$,

$\overrightarrow{DA} = \frac{1}{2}(\underline{s} + \underline{p})$

Again, $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \underline{p} + \underline{q}$

and $\overrightarrow{RP} = \overrightarrow{RS} + \overrightarrow{SP} = \underline{r} + \underline{s}$

But $(\underline{p} + \underline{q}) + (\underline{r} + \underline{s}) = \overrightarrow{PR} + \overrightarrow{RP} = \overrightarrow{PR} - \overrightarrow{PR} = 0$

That is $(\underline{p} + \underline{q}) = -(\underline{r} + \underline{s})$

∴ $\overrightarrow{AB} = \frac{1}{2}(\underline{p} + \underline{q}) = -\frac{1}{2}(\underline{r} + \underline{s}) = -\overrightarrow{CD} = \overrightarrow{DC}$

Here, the line of supports of $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are either same or parallel. Here line of supports are not same.

∴ Line of supports are parallel. ∴ $\overrightarrow{AB}$ ∥ $\overrightarrow{DC}$.

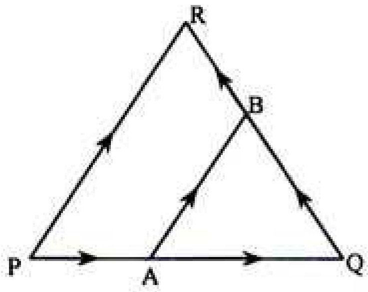
Now, $|\overrightarrow{AB}| = |\overrightarrow{DC}|$ ∴ AB = DC

∴ AB and DC are equal and parallel.

Similarly, BC and AD are equal and parallel.

∴ABCD is a parallelogram. **(Proved)**

c



Proof: According to triangle law of vector addition, from ΔABQ,

$\overrightarrow{AQ} + \overrightarrow{QB} = \overrightarrow{AB}$ ... ... (i)

Again, in ΔPQR, $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

Or, $2\overrightarrow{AQ} + 2\overrightarrow{QB} = \overrightarrow{PR}$ [∵ A and B are the midpoints of PQ and QR respectively]

Or, $2(\overrightarrow{AQ} + \overrightarrow{QB}) = \overrightarrow{PR}$

Or, $2\overrightarrow{AB} = \overrightarrow{PR}$

Or, $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{PR}$

Or, $|\overrightarrow{AB}| = \frac{1}{2}|\overrightarrow{PR}|$

$\therefore AB = \frac{1}{2}PR$.

Again, the line of supports of $\overrightarrow{AB}$ and $\overrightarrow{PR}$ are either same or parallel. Here line of supports are not same.

∴ Line of supports of $\overrightarrow{AB}$ and $\overrightarrow{PR}$ are parallel.

$\therefore AB \parallel PR$ and $AB = \frac{1}{2}PR$ **(Proved)**

Question ▶ 7 The vertices of the quadrilateral ABCD, arranged in anti-clockwise order are A(6, – 4), B(2, 2), C(–2,2), D(–6, –4). [D.B.16]

a. Find the length of BD. 2

b. Determine the length of the diagonal of the square where the area of square is equal to the area of the quadrilateral ABCD. 4

c. If ABCD is a trapezium and P and Q are the middle points of AB and CD respectively, then prove with the help of vectors that, $PQ \parallel AD \parallel BC$ and $PQ = \frac{1}{2}(AD + BC)$. 4

Solution to the question no. 7

a Distance between the two points B(2, 2) and D(–6, –4) that is, length of BD $= \sqrt{(-6-2)^2 + (-4-2)^2}$ unit

$= \sqrt{64 + 36}$ unit $= \sqrt{100}$ unit

= 10 unit**(Ans)**

b The vertices A(6, –4), B(2, 2), C(–2, 2) and D(–6, –4) are taken in anti-clockwise order.the area of the quadrilateral ABCD $= \frac{1}{2}$

$\begin{vmatrix} 6 & 2 & -2 & -6 & 6 \\ -4 & 2 & 2 & -4 & -4 \end{vmatrix}$ square unit

$= \frac{1}{2}(12 + 4 + 8 + 24 + 8 + 4 + 12 + 24)$ square unit

$= \frac{1}{2} \times 96$ square unit = 48 square unit

According to the question, area of square = area of the quadrilateral ABCD = 48 square unit

∴ Length of one side of square $= \sqrt{48}$ unit $= \sqrt{16 \times 3}$

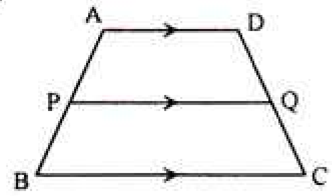
$= 4\sqrt{3}$ unit

∴ Length of diagonal of square $= \sqrt{2} \times$ Length of one side

$= \sqrt{2} \times 4\sqrt{3}$ unit

$= 4\sqrt{6}$ unit**(Ans.)**

c Here, P and Q are midpoints of non-parallel sides AB and CD of a trapezium ABCD.It is required to prove that, $PQ \parallel AD \parallel BC$ and $PQ = \frac{1}{2}(AD + BC)$



Proof: let, position vectors ofA, B, C and D are $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ respectively

$\therefore \overrightarrow{BC} = \underline{c} - \underline{b}$ and $\overrightarrow{AD} = \underline{d} - \underline{a}$

∴ Position vector of point P$= \frac{1}{2}(\underline{a} + \underline{b})$

[∵ P is the midpoint of AB]

Position vector of point Q $= \frac{1}{2}(\underline{c} + \underline{d})$

[∵ Q is the midpoint of CD]

Now, $\overrightarrow{PQ} = \frac{1}{2}(\underline{c} + \underline{d}) - \frac{1}{2}(\underline{a} + \underline{b}) = \frac{1}{2}(\underline{c} + \underline{d} - \underline{a} - \underline{b})$

$= \frac{1}{2}\{(\underline{c} - \underline{b}) + (\underline{d} - \underline{a})\} = \frac{1}{2}(\overrightarrow{BC} + \overrightarrow{AD})$

But as BC and AD are parallel so $(\overrightarrow{BC} + \overrightarrow{AD})$ will be parallel to BC and AD. So PQ vector will be parallel to BC and AD.

$\overrightarrow{PQ} = \frac{1}{2}(\overrightarrow{AD} + \overrightarrow{BC})$ or, $|\overrightarrow{PQ}| = \frac{1}{2}|\overrightarrow{BC} + \overrightarrow{AD}|$

So, $PQ = \frac{1}{2}(AD + BC)$

So, $PQ \parallel AD \parallel BC$ and $PQ = \frac{1}{2}(AD + BC)$ **(Proved)**

Question ▶ 8 The points A(7, 2), B(– 4, 2), C(– 4, – 3) and D(7, –3) are respectively the four vertices of a quadrilateral. [R.B.16]

a. Find the equation of the straight line AC. 2

b. Ascertain whether the quadrilateral is a parallelogram or a rectangle. 4

c. If the middle-points of the quadrilateral mentioned in the stem are P, Q, R & S respectively, then prove by vector method PQRS is a paralleogram. 4

Solution to the question no. 8

a Equation of a straight line passing through (7, 2) and C(–4,–3), $\frac{x-7}{7-(-4)} = \frac{y-2}{2-(-3)}$

Or, $\frac{x-7}{7+4} = \frac{y-2}{2+3}$

Or, $5x - 35 = 11y - 22$

Or, $5x - 11y - 35 + 22 = 0$

$\therefore 5x - 11y - 13 = 0$ **(Ans.)**

b Given, four vertices of the quadrilateral ABCD, A(7, 2), B(–4, 2), C(–4, –3) and D(7,–3)

∴ Length of side AB $= \sqrt{(7+4)^2 + (2-2)^2}$

$= \sqrt{(11)^2 + 0} = 11$ unit

Length of side BC $= \sqrt{(-4+4)^2 + (2+3)^2}$

$= \sqrt{0 + 5^2} = 5$ unit

Length of side CD $=\sqrt{(-4-7)^2+(-3+3)^2}$
$=\sqrt{121+0}=11$ unit

Length of side AD $=\sqrt{(7-7)^2+(2+3)^2}$
$=\sqrt{0+5^2}=5$ unit

Length of side AC $=\sqrt{(7+4)^2+(2+3)^2}$
$=\sqrt{121+25}=\sqrt{146}$ unit

Length of side BD $=\sqrt{(-4-7)^2+(2+3)^2}$
$=\sqrt{121+25}=\sqrt{146}$ unit

So, we get AB = CD and BC = AD
Again, diagonal AC = diagonal BD.
$\therefore$ ABCD is a rectangle**(Ans.)**

c See Text book chapter 12, example-5, page- 281

Question ▶ 9 The points P(7, 2), Q(– 4, 2), R(– 4,–3) and S(7, –3) are respectively the four vertices of a quadrilateral. [C.B.16]

a. Find the slope of the line PQ. 2
b. Ascertain whether the quadrilateral is a rectangle or a parallelogram. 4
c. If the middle points of the quadrilateral mentioned in the stem are D, E, F and G respectively, then prove by vector method DEFG is a parallelogram. 4

Solution to the question no. 9

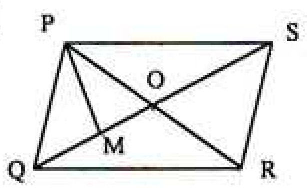
a Given, P(7, 2) andQ(–4, 2)

Slope of side PQ $=\dfrac{2-2}{7-(-4)}=\dfrac{0}{7+4}=0$ **(Ans.)**

b See ques. solution 2(b); page 517

c See text book, chapter-12, example-5; page 281

Question ▶ 10 [Dj.B.16]



In the figure PQRS is a parallelogram.
a. State the Apollonius theorem. 2
b. Prove that, $PQ^2+PS^2=2(PO^2+QO^2)$. 4
c. By vector method prove that, PO = RO and QO = SO. 4

Solution to the question no. 10

a **Theorem of Apollonius:** The sum of the areas of the squares drawn on any two sides of a triangle is equal to twice the sum of area of the squares drawn on the median of the third side and on either half of that side.

b The median PO of ΔPSQ bisect the side SQ. It is to be proved that: $PQ^2+PS^2=2(PO^2+QO^2)$

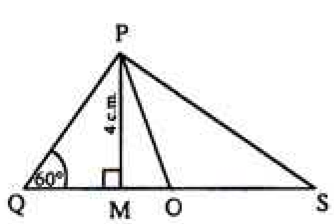


Figure-1

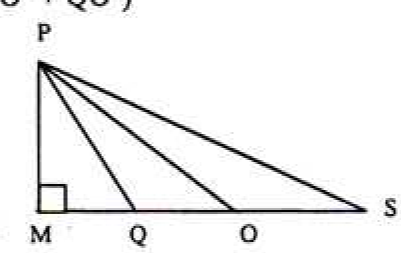


Figure-2

Construction: We draw a perpendicular PM on the side SQ (figure 1) and on the extended side of *BC* (figure 2)
Proof: ∠POS is an obtuse angle of ΔPSO and OM is the orthogonal projection of the line PO on the SO extended [in both figures]

$\therefore$ As per the extension of the theorem of Pythagoras we get,
$PS^2=PO^2+SO^2+2SO.OM$ ...... (i)
Again, ∠POQ is an acute angle of ΔPQO and OM is the orthogonal projection of the line PO on the line OQ (figure 1) and on the line OQ extended (figure 2)
$\therefore$ As per the extension of the theorem of Pythagoras in the case of the acute angle. We get,

$PQ^2=PO^2+QO^2-2QO.\ OM$...........(ii)

Now adding the equations (i) and (ii), we get,

$PS^2+PQ^2=2PO^2+SO^2+QO^2+2SO.OM-2QO.OM$
$=2PO^2+QO^2+QO^2+2QO.OM-2QO.OM$ [$\because$ SO = QO]
$=2(PO^2+QO^2)$
$\therefore PQ^2+PS^2=2(PO^2+QO^2)$ **(Proved)**

c See text book, chapter-12, example-4; page 280.

Question ▶ 11 The vertices of ΔABC are A(2,–4), B(– 4,4) and C(3, a) respectively. Where a > 0. [Ctg.B.16]

a. If AC = BC, find the value of a. 2
b. Determine the equation of straight line AB and its slope. 4
c. With the help of vectors, prove that the line segment joining the middle points of any two sides of a triangle is parallel to and half of the third side. 4

Solution to the question no. 11

a Given,
vertices of ABC triangle A(2, – 4), B(– 4, 4) and C(3, a)
According to question, AC = BC

Or, $\sqrt{(3-2)^2+(a+4)^2}=\sqrt{(3+4)^2+(a-4)^2}$
Or, $\sqrt{1+a^2+8a+16}=\sqrt{49+a^2-8a+16}$
Or, $a^2+8a+17=a^2-8a+65$ [Squaring both sides]
Or, $16a=65-17$ or, $a=\dfrac{48}{16}$
$\therefore$ a = 3 **(Ans.)**

b Equation of a straight line passing through A(2, –4) and B(– 4, 4), $\dfrac{x-2}{2-(-4)}=\dfrac{y-(-4)}{-4-4}$

Or, $\dfrac{x-2}{6}=\dfrac{y+4}{-8}$
Or, $-8x+16=6y+24$
Or, $-8x-6y+16-24=0$
Or, $-8x-6y-8=0$
Or, $-2(4x+3y+4)=0$
$\therefore 4x+3y+4=0$ **(Ans.)**

Again, slope of side AB $=\dfrac{4-(-4)}{-4-2}=\dfrac{8}{-6}=-\dfrac{4}{3}$**(Ans.)**

c See text book, chapter-12, example-3; page 279.

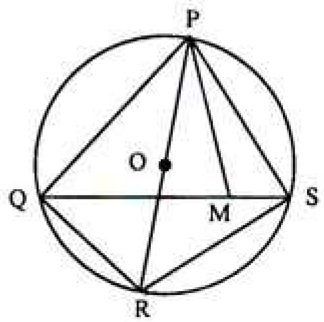
Question ▶ 12 PQRS is a cyclic quadrilateral and PQ, QS are two diagonals. [S.B.16]

a. What is the position of the centre of nine point circle and find its radius? 2
b. Prove that, PR·QS = PQ·RS + QR·PS. 4
c. Prove with the help of vectors that, the straight lines joining the middle points of the adjacent sides of a quadrilateral PQRS from of a parallelogram. 4

Solution to the question no. 12

a The centre of the nine point circle is the middle point of the line segment joining the orthocenter and the circumcenter. Again, The radius of the nine point circle is half of the circumradius.

b Suppose PQRS is a cyclic Quadrilateral. PR and QS are its diagonals and PQ, RS and QR, PS are its two pairs of opposite sides.It is to be proved that, PR . QS = PQ . RS + QR . PS.



Construction: Without loss of generality we can assume that ∠QPR is smaller than ∠SPR; we draw ∠SPM making it equal to ∠QPR at the point P

Proof : By construction∠QPR=∠SPM

Adding ∠RPM to both sides we get,

∠QPR+∠RPM =∠SPM + ∠RPM

So, ∠QPM=∠RPS

Now in ΔPQM and ΔPRS,

∠PQS =∠PRS [angles on the same segment]

and remaining∠PMQ = remaining∠PSR

∴ ΔPQM and∠PRS are equiangular

$\frac{QM}{RS} = \frac{PQ}{PR}$

So,PR . QM = PQ .RS .... (i)

Again, in ΔPQR and ΔPMS,

∠QPR =∠SPM [by construction]

∠PSM =∠PRQ [angles on the same segment]

And remaining ∠PQR = remaining ∠PMS

∴ΔPQR and ΔPMS are equiangular

$\therefore \frac{PS}{PR} = \frac{MS}{QR}$

So,PR . MS = QR . PS ....(ii)

Adding (i) and (ii) we get,

PR . QM + PR. MS = PQ . RS + QR . PS

Or, PR (QM + MS) = PQ . RS + QR . PS

Or, PR . QS = PQ . RS + QR . PS [∵ QM + MS = QS]

∴ PR.QS = PQ . RS + QR . PS **(Proved)**

c See text book, chapter-12, example-5; page 281.

Question ▶ 13 A(–5, 0), B(5, 0), C(5, 5) and D(–5, 5) are the vertices of the quadrilateral ABCD turned round of the anti-clockwise. [B.B.16]

a. Find the area of the quadrilateral ABCD. 2

b. Show that the quadrilateral ABCD is a rectangle. 4

c. If S and T are the middle point of the sides AB and AC, prove by the vector method that ST || BC and ST = $\frac{1}{2}$ BC. 4

Solution to the question no. 13

a The vertices A(–5, 0), B(5, 0), C(5, 5) and D(–5, 5) are taken in anti-clockwise order. The area of the quadrilateral ABCD

$$= \frac{1}{2}\begin{vmatrix} -5 & 5 & 5 & -5 & -5 \\ 0 & 0 & 5 & 5 & 0 \end{vmatrix}$$

$$= \frac{1}{2}(0 + 25 + 25 + 0 - 0 - 0 + 25 + 25)$$

$$= \frac{1}{2} \times 100 = 50 \text{ square unit } \textbf{(Ans.)}$$

b Given,A(–5, 0), B(5, 0), C(5, 5) and D(–5, 5)

So,Length of side AB $= \sqrt{(5+5)^2 + (0-0)^2}$

$= \sqrt{(10)^2 + (0)^2}$

$= \sqrt{100} = 10$ unit

Length of side BC $= \sqrt{(5-5)^2 + (5-0)^2}$

$= \sqrt{0^2 + 5^2} = \sqrt{25}$

= 5 unit

Length of side CD $= \sqrt{(-5-5)^2 + (5-5)^2}$

$= \sqrt{(-10)^2 + 0^2}$

$= \sqrt{100} = 10$ unit

and Length of side AD $= \sqrt{(-5+5)^2 + (5-0)^2}$

$= \sqrt{0^2 + 5^2}$

$= \sqrt{25} = 5$ unit

Again, Length of diagonal AC

$= \sqrt{(-5-5)^2 + (0-5)^2}$

$= \sqrt{10^2 + 5^2} = \sqrt{100 + 25}$

$= \sqrt{125} = 5\sqrt{5}$ unit

and Length of diagonal BD

$= \sqrt{(-5-5)^2 + (5-0)^2}$

$= \sqrt{(-10)^2 + 5^2}$

$= \sqrt{100 + 25}$

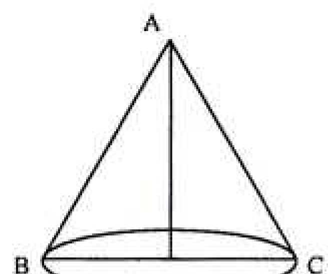
$= \sqrt{125} = 5\sqrt{5}$ unit

Here, AB = CD; BC = AD and diagonal AC = diagonal BD

∴ ABCD is a rectangle**(Shown)**

c See text book, chapter-12, example-3; page 279.

Question ▶ 14



In the figure, height is 7.50 metres and area of the land 2000 sq. metres. *[Mirzapur Cadet College, Tangail]*

a. Find the volume of the cylinder, if diameter is 6 cm and height is 8 cm. 2

b. In the above figure, how much canvas will be required to enclose a land. 4

c. In the above ΔABC, D and E are the mid-points of AB and AC respectively. Prove by vector method, DE || BC and DE = $\frac{1}{2}$ BC. 4

Solution to the question no. 14

a Given, diameter of cylinder = 6 cm

∴ radius of cylinder, r = 3 cm

and height, h = 8 cm

∴ Volume of cylinder, V $= \pi r^2 h$ cubic unit

$= 3.1416 \times (3)^2 \times 8$ cubic cm

= 226.19 cubic cm **(Ans.)**

b Given, height of the tent, h = 7.5 m.

and area of the land = 2000 sq. m.

So, area of the base of the cone is 2000 sq. m.

Let, radius of the base, r = x m.

According to the question, $\pi x^2 = 2000$

[∵ area of the base of cone = πr^2]

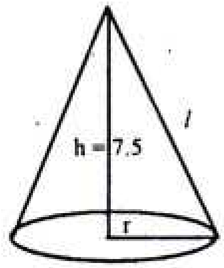
Or, $x^2 = \frac{2000}{3.1416}$ [∵ $\pi = 3.1416$]

Or, $x^2 = 636.6183$

∴ $x = 25.2313$

We know,

Slant height of the cone,



$\ell = \sqrt{h^2 + r^2}$ unit

$= \sqrt{(7.5)^2 + (25.2313)^2}$ m.

$= 26.3224$ m.

Total required canvas will be equal to the area of the curved surface of the cone.

∴ Canvas of the tent = $\pi r \ell$ sq. m.

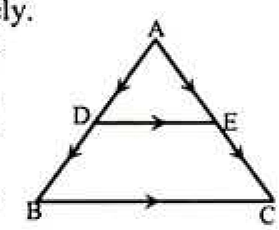
$= (3.1416 \times 25.2313 \times 26.3224)$ sq. m.

$= 2086.4885$ sq. m.

$= 2086.49$ sq. m. (approx.) **(Ans.)**

c Given the middle points of the sides AB and AC of the triangle ABC are D and E respectively.

Join D, E. It is required to prove with the help of vectors that $DE = \frac{1}{2}$ BC and DE || BC



Proof: D and E are the middle points of AB and AC respectively.

$\therefore \overrightarrow{DB} = \overrightarrow{AD} = \frac{1}{2}\overrightarrow{AB}$ and $\overrightarrow{AE} = \overrightarrow{EC} = \frac{1}{2}\overrightarrow{AC}$

According to the triangle law we get,

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

Or, $\overrightarrow{BC} = -\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AC} - \overrightarrow{AB}$ .... (i)

and $\overrightarrow{DE} = \overrightarrow{DA} + \overrightarrow{AE}$

$= -\overrightarrow{AD} + \overrightarrow{AE}$

$= -\frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC}$ [∵ $\overrightarrow{AD} = \frac{1}{2}\overrightarrow{AB}$, $\overrightarrow{AE} = \frac{1}{2}\overrightarrow{AC}$]

$= \frac{1}{2}(\overrightarrow{AC} - \overrightarrow{AB}) = \frac{1}{2}\overrightarrow{BC}$ [from the equation (i)]

So, $|\overrightarrow{DE}| = \frac{1}{2}|\overrightarrow{BC}|$

∴ $DE = \frac{1}{2}BC$ and the support line of $\overrightarrow{DE}$ and $\overrightarrow{BC}$ will be same or parallel.

But, D and E are the middle points of AB and AC respectively. So, the support line of $\overrightarrow{DE}$ and $\overrightarrow{BC}$ can not be same.

∴ DE || BC

That is, $DE = \frac{1}{2}BC$ and DE || BC **(Proved)**

Question ▶ 15 From vector geometry—

[Joypurhat Girls' Cadet College, Joypurhat]

a. Define position vector and give an example. 2

b. By the vector method, prove that, the joining line segment of two middle points of two sides of a triangle is half of third side and parallel to that side. 4

c. Prove by the vector method, the diagonals of a rhombus bisect each other. 4

Solution to the question no. 15

a See chapter-12, 12.8, page – 277 from text book.

b See chapter-12, example-3, Page-279 from text book

b Similar to example-4, chapter-12, from text book. Page–280

Question ▶ 16 D, E and F are the middle points of the sides BC, CA and AB of the triangle ABC respectively.

[Rangpur Cadet College, Rangpur]

a. If both a, b are non-zero and non-parallel vectors and if ma + nb = 0, then show that m = n = 0. 2

b. Prove that $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \overrightarrow{0}$ 4

c. Prove with the help of vectors that the straight line drawn through F parallel to BC must go through E. 4

Solution to the question no. 16

a Given, $\underline{a}$, $\underline{b}$ are two nonzero non–parallel vectors and $m\underline{a} + n\underline{b} = \underline{0}$. It is required to show that, m = n = 0

Given, $m\,\underline{a} + n\,\underline{b} = \underline{0}$

Or, $m\,\underline{a} + n\,\underline{b} - n\,\underline{b} = \underline{0} - n\,\underline{b}$

[adding ($- n\,\underline{b}$) in both sides]

Or, $m\,\underline{a} + \underline{0} = -n\,\underline{b}$

∴ $m\,\underline{a} = -n\,\underline{b}$

If m and n are nonzero, then the direction of $\underline{a}$ and $\underline{b}$ will be

(i) opposite if the sign of m and n are same.

(ii) same if the sign of m and n are opposite.

In both cases, $\underline{a}$ and $\underline{b}$ will be parallel which is impossible, because it is given that, $\underline{a}$ and $\underline{b}$ are two non–parallel vectors.

∴ m and n can not be nonzero.

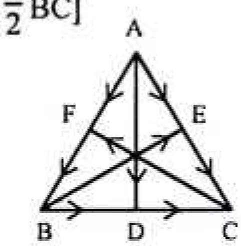
That is, m = n = 0. **(Shown)**

b By the triangle law, we get from ΔABD,

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

∴ $\overrightarrow{AD} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$ .... (i)

[D is the middle point of BC, so $\overrightarrow{BD} = \frac{1}{2}\overrightarrow{BC}$]



In ΔACF, $\overrightarrow{AF} = \overrightarrow{AC} + \overrightarrow{CF}$

∴ $\overrightarrow{CF} = \overrightarrow{AF} - \overrightarrow{AC}$ [$\overrightarrow{AC} = -\overrightarrow{CA}$]

∴ $\overrightarrow{CF} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{AC}$ .... (ii)

[F is the middle point of AB, so $\overrightarrow{AF} = \frac{1}{2}\overrightarrow{AB}$]

and from the triangle ΔABE, $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE}$

Or, $\overrightarrow{BE} = \overrightarrow{AE} - \overrightarrow{AB}$

∴ $\overrightarrow{BE} = \frac{1}{2}\overrightarrow{AC} - \overrightarrow{AB}$ .... (iii)

[E is the middle point of AC, so $\overrightarrow{AE} = \frac{1}{2}\overrightarrow{AC}$]

Now, by adding the equations (i), (ii) and (iii) we get,

$\overrightarrow{AD} + \overrightarrow{CF} + \overrightarrow{BE} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} + \frac{1}{2}\overrightarrow{AB} - \overrightarrow{AC} + \frac{1}{2}\overrightarrow{AC} - \overrightarrow{AB}$

Or, $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} - \frac{1}{2}\overrightarrow{AC}$

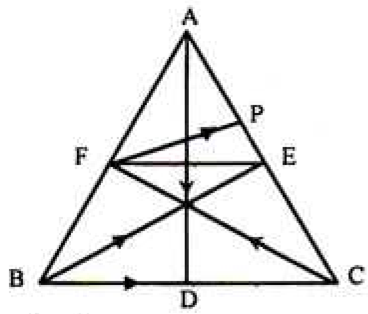
$= \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) - \frac{1}{2}\overrightarrow{AC}$

$= \frac{1}{2}\overrightarrow{AC} - \frac{1}{2}\overrightarrow{AC} = \underline{0}$

∴ $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \overrightarrow{0}$ **(Proved)**

c Let, F is the middle point of the side AB of the triangle ABC. The line drawn parallel to BC intersects the side AC at the point E. It is required to prove that, E is the middle point of AC.

Let, not E rather P is the middle point of AC.



Then, $\overrightarrow{AF} = \frac{1}{2}\overrightarrow{AB}$ [∵ F is the middle point of AB and $\overrightarrow{AP}$

$= \frac{1}{2}\overrightarrow{AC}$ [∵ P is the middle point of $\overrightarrow{AC}$]

$\therefore \overrightarrow{FP} = \overrightarrow{FA} + \overrightarrow{AP} = -\overrightarrow{AF} + \overrightarrow{AP}$ [$\therefore \overrightarrow{FA} = -\overrightarrow{AF}$]

$= \overrightarrow{AP} - \overrightarrow{AF} = \frac{1}{2}\overrightarrow{AC} - \frac{1}{2}\overrightarrow{AB}$

$= \frac{1}{2}(\overrightarrow{AC} - \overrightarrow{AB}) = \frac{1}{2}\overrightarrow{BC}$

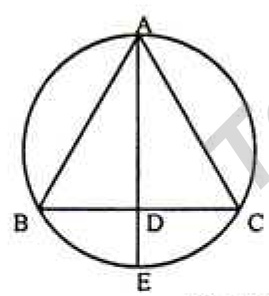
$\therefore \overrightarrow{FP} = \frac{1}{2}\overrightarrow{BC}$

That is, FP || BC. But FE || BC (given)

So, the both lines $\overrightarrow{FE}$ and $\overrightarrow{FP}$ pass through the point F and parallel to $\overrightarrow{BC}$. So, they (that is, $\overrightarrow{FE}$ and $\overrightarrow{FP}$) must be coincide to each other.

∴ E and P will be the same point. That is, E is the middle point of AC. **(Proved)**

Question ▶ 17



[Jhenidah Cadet College, Jhenidah]

a. If the length of side of an equilateral triangle is 4 unit, find the circumradius of that triangle. 2

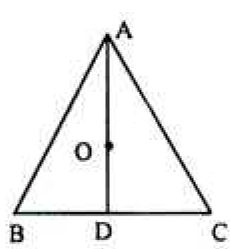
b. Under what condition AD^2 = AB. AC – BC.CD? Explain geometrically. 4

c. If P, Q and R are mid-point of AB, AC and BC respectively, prove that $\overrightarrow{AR} + \overrightarrow{BQ} + \overrightarrow{CP} = 0$ 4

Solution to the question no. 17

a Let, ΔABC is an equilateral triangle and AB = BC = AC = 4 cm

∴ median AD is perpendicular to BC



$\therefore AD^2 = AB^2 - BD^2$

$= 4^2 - (2)^2$

$\therefore AD = \sqrt{16-4}$

$= \sqrt{12}$

$= 2\sqrt{3}$

Again O is the centroid of ΔABC

then $\frac{OD}{OA} = \frac{1}{2}$

Or, $\frac{OD + OA}{OA} = \frac{1+2}{2}$

Or, $\frac{AD}{OA} = \frac{3}{2}$

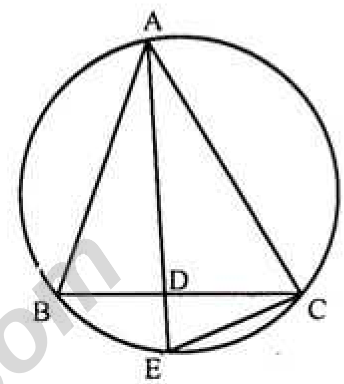
Or, $OA = \frac{2}{3} \times 2\sqrt{3}$ [∵ $AD = 1\sqrt{3}$]

$= \frac{4\sqrt{3}}{3}$

∴ Circumradius = 2.309 cm (appr.) **(Ans).**

b If AE is the bisector of the angle ∠A, then AD^2 = AB.AC – BD.CD

Solution:



Particular Enunciation: Given, the bisector of the angle A of the triangle ABC intersects BC at D and the circumcircle at E. It is to be shown that AD^2 = AB. AC – BD.DC.

Construction: Join C and E.

Proof: In ΔABD and ΔACE,

∠BAD = ∠CAE [∵ AD is the bisector of ∠A]

and ∠ABD = ∠AEC [∵ angles subtended by the same arc are equal]

∴ remaining ∠ADB = remaining ∠ACE

∴ The triangles are equiangular.

[∵ If two triangles are equiangular, their corresponding sides are proportional.]

Therefore, AB. AC = AD. AE .... (i)

Again, in ΔABD and ΔCDE,

∠ABD = ∠CED

[∵ angle subtended by the same arc are equal]

and ∠ADB = ∠CDE [∵ opposite angle]

∴ Remaining ∠BAD = remaining ∠DCE

∴ The triangles are equiangular.

$\therefore \frac{BD}{DE} = \frac{AD}{DC}$

[∵ If two triangles are equiangular, their corresponding sides are proportional.]

Therefore, AD. DE = BD. DC ... ... (ii)

Now, from equation (i), we get

AB. AC = AD.AE

= AD (AD + DE) [∵ AE = AD + DE]

= AD.AD + AD.DE

= AD^2 + AD. DE

Or, AD^2 = AB. AC – AD. DE

∴ AD^2 = AB.AC – BD. DC [putting the value from (ii)]

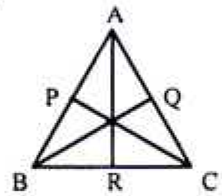
Therefore, AD^2 = AB.AC – BD. DC **(Shown)**

c By the triangle law, we get from ΔABR,

$\overrightarrow{AR} = \overrightarrow{AB} + \overrightarrow{BR}$

$\therefore \overrightarrow{AR} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$ .... (i)

[R is the middle point of BC, so $\overrightarrow{BR} = \frac{1}{2}\overrightarrow{BC}$]



In ΔACP, $\overrightarrow{AP} = \overrightarrow{AC} + \overrightarrow{CP}$

$\therefore \overrightarrow{CP} = \overrightarrow{AP} - \overrightarrow{AC}$ [$\overrightarrow{AC} = -\overrightarrow{CA}$]

$\therefore \overrightarrow{CP} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{AC}$ .... (ii)

[P is the middle point of AB, so $\overrightarrow{AP} = \frac{1}{2}\overrightarrow{AB}$]

and from the triangle ΔABQ, $\overrightarrow{AQ} = \overrightarrow{AB} + \overrightarrow{BQ}$

Or, $\overrightarrow{BQ} = \overrightarrow{AQ} - \overrightarrow{AB}$

$\therefore \overrightarrow{BQ} = \frac{1}{2}\overrightarrow{AC} - \overrightarrow{AB}$ .... (iii)

[Q is the middle point of AC, so $\overrightarrow{AQ} = \frac{1}{2}\overrightarrow{AC}$]

Now, by adding the equations (i), (ii) and (iii) we get,

$\overrightarrow{AR} + \overrightarrow{CP} + \overrightarrow{BQ} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} + \frac{1}{2}\overrightarrow{AB} - \overrightarrow{AC} + \frac{1}{2}\overrightarrow{AC} - \overrightarrow{AB}$

Or, $\overrightarrow{AR} + \overrightarrow{BQ} + \overrightarrow{CP} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} - \frac{1}{2}\overrightarrow{AC}$

$= \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) - \frac{1}{2}\overrightarrow{AC}$

$= \frac{1}{2}\overrightarrow{AC} - \frac{1}{2}\overrightarrow{AC} = \underline{0}$

$\therefore \overrightarrow{AR} + \overrightarrow{BQ} + \overrightarrow{CP} = \overrightarrow{0}$ **(Proved)**

Question ▶ 18 The co-ordinate of three points P, Q and R are (a, a + 1), (– 6, –3) and (5, – 1) respectively.

[Barishal Cadet College, Barishal]

a. Find the length of PQ and PR. 2

b. If PQ = 2PR then find the probable value of 'a'. 4

c. If L and M be the middle point of PQ and PR in the triangle PQR, then prove that $\overrightarrow{LM} = \frac{1}{2}\overrightarrow{QR}$ 4

Solution to the question no. 18

a The co-ordinates of P,Q and R are (a. a + 1), (– 6, – 3), (5, – 1), respectivety.

$PQ = \sqrt{(a+6)^2 + (a+1+3)^2}$

$= \sqrt{a^2 + 12a + 36 + (a+4)^2}$

$= \sqrt{a^2 + 12a + 36 + a^2 + 8a + 16}$

$= \sqrt{2a^2 + 20a + 52}$

$PR = \sqrt{(a-5)^2 + (a+1+1)^2}$

$= \sqrt{(a-5)^2 + (a+2)^2}$

$= \sqrt{a^2 - 10a + 25 + a^2 + 4a + 4}$

$= \sqrt{2a^2 - 6a + 29}$ **(Ans.)**

b From 'a' we get

$PQ = \sqrt{2a^2 + 20 + 52}$

$PR = \sqrt{2a^2 - 6a + 29}$

Given, PQ = 2PR

Or, $\sqrt{2a^2 + 20a + 52} = 2\sqrt{2a^2 - 6a + 29}$

Or, $2a^2 + 20a + 52 = 4(2a^2 - 6a + 29)$

Or, $2a^2 + 20a + 52 = 8a^2 - 24a + 116$

Or, $8a^2 - 24a + 116 - 2a^2 - 20a - 52 = 0$

Or, $6a^2 - 44a + 64 = 0$

Or, $3a^2 - 22a + 32 = 0$

Or, $3a^2 - 6a - 16a + 32 = 0$

Or, $3a(a-2) - 16(a-2) = 0$

Or, $(a-2)(3a-16) = 0$

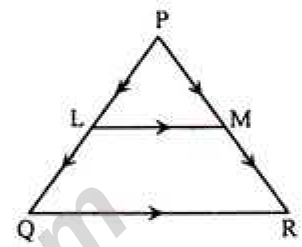
Either, $a - 2 = 0$ $\therefore a = 2$

Or, $3a - 16 = 0$ Or, $3a = 16$ $\therefore a = \frac{16}{3}$

So the value of a is 2 or $\frac{16}{3}$ **(Ans:)**

c Let, in ΔPQR, L and M are the middle points of sides PQ and PR respectively. L and M are joined.

Required to show that, LM || QR and $LM = \frac{1}{2}QR$.



Proof: L and M are the middle points of PQ and PR respectvely.

$\therefore \overrightarrow{LQ} = \overrightarrow{PL} = \frac{1}{2}\overrightarrow{PQ}$

and $\overrightarrow{PM} = \overrightarrow{MR} = \frac{1}{2}\overrightarrow{PR}$

In ΔPQR

According to triangle law,

$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

$\therefore \overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ}$............(i)

According to triangle law, in ΔPLM,

$\overrightarrow{PM} = \overrightarrow{PL} + \overrightarrow{LM}$

$\therefore \overrightarrow{LM} = \overrightarrow{PM} - \overrightarrow{PL}$

$= \frac{1}{2}\overrightarrow{PR} - \frac{1}{2}\overrightarrow{PQ}$ [$\because \overrightarrow{PM} = \frac{1}{2}\overrightarrow{PR}$ and $\overrightarrow{PL} = \frac{1}{2}\overrightarrow{PQ}$]

$= \frac{1}{2}(\overrightarrow{PR} - \overrightarrow{PQ})$

$= \frac{1}{2}\overrightarrow{QR}$ [from (i)]

$\therefore |\overrightarrow{LM}| = \frac{1}{2}|\overrightarrow{QR}|$

$\therefore LM = \frac{1}{2}QR$ and either $\overrightarrow{LM}$ and $\overrightarrow{QR}$ lies on the same line or they are parallel.

But LM and QR are seprate lines.

$\therefore$ LM || QR and $LM = \frac{1}{2}QR$. **(Proved)**

Question ▶ 19 The points A (7,2), B (–4,2), C (–4, –3) and D (7, – 3) are respectively the four vertices of a quadrilateral.

[RAJUK Uttara Model College, Dhaka]

a. Find the equation of the straight line AC. 2

b. Ascertain whether the quadrilateral is a paralleogram or a rectangle also find the area of the quadrilateral by using method II. 4

c. If the middle points of the quadrilateral mentioned in the stem are respectively P,Q, R and S then prove by using vector method PQRS is a parallelogram. 4

Solution to the question no. 19

a Equation of a straight line passing through A (7, 2) and C(–4,–3), $\frac{x-7}{7-(-4)} = \frac{y-2}{2-(-3)}$

Or, $\frac{x-7}{7+4} = \frac{y-2}{2+3}$

Or, $5x - 35 = 11y - 22$

Or, $5x - 11y - 35 + 22 = 0$

$\therefore$ $5x - 11y - 13 = 0$ **(Ans.)**

b Given, four vertices of the quadrilateral ABCD, A(7, 2), B(–4, 2), C(–4, –3) and D(7,–3)

$\therefore$ Length of side AB $= \sqrt{(7+4)^2 + (2-2)^2}$

$= \sqrt{(11)^2 + 0} = 11$ unit

Length of side BC $= \sqrt{(-4+4)^2 + (2+3)^2}$

$= \sqrt{0 + 5^2} = 5$ unit

Length of side CD $= \sqrt{(-4-7)^2 + (-3+3)^2}$

$= \sqrt{121 + 0} = 11$ unit

Length of side AD $= \sqrt{(7-7)^2 + (2+3)^2}$

$= \sqrt{0 + 5^2} = 5$ unit

Length of side AC $= \sqrt{(7+4)^2 + (2+3)^2}$

$= \sqrt{121 + 25} = \sqrt{146}$ unit

Length of side BD $= \sqrt{(-4-7)^2 + (2+3)^2}$

$= \sqrt{121 + 25} = \sqrt{146}$ unit

So, we get AB = CD and BC = AD

Again, diagonal AC = diagonal BD.

$\therefore$ ABCD is a rectangle**(Ans.)**

Area of ABCD $= \frac{1}{2}\begin{vmatrix} 7 & -4 & -4 & 7 & 7 \\ 2 & 2 & 2 & -3 & 2 \end{vmatrix}$

$= \frac{1}{2}[(14 + 12 + 12 + 14) - (-8 - 8 - 21 - 21)]$

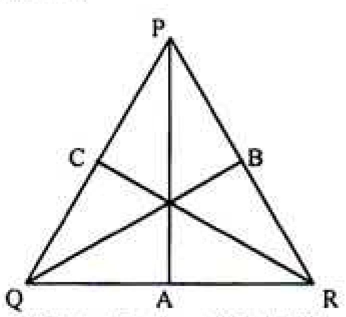
$= \frac{1}{2}[52 + 58]$

$= \frac{1}{2} \times 110$

= 55 sq unit **(Ans.)**

c See text book chapter 12, example-5.

Question ▸ 20 A, B and C are the mid points respectively of the sides QR, RP and PQ



[Dhaka Residential Model School and College, Dhaka]

a. Express the vector $\overrightarrow{PQ}$ interms of $\overrightarrow{BQ}$ and $\overrightarrow{CR}$. 2

b. Show that, $\overrightarrow{PA} + \overrightarrow{QB} + \overrightarrow{RC} = \underline{0}$. 4

c. Prove with the help of vectors that the straight line drawn from the point C and parallel to QR must passes through the point B. 4

Solution to the question no. 20

a $\overrightarrow{PQ} + \overrightarrow{QB} = \overrightarrow{PB}$

Or, $\overrightarrow{PQ} - \overrightarrow{BQ} = \overrightarrow{PB}$

Or, $\overrightarrow{PQ} = \overrightarrow{BQ} + \overrightarrow{PB}$ ....(i)

Again, $\overrightarrow{PC} + \overrightarrow{CR} = \overrightarrow{PR}$

Or, $\frac{1}{2}\overrightarrow{PQ} + \overrightarrow{CR} = 2\,\overrightarrow{PB}$

Or, $\overrightarrow{PB} = \frac{1}{4}\overrightarrow{PQ} + \frac{1}{2}\overrightarrow{CR}$

Now, From equation (i), we get,

$\overrightarrow{PQ} = \overrightarrow{BQ} + \overrightarrow{PB}$

$= \overrightarrow{BQ} + \frac{1}{4}\overrightarrow{PQ} + \frac{1}{2}\overrightarrow{CR}$

Or, $\overrightarrow{PQ} - \frac{1}{4}\overrightarrow{PQ} = \overrightarrow{BQ} + \frac{1}{2}\overrightarrow{CR}$

$\frac{3}{4}\overrightarrow{PQ} = \overrightarrow{BQ} + \frac{1}{2}\overrightarrow{CR}$

$\therefore \overrightarrow{PQ} = \frac{4\overrightarrow{BQ}}{3} + \frac{2}{3}\overrightarrow{CR}$ **(Ans.)**

b By the triangle law, we get from ΔPQA,

$\overrightarrow{PA} = \overrightarrow{PQ} + \overrightarrow{QA}$

$\therefore \overrightarrow{PA} = \overrightarrow{PQ} + \frac{1}{2}\overrightarrow{QR}$ .... (i)

[A is the middle point of QR, so $\overrightarrow{QA} = \frac{1}{2}\overrightarrow{QR}$]

In ΔPRC, $\overrightarrow{PC} = \overrightarrow{PR} + \overrightarrow{RC}$

$\therefore \overrightarrow{RC} = \overrightarrow{PC} - \overrightarrow{PR}$

$\therefore \overrightarrow{RC} = \frac{1}{2}\overrightarrow{PQ} - \overrightarrow{PR}$ .... (ii)

[C is the middle point of PQ, so $\overrightarrow{PC} = \frac{1}{2}\overrightarrow{PQ}$]

and from the triangle ΔPQB, $\overrightarrow{PB} = \overrightarrow{PQ} + \overrightarrow{QB}$

Or, $\overrightarrow{QB} = \overrightarrow{PB} - \overrightarrow{PQ}$

$\therefore \overrightarrow{QB} = \frac{1}{2}\overrightarrow{PR} - \overrightarrow{PQ}$ .... (iii)

[B is the middle point of PR, so $\overrightarrow{PB} = \frac{1}{2}\overrightarrow{PR}$]

Now, by adding the equations (i), (ii) and (iii) we get,

$\overrightarrow{PA} + \overrightarrow{RC} + \overrightarrow{QB} = \overrightarrow{PQ} + \frac{1}{2}\overrightarrow{QR} + \frac{1}{2}\overrightarrow{PQ} - \overrightarrow{PR} + \frac{1}{2}\overrightarrow{PR} - \overrightarrow{PQ}$

Or, $\overrightarrow{PA} + \overrightarrow{QB} + \overrightarrow{RC} = \frac{1}{2}\overrightarrow{PQ} + \frac{1}{2}\overrightarrow{QR} - \frac{1}{2}\overrightarrow{PR}$

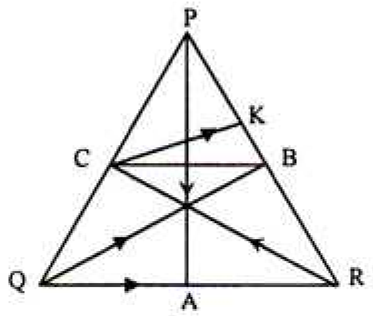
$= \frac{1}{2}(\overrightarrow{PQ} + \overrightarrow{QR}) - \frac{1}{2}\overrightarrow{PR}$

$= \frac{1}{2}\overrightarrow{PR} - \frac{1}{2}\overrightarrow{PR} = \underline{0}$

$\therefore \overrightarrow{PA} + \overrightarrow{QB} + \overrightarrow{RC} = \underline{0}$ **(Proved)**

c Let, C is the middle point of the side PQ of the triangle PQR. The line drawn parallel to QR intersects the side PR at the point B. It is required to prove that, B is the middle point of PR.

Proof: Let, not B rather K is the middle point of PR.



Then, $\overrightarrow{PC} = \frac{1}{2}\overrightarrow{PQ}$ [$\because$ C is the middle point of PQ]

and $\overrightarrow{PK} = \frac{1}{2}\overrightarrow{PR}$ [$\because$ K is the middle point of $\overrightarrow{PR}$]

$\therefore \overrightarrow{CK} = \overrightarrow{CP} + \overrightarrow{PK} = -\overrightarrow{PC} + \overrightarrow{PK}$ [$\therefore \overrightarrow{CP} = -\overrightarrow{PC}$]

$= \overrightarrow{PK} - \overrightarrow{PC} = \frac{1}{2}\overrightarrow{PR} - \frac{1}{2}\overrightarrow{PQ}$

$= \frac{1}{2}(\overrightarrow{PR} - \overrightarrow{PQ}) = \frac{1}{2}\overrightarrow{QR}$

$\therefore \overrightarrow{CK} = \frac{1}{2}\overrightarrow{QR}$

That is, CK || QR. But CB || QR (given)

So, the both lines $\overrightarrow{CB}$ and $\overrightarrow{CK}$ pass through the point C and parallel to $\overrightarrow{QR}$. So, they (that is, $\overrightarrow{CB}$ and $\overrightarrow{CK}$) must coincide to each other.

$\therefore$ B and K will be the same point. That is, B is the middle point of PR. **(Proved)**

Question ▸ 21 A(–2, –1), B(5, 4), C(6,7) and D(–1, 2) are the coordinates of the four vertices of a quadrilateral. The diagonals of the quadrilateral intersect each other at the point O. *[Milestone College, Dhaka]*

a. Find the equation of the straight line with slope 2 passing through the point A. 2

b. Show that, the quadrilateral ABCD is a parallelogram. 4

c. With the help of vectors, prove that the point O bisects the diagonals. 4

Solution to the question no. 21

a Given points A (– 2, – 1)

The equation of straight line with slope 2 and passing through A (– 2, – 1) is

$y - (-1) = 2\{x - (-2)\}$

Or, $y + 1 = 2(x + 2)$

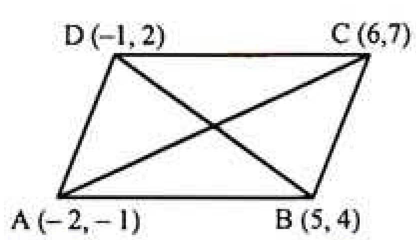
Or, $y + 1 = 2x + 4$

Or, $2x + 4 - y - 1 = 0$

$\therefore 2x - y + 3 = 0$ **(Ans.)**

b Let, A(– 2, – 1), B(5, 4)

C(6, 7) and D(– 1, 2) are the vertex of a quadrilateral ABCD



$AB = \sqrt{(-2-5)^2 + (-1-4)^2} = \sqrt{49 + 25} = \sqrt{74}$

$BC = \sqrt{(5-6)^2 + (4-7)^2} = \sqrt{1 + 9} = \sqrt{10}$

$CD = \sqrt{(6+1)^2 + (7-2)^2} = \sqrt{49 + 25} = \sqrt{74}$

$DA = \sqrt{(-1+2)^2 + (2+1)^2} = \sqrt{1 + 9} = \sqrt{10}$

$AC = \sqrt{(-2-6)^2 + (-1-7)^2} = \sqrt{64 + 64} = \sqrt{128}$

$BD = \sqrt{(5+1)^2 + (4-2)^2} = \sqrt{36 + 4} = \sqrt{40}$

Since, side AB = CD and side BC = DA but diagonal AC $\neq$ BD.

So, the quadrilateral ABCD is a parallelogram. **(Proved)**

c See example-4 of chapter-12 in Higher Math text book. Page-280.

Question ▸ 22 The diagonal AC and BD of a parallelogram ABCD intersects at O. *[Saint Joseph Higher Secondary School, Dhaka]*

a. State the law of triangle of vectors. 2

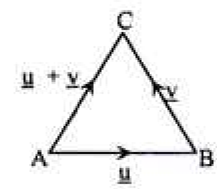
b. Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in terms of $\overrightarrow{AD}$ and $\overrightarrow{BD}$. 4

c. Prove using vector method that the diagonals of a parallelogram bisect each other. 4

Solution to the question no. 22

a **Triangle law of vectors :**

Let, the vectors $\underline{u}$ and $\underline{v}$ are represented by the sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of triangle ABC such that the terminal point of $\underline{u}$ is the initial point of $\underline{v}$.

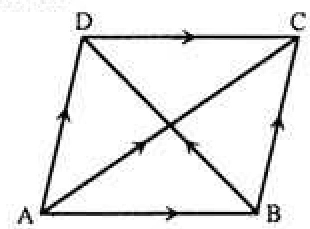


Then the side $\overrightarrow{AC}$ represent the sum of vectors $\underline{u} + \underline{v}$.

i. e. $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

This is known as triangle law of vectors.

b Given, ABCD is a parallelogram. $\overrightarrow{AC}$ and $\overrightarrow{BD}$ are two diagonals of it. We have to express the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in terms of $\overrightarrow{AD}$ and $\overrightarrow{BD}$



In the triangle ΔABD,

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$ [triangle law]

$\therefore \overrightarrow{AB} = \overrightarrow{AD} - \overrightarrow{BD}$............. (i)

Again in triangle ΔACD, $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$ [triangle law]

$= \overrightarrow{AD} + \overrightarrow{AB}$

[since, ABCD is parallelogram, so $\overrightarrow{DC} = \overrightarrow{AB}$]

$= \overrightarrow{AD} + \overrightarrow{AD} - \overrightarrow{BD}$ [$\because \overrightarrow{AB} = \overrightarrow{AD} - \overrightarrow{BD}$]

$\therefore \overrightarrow{AC} = 2\overrightarrow{AD} - \overrightarrow{BD}$ .... (ii)

c See example–4 of chaper–12 in your text book.

Question ▸ 23 A(P, 3P), B(P^2, 2P), C(P – 2, P) and D(1, 1) are the four different point. *[BAF Shaheen College, Tejgaon, Dhaka]*

a. Find the value of P, if the line joining the points B and C has slope $\frac{1}{2}$ 2

b. If the lines AB and CD are parallel, then find the admissible value of P. 4

c. With negative value of P from (b), the straight line joining the middle points of the non-parallel sides R and S of the trapezium ABCD, prove with the help of vectors that, RS || AB || CD and RS = $\frac{1}{2}$(AB + CD). 4

Solution to the question no. 23

a Coordinates of points B and C are $(p^2, 2p)$ and $(p - 2, p)$ respectively.

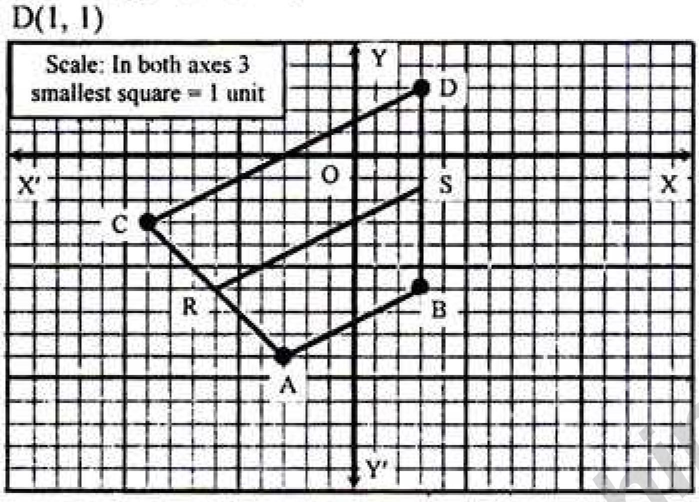
$\therefore$ Slope of the line BC $= \frac{p - 2p}{p - 2 - p^2} = \frac{-p}{p - 2 - p^2}$

According to question, $\frac{-p}{p - 2 - p^2} = \frac{1}{2}$

Or, $-2p = p - 2 - p^2$
Or, $p^2 - 3p + 2 = 0$
Or, $p^2 - 2p - p + 2 = 0$
Or, $p(p - 2) - 1(p - 2) = 0$
Or, $(p - 2)(p - 1) = 0$
$\therefore p = 1, 2$ **(Ans.)**

b See example-5 of exercise-11.3 from your textbook. page-259
[N.B. t will be replaced with p]

c From 'b' we get, $p = -1$
$\therefore A(p, 3p) \equiv (-1, -3)$
$B(p^2, 2p) \equiv (1, -2)$
$C(p - 2, p) \equiv (-3, -1)$
$D(1, 1)$



Let, ABCD be the trapezium whose sides AC and BD are non-parallel and the sides AB and CD are parallel. The midpoints of sides AC and BD are R and S respectively. R, S are joined. It is required to prove that, RS || AB || CD and $RS = \frac{1}{2}(AB + CD)$.

Proof: Let, the position vectors of the points A, B, C and D with respect to origin be $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ respectively.

$\therefore \overrightarrow{AB} = \underline{b} - \underline{a}$, $\overrightarrow{CD} = \underline{d} - \underline{c}$

$\therefore$ The position vector of point R $= \frac{1}{2}(\underline{a} + \underline{c})$

and the position vector of point S $= \frac{1}{2}(\underline{b} + \underline{d})$

$\therefore \overrightarrow{RS} = \frac{1}{2}(\underline{b} + \underline{d}) - \frac{1}{2}(\underline{a} + \underline{c})$

$= \frac{1}{2}(\underline{b} + \underline{d} - \underline{a} - \underline{c})$

$= \frac{1}{2}\{(\underline{b} - \underline{a}) + (\underline{d} - \underline{c})\}$

$\therefore \overrightarrow{RS} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{CD})$

$\therefore |\overrightarrow{RS}| = \frac{1}{2}|(\overrightarrow{AB} + \overrightarrow{CD})|$

So, $RS = \frac{1}{2}(AB + CD)$

Since $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel so $\overrightarrow{AB} + \overrightarrow{CD}$ will be parallel to $\overrightarrow{AB}$ and $\overrightarrow{CD}$. So the vector $\overrightarrow{RS}$ will be parallel to $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

Therefore, RS || AB || CD and $RS = \frac{1}{2}(AB + CD)$ **(Proved)**

Question ▶ 24 D and E are respectively the middle points of the sides AB and AC of the ΔABC.

[BAF Shaheen College, Kurmitola, Dhaka]

a. Express $(\overrightarrow{AD} + \overrightarrow{DE})$ in terms of $\overrightarrow{AC}$. 2

b. Prove with the help of vectors that, BC || DE and $DE = \frac{1}{2}$ BC. 4

c. If M and N are the middle points of the diagonals of the trapezium DBEC, then prove with the help of vectors that, MN || DE || BC and $MN = \frac{1}{2}(BC - DE)$. 4

Solution to the question no. 24

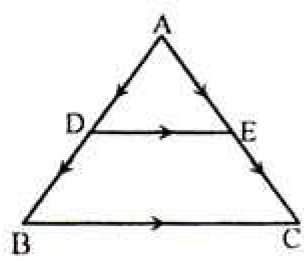
a In the triangle ΔADE,

$\overrightarrow{AD} + \overrightarrow{DE} = \overrightarrow{AE}$ [triangle law]

$= \frac{1}{2}\overrightarrow{AC}$ [since, E is the middle point of AC]

So, $\overrightarrow{AD} + \overrightarrow{DE} = \frac{1}{2}\overrightarrow{AC}$.

b Given, the middle points of the sides AB and AC of the triangle ABC are D and E respectively.
Join D, E. It is required to prove with the help of vectors that $DE = \frac{1}{2}BC$ and DE || BC



Proof: D and E are the middle points of AB and AC respectively.

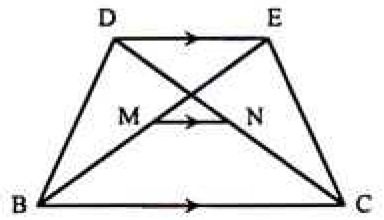
$\therefore \overrightarrow{DB} = \overrightarrow{AD} = \frac{1}{2}\overrightarrow{AB}$ and $\overrightarrow{AE} = \overrightarrow{EC} = \frac{1}{2}\overrightarrow{AC}$

According to the triangle law we get,
$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$
or, $\overrightarrow{BC} = -\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AC} - \overrightarrow{AB}$ .... (i)
and $\overrightarrow{DE} = \overrightarrow{DA} + \overrightarrow{AE}$
$= -\overrightarrow{AD} + \overrightarrow{AE}$
$= -\frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC}$ [$\because \overrightarrow{AD} = \frac{1}{2}\overrightarrow{AB}, \overrightarrow{AE} = \frac{1}{2}\overrightarrow{AC}$]
$= \frac{1}{2}(\overrightarrow{AC} - \overrightarrow{AB}) = \frac{1}{2}\overrightarrow{BC}$ [from the equation (i)]

So, $|\overrightarrow{DE}| = \frac{1}{2}|\overrightarrow{BC}|$

$\therefore DE = \frac{1}{2}BC$ and the support line of $\overrightarrow{DE}$ and $\overrightarrow{BC}$ will be same or parallel.
But, D and E are the middle points of AB and AC respectively. So, the support line of $\overrightarrow{DE}$ and $\overrightarrow{BC}$ can not be same.
$\therefore$ DE || BC
That is, $DE = \frac{1}{2}BC$ and DE || BC **(Proved)**

c Let, in the trapezium BCED, DE || BC and the middle points of CD and BE are N and M respectively. Let us join M, N.



It is required to prove that MN $= \frac{1}{2}$(BC – DE) and MN || DE || BC.

Proof: Let the position vectors of points B,C,E,D are $\underline{b}$, $\underline{c}$, $\underline{e}$, $\underline{d}$ respectively with respect to the origin.

$\overrightarrow{BC} = \underline{c} - \underline{b}$

$\overrightarrow{DE} = \underline{e} - \underline{d}$

$\therefore$ The position vector of M $= \frac{1}{2}(\underline{b} + \underline{e})$

[$\because$ M is the middle point of BE]

and the position vector of N $= \frac{1}{2}(\underline{b} + \underline{d})$

[$\because$ N is the middle point of CD]

$\therefore \overrightarrow{MN} = \frac{1}{2}(\underline{c} + \underline{d}) - \frac{1}{2}(\underline{b} + \underline{e}) = \frac{1}{2}(\underline{c} + \underline{d} - \underline{b} - \underline{e})$

$= \frac{1}{2}\{(\underline{c} - \underline{b}) - (\underline{e} - \underline{d})\} = \frac{1}{2}(\overrightarrow{BC} - \overrightarrow{DE})$

Since, DE || BC, So the vector $(\overrightarrow{BC} - \overrightarrow{DE})$ is parallel to both $\overrightarrow{BC}$ and $\overrightarrow{DE}$. So the vector $\overrightarrow{MN}$ will be parallel to both $\overrightarrow{BC}$ and $\overrightarrow{DE}$.

Because, $\overrightarrow{MN} = \frac{1}{2}(\overrightarrow{BC} - \overrightarrow{DE})$

$\therefore |\overrightarrow{MN}| = \frac{1}{2}|(\overrightarrow{BC} - \overrightarrow{DE})| = \frac{1}{2}(|\overrightarrow{BC}| - |\overrightarrow{DE}|)$

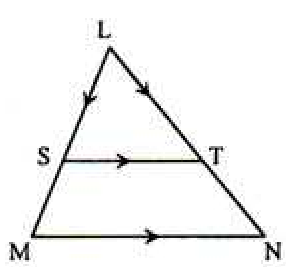
$\therefore$ MN $= \frac{1}{2}$(BC – DE)

That is, MN || DE || BC

and MN $= \frac{1}{2}$(BC – DE) **(Proved)**

[NB: In the question of text book, ABCD will be repleced by BCED and also MN | | AD | | BC will be replaced by MN | | DE | | BC]

Question ▶ 25



In figure, S and T are the mid-points of the sides LM and LN of LMN respectively.

[Bangladesh International School & College, Dhaka]

a. What do you mean by the triangular of law of vector addition? 2

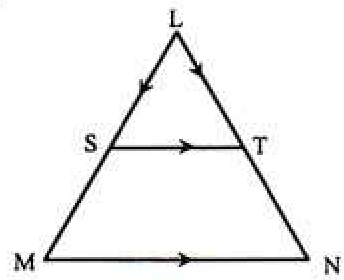
b. Show, by vector method, that ST||MN and ST = $\frac{1}{2}$MN. 4

c. If D and E are the mid-points of the sides SM and TN of trapezium SMNT respectively, then prove with the help of vectors that DE||ST||MN and DE= $\frac{1}{2}$(ST + MN). 4

Solution to the question no. 25

a See chapter-12, page-271 from yours text book.

b



Given, in ΔLMN, the midpoints of side LM and LN are respectively S and T. Join S, T. It is required to prove that, ST || MN and ST $= \frac{1}{2}$ MN

Proof: According to the triangle law of vector addition,

$\overrightarrow{LS} + \overrightarrow{ST} = \overrightarrow{LT} \therefore \overrightarrow{LT} - \overrightarrow{LS} = \overrightarrow{ST}$ .... (i)

and $\overrightarrow{LM} + \overrightarrow{MN} = \overrightarrow{LN} \therefore \overrightarrow{LN} - \overrightarrow{LM} = \overrightarrow{MN}$ .... (ii)

But $\overrightarrow{LN} = 2.\ \overrightarrow{LT}$ and $\overrightarrow{LM} = 2.\ \overrightarrow{LS}$

[$\because$ S and T is the midpoints of LM and LN respectively]

$\therefore$ From (ii) we get, $2\overrightarrow{LT} - 2\overrightarrow{LS} = \overrightarrow{MN}$

or, $2(\overrightarrow{LT} - \overrightarrow{LS}) = \overrightarrow{MN}$

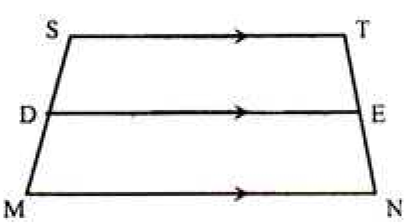
or, $2\overrightarrow{ST} = \overrightarrow{MN}$ [from (i)]

$\therefore \overrightarrow{ST} = \frac{1}{2}\overrightarrow{MN}$

or, $|\overrightarrow{ST}| = \frac{1}{2}|\overrightarrow{MN}|$ or, ST $= \frac{1}{2}$MN **(Proved)**

Again, the support line of $\overrightarrow{ST}$ and $\overrightarrow{MN}$ will be same or parallel. But here, support line is not same. So $\overrightarrow{ST}$ and $\overrightarrow{MN}$ are parallel that is ST || MN. **(Proved)**

c



Here, D and E are midpoints of non-parallel sides SM and NT of a trapezium SMNT.It is required to prove that, DE || ST || MN and DE $= \frac{1}{2}$(ST + MN)

Proof: let, position vector of S, M, N and T are $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ respectively

$\therefore$ $\overrightarrow{MN} = \underline{c} - \underline{b}$ and $\overrightarrow{ST} = \underline{d} - \underline{a}$

$\therefore$ Position vector of point D= $\frac{1}{2}(\underline{a} + \underline{b})$

[$\because$ D is the midpoint of SM]

Position vector of point E $= \frac{1}{2}(\underline{c} + \underline{d})$

[$\because$ E is the midpoint of NT]

Now, $\overrightarrow{DE} = \frac{1}{2}(\underline{c} + \underline{d}) - \frac{1}{2}(\underline{a} + \underline{b}) = \frac{1}{2}(\underline{c} + \underline{d} - \underline{a} - \underline{b})$

$= \frac{1}{2}\{(\underline{c} - \underline{b}) + (\underline{d} - \underline{a})\} = \frac{1}{2}(\overrightarrow{MN} + \overrightarrow{ST})$

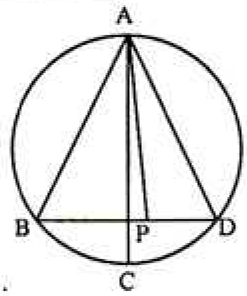
But as MN and ST are parallel so $(\overrightarrow{MN}+\overrightarrow{ST})$ will be parallel to MN and ST. So DE vector will be parallel to MN and ST.

$\overrightarrow{DE}=\frac{1}{2}(\overrightarrow{ST}+\overrightarrow{MN})$ or, $|\overrightarrow{DE}|=\frac{1}{2}|\overrightarrow{MN}+\overrightarrow{ST}|$

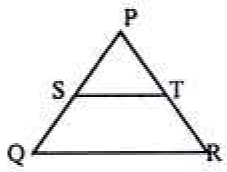
So, $DE=\frac{1}{2}(ST+MN)$

So, DE || ST || MN and $DE=\frac{1}{2}(ST+MN)$ **(Proved)**

Question ▶ 26



Stem–1



Stem–2

[Mirpur Girls' Ideal Laboratory Institute, Dhaka]

a. What do you understand a nine point circle? 2

b. Prove that AC.BD = AB.CD + BC.AD 4

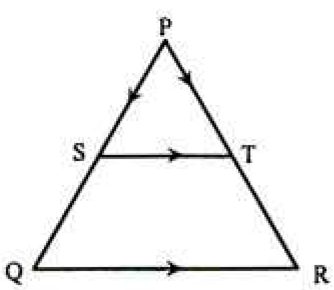
c. S and T are the mid points of PQ and PR. Prove that ST || QR and $ST=\frac{1}{2}QR$ (Prove with the help of vector). 4

Solution to the question no. 26

a See from your textbook, exercise-3.2, page-77

b See theorem-12 (Ptolemy's Theorem) from your textbook, exercise-3.2, page-78.

c



Given, in ΔPQR, the midpoints of side PQ and PR are respectively S and T. Join S, T. It is required to prove that, ST || QR and $ST=\frac{1}{2}QR$

Proof: According to the triangle law of vector addition,

$\overrightarrow{PS}+\overrightarrow{ST}=\overrightarrow{PT}$ ∴ $\overrightarrow{PT}-\overrightarrow{PS}=\overrightarrow{ST}$ .... (i)

and $\overrightarrow{PQ}+\overrightarrow{QR}=\overrightarrow{PR}$ ∴ $\overrightarrow{PR}-\overrightarrow{PQ}=\overrightarrow{QR}$ .... (ii)

But $\overrightarrow{PR}=2.\overrightarrow{PT}$ and $\overrightarrow{PQ}=2.\overrightarrow{PS}$

[∵ S and T is the midpoints of PQ and PR respectively]

∴ From (ii) we get, $2\overrightarrow{PT}-2\overrightarrow{PS}=\overrightarrow{QR}$

Or, $2(\overrightarrow{PT}-\overrightarrow{PS})=\overrightarrow{QR}$

Or, $2\overrightarrow{ST}=\overrightarrow{QR}$ [from (i)]

∴ $\overrightarrow{ST}=\frac{1}{2}\overrightarrow{QR}$

Or, $|\overrightarrow{ST}|=\frac{1}{2}|\overrightarrow{QR}|$ or, $ST=\frac{1}{2}QR$ **(Proved)**

Again, the support line of $\overrightarrow{ST}$ and $\overrightarrow{QR}$ will be same or parallel. But here, support line is not same. So $\overrightarrow{ST}$ and $\overrightarrow{QR}$ are parallel that is ST || QR. **(Proved)**

Question ▶ 27 A(0, – 1), B (–2,3), C (6,7) and D (8.3) are the four vertices of a quadrilateral in the xy-plane.

[Baridhara Scholars' Institution (BSI), Dhaka]

a. Draw the quadrilateral on the graph paper. 2

b. Show that the quadrilateral is a rectangle. Find the area of the quadrilateral ABCD. 4

c. If P,Q,R,S be the midpoints of the sides of the quadrilateral ABCD, then by the vector method show that PQRS is a parallelogram. 4

Solution to the question no. 27

a See from text book. chapter-11, page-240, example-4

b Let, A (0, –1), B (8, 3), C (6, 7) and D (–2, 3)

Now,

Length of the side $AB=\sqrt{(0-8)^2+(-1-3)^2}$
$=\sqrt{64+16}=\sqrt{80}$ unit

Length of the side $BC=\sqrt{(8-6)^2+(3-7)^2}$
$=\sqrt{2^2+(-4)^2}$
$=\sqrt{4+16}=\sqrt{20}$ unit

Length of the side $CD=\sqrt{(6+2)^2+(7-3)^2}$
$=\sqrt{64+16}=\sqrt{80}$ unit

And length of the side $AD=\sqrt{(0+2)^2+(-1-3)^2}$
$=\sqrt{4+16}=\sqrt{20}$ unit **(Ans.)**

Again, length of the diagonal AC
$=\sqrt{(0-6)^2+(-1-7)^2}=\sqrt{36+64}$
$=\sqrt{100}=10$ unit **(Ans.)**

And length of the diagonal $BD=\sqrt{(8+2)^2+(3-3)^2}$
$=\sqrt{10^2}=10$ unit

It is seen that, AB = CD, BC = AD and diagonals AC = diagonal BD.

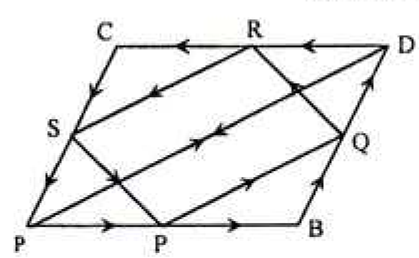
∴ The points A, B, C, D are the vertices of a rectangle.

Here, $AD=BC=4\sqrt{5}$

$AB=CD=2\sqrt{5}$

∴ Area of the rectangle = AB × CD
$=2\sqrt{5}\times4\sqrt{5}$
$=8.5$
= 40 sq. unit. **(Ans.)**

c



Let, the midpoints of the sides of the quadrilateral ABCD be P, Q, R and S respectively. It is required to prove that, PQRS is a parallelogram.

Proof: Let, $\overrightarrow{AB}=\underline{a}$, $\overrightarrow{BC}=\underline{b}$, $\overrightarrow{CD}=\underline{c}$, $\overrightarrow{DA}=\underline{d}$

Thus,

$\overrightarrow{PQ}=\overrightarrow{PB}+\overrightarrow{BQ}=\frac{1}{2}(\overrightarrow{AB}+\overrightarrow{BC})=\frac{1}{2}(a+b)$

Similarly, $\overrightarrow{RQ} = \frac{1}{2}(b + c)$, $\overrightarrow{RS} = \frac{1}{2}(c + d)$,

$SP = \frac{1}{2}(d + a)$

Again, $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = a + b$

and $\overrightarrow{CA} = \overrightarrow{CD} + \overrightarrow{DA} = c + d$

But $(a + b) + (c + d) = \overrightarrow{AC} + \overrightarrow{CA} = \overrightarrow{AC} - \overrightarrow{AC} = 0$

That is $(a + b) = -(c + d)$

$\therefore \overrightarrow{PQ} = \frac{1}{2}(a + b) = -\frac{1}{2}(c + d) = -\overrightarrow{RS} = \overrightarrow{SR}$

Here, the line of supports of $\overrightarrow{PQ}$ and $\overrightarrow{SR}$ are either same or parallel. Here line of supports are not same.

$\therefore$ Line of supports are parallel. $\therefore \overrightarrow{PQ} \parallel \overrightarrow{SR}$.

Now, $|\overrightarrow{PQ}| = |\overrightarrow{SR}|$ $\therefore PQ = SR$

$\therefore$ PQ and SR are equal and parallel.

Similarly, QR and PS are equal and parallel.

$\therefore$ PQRS is a parallelogram. **(Proved)**

Question ▸ 28 The vertices of a quadrilateral are A(1,1), B (4,4), C (4,8) and D(1,5). *[Chetona Model Academy (CMA), Dhaka]*

a. Find the equation of straight line passing through the points (5,3) and (2,–2) 2

b. Show that, ABCD is a parallelogram. Also find the area. 4

c. If P,Q, R, S are the middle points of the adjacent sides of the given quadrilateral ABCD, prove by vector methods PQRS is a parallelogram. 4

Solution to the question no. 28

a The equation passing through (5, 3) and (2, –2) is

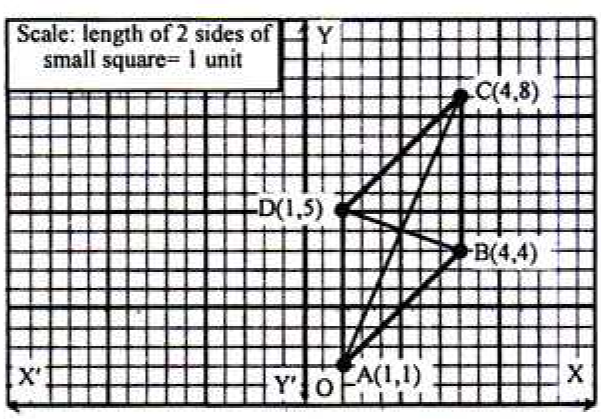
$\frac{y-3}{3+2} = \frac{x-5}{5-2}$

Or, $\frac{y-3}{5} = \frac{x-5}{3}$

Or, $5x - 25 = 3y - 9$

Or, $5x - 3y = 16$ **(Ans.)**

b A parallelogram is drawn by plotting the points A (1, 1), B (4, 4), C (4, 8) and D (1, 5) on xy plane.



Length of the side $AB = \sqrt{(1-4)^2 + (1-4)^2}$
$= \sqrt{(-3)^2 + (-3)^2} = 3\sqrt{2}$ unit

Length of the side $DC = \sqrt{(1-4)^2 + (5-8)^2}$
$= \sqrt{(-3)^2 + (-3)^2} = 3\sqrt{2}$ unit

Length of the side $AD = \sqrt{(1-1)^2 + (1-5)^2}$
$= \sqrt{0^2 + (-4)^2} = 4$ unit

and length of the side $BC = \sqrt{(4-4)^2 + (4-8)^2}$
$= \sqrt{0^2 + (-4)^2} = 4$ unit

Again, length of the diagonal $AC = \sqrt{(1-4)^2 + (1-8)^2}$
$= \sqrt{(-3)^2 + (-7)^2}$
$= \sqrt{9 + 49} = \sqrt{58}$ unit

and length of the diagonal $BD = \sqrt{(4-1)^2 + (4-5)^2}$
$= \sqrt{3^2 + (-1)^2} = \sqrt{10}$ unit

Here, AB = DC and AD = BC;

But, diagonal AC ≠ diagonal BD.

$\therefore$ The four points A, B, C, D are the vertices of a parallelogram. **(Shown)**

Determination of the area of parallelogram:

Now, half of the perimeter of ΔABD

$= \frac{AB + AD + BD}{2}$ unit

$= \frac{(3\sqrt{2} + 4 + \sqrt{10})}{2} = 5.70$ unit

$\therefore$ Area of ΔABD

$= \sqrt{5.70(5.70 - 3\sqrt{2})(5.70 - 4)(5.70 - \sqrt{10})}$ sq. unit

$= \sqrt{5.70(1.457)(1.70)(2.538)}$ sq. unit

$= \sqrt{35.832}$ sq. unit = 5.986 sq. unit (approx.)

$\therefore$ Area of the parallelogram = 2 × Area of the ΔABD
= 2 × 5.986 sq. unit (approx.)
= 11.972 sq. unit (approx.) **(Ans.)**

Alternative Solution:

Area of the parallelogram ABCD

$= \frac{1}{2}\begin{vmatrix} 1 & 4 & 4 & 1 & 1 \\ 1 & 4 & 8 & 5 & 1 \end{vmatrix}$ sq. unit

$= \frac{1}{2}(4 + 32 + 20 + 1 - 4 - 16 - 8 - 5)$

$= \frac{1}{2}(57 - 33) = 12$ sq. unit

$\therefore$ Area of the parallelogram ABCD = 12 sq. unit **(Ans.)**

c See example 5 (chapter-12) of textbook (Page-281)

Question ▸ 29 The position vectors of A, B, C and D are respectively $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ *[Chetona Model Academy (CMA), Dhaka]*

a. Show that, $\overrightarrow{AB} = \underline{b} - \underline{a}$ 2

b. Show that, ABCD will be a parallelogram if and only if $\underline{b} - \underline{a} = \underline{c} - \underline{d}$ 4

c. If the line segment AB is divided internally in the ratio m:n at the point c, show that, the position vector of c is $\underline{c} = \frac{m\underline{b} + n\underline{a}}{m + n}$ 4

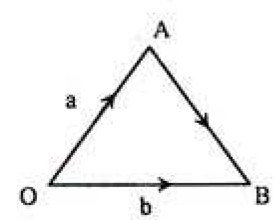
Solution to the question no. 29

a Let, on a plane the position vector of the point A with respect to the point O is $\overrightarrow{OA} = \underline{a}$ and the position vector of B is $\overrightarrow{OB} = \underline{b}$.

Then, $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

Or, $\underline{a} + \overrightarrow{AB} = \underline{b}$

$\therefore \overrightarrow{AB} = \underline{b} - \underline{a}$ **(Shown)**



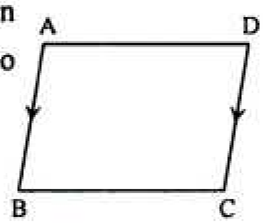
b Given, the position vectors of the points A, B, C, D are $\underline{a}$, $\underline{b}$, $\underline{c}$, $\underline{d}$ respectively.

It is required to show that, ABCD will be a parallelogram if and only if $\underline{b} - \underline{a} = \underline{c} - \underline{d}$.

The position vectors of A, B, C and D are $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ respectively with respect to the origin.

$\therefore \overrightarrow{AB} = \underline{b} - \underline{a}$ and $\overrightarrow{DC} = \underline{c} - \underline{d}$

Let, ABCD be a parallelogram. Then AB and DC are parallel and equal to each other.



$\therefore \overrightarrow{AB} = \overrightarrow{DC}$

$\therefore \underline{b} - \underline{a} = \underline{c} - \underline{d}$

Conversely, let, $\underline{b} - \underline{a} = \underline{c} - \underline{d}$

$\therefore \overrightarrow{AB} = \overrightarrow{DC}$

So, AB and DC are parallel and equal to each other. That is, ABCD is a parallelogram.

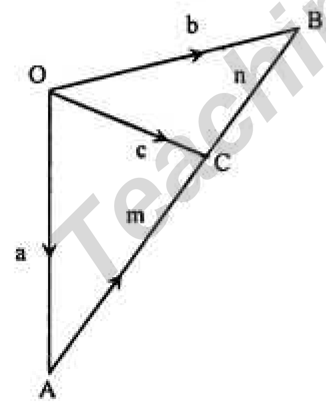
$\therefore$ ABCD will be a parallelogram if and only if $\underline{b} - \underline{a} = \underline{c} - \underline{d}$. **(Shown)**

c Let, the position vectors of A and B with respect to the origin O are respectively $\underline{a}$ and $\underline{b}$. If the line segment AB is divided internally in the ratio m : n at the point C, we have to show that the position vector of C is $\underline{c} = \dfrac{n\underline{a} + m\underline{b}}{m + n}$

[$\because$ the line segment AB has divided internally in the ratio m : n at the point C]

Or, $\dfrac{|\overrightarrow{AC}|}{|\overrightarrow{CB}|} = \dfrac{m}{n}$

Or, $\dfrac{|\overrightarrow{CB}|}{|\overrightarrow{AC}|} = \dfrac{n}{m}$ [taking inverse]



Or, $\dfrac{|\overrightarrow{CB}| + |\overrightarrow{AC}|}{|\overrightarrow{AC}|} = \dfrac{n + m}{m}$

[by componendo]

Or, $\dfrac{|\overrightarrow{AB}|}{|\overrightarrow{AC}|} = \dfrac{m + n}{m}$

Or, $\dfrac{|\overrightarrow{AC}|}{|\overrightarrow{AB}|} = \dfrac{m}{m + n}$; [taking inverse]

Or, $|\overrightarrow{AC}| = \left(\dfrac{m}{m + n}\right) |\overrightarrow{AB}|$

Or, $\overrightarrow{AC} = \left(\dfrac{m}{m + n}\right) \overrightarrow{AB}$

[$\because$ the direction of $\overrightarrow{AC}$ and $\overrightarrow{AB}$ are same]

Or, $\underline{c} - \underline{a} = \dfrac{m}{m + n}(\underline{b} - \underline{a})$

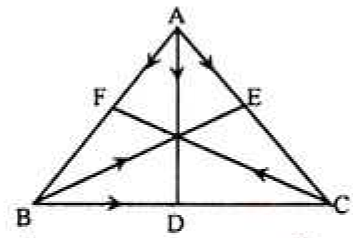
Or, $\underline{c} = \dfrac{m}{m + n}(\underline{b} - \underline{a}) + \underline{a}$

Or, $\underline{c} = \dfrac{m\underline{b} - m\underline{a} + m\underline{a} + n\underline{a}}{m + n}$

$\therefore \underline{c} = \dfrac{n\underline{a} + m\underline{b}}{m + n}$ **(Proved)**

Question ▶ 30 D, E and F are the middle points of the sides BC, CA and AB respectively of ΔABC.

[Rajshahi Cantonment Public School & College, Rajshahi]



a. Express $\overrightarrow{AB}$ in terms of vector $\overrightarrow{BE}$ and $\overrightarrow{CF}$. 2

b. Prove that, $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF}; = 0$ 4

c. Prove with the help of vectors that the straight line drawn through F parallel to BC must go through E. 4

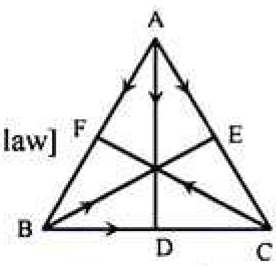
Solution to the question no. 30

a $\overrightarrow{AB} + \overrightarrow{BE} = \overrightarrow{AE}$

Or, $\overrightarrow{AB} = \overrightarrow{AE} - \overrightarrow{BE}$; [triangle law]

$= \frac{1}{2}\overrightarrow{AC} - \overrightarrow{BE}$ [E is the middle point of AC,

so $\overrightarrow{AE} = \frac{1}{2}\overrightarrow{AC}$ and $\overrightarrow{EB} = -\overrightarrow{BE}$]



$= \frac{1}{2}(\overrightarrow{AF} - \overrightarrow{CF}) - \overrightarrow{BE}$ [triangle law]

$= \frac{1}{2}\left(\frac{1}{2}\overrightarrow{AB} - \overrightarrow{CF}\right) - \overrightarrow{BE}$

[F is the middle point of AB, so $\overrightarrow{AF} = \frac{1}{2}\overrightarrow{AB}$]

Or, $\overrightarrow{AB} = \frac{1}{4}\overrightarrow{AB} - \frac{1}{2}\overrightarrow{CF} - \overrightarrow{BE}$

Or, $4\overrightarrow{AB} = \overrightarrow{AB} - 2\overrightarrow{CF} - 4\overrightarrow{BE}$;

[multiplying both sides by 4]

Or, $4\overrightarrow{AB} - \overrightarrow{AB} = \overrightarrow{AB} - 2\overrightarrow{CF} - 4\overrightarrow{BE} - \overrightarrow{AB}$

[adding $(-\overrightarrow{AB})$ in both sides]

Or, $3\overrightarrow{AB} = -2\overrightarrow{CF} - 4\overrightarrow{BE}$

$\therefore \overrightarrow{AB} = -\frac{2}{3}\overrightarrow{CF} - \frac{4}{3}\overrightarrow{BE}$; [multiplying both sides by $\frac{1}{3}$]

b By the triangle law, we get from ΔABD,

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$\therefore \overrightarrow{AD} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$ .... (i)

[D is the middle point of BC, so $\overrightarrow{BD} = \frac{1}{2}\overrightarrow{BC}$]

In ΔACF, $\overrightarrow{AF} = \overrightarrow{AC} + \overrightarrow{CF}$

$\therefore \overrightarrow{CF} = \overrightarrow{AF} - \overrightarrow{AC}$ [$\overrightarrow{AC} = -\overrightarrow{CA}$]

$\therefore \overrightarrow{CF} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{AC}$ .... (ii)

[F is the middle point of AB, so $\overrightarrow{AF} = \frac{1}{2}\overrightarrow{AB}$]

and from the triangle ΔABE, $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE}$

Or, $\overrightarrow{BE} = \overrightarrow{AE} - \overrightarrow{AB}$

$\therefore \overrightarrow{BE} = \frac{1}{2}\overrightarrow{AC} - \overrightarrow{AB}$ .... (iii)

[E is the middle point of AC, so $\overrightarrow{AE} = \frac{1}{2}\overrightarrow{AC}$]

Now, by adding the equations (i), (ii) and (iii) we get,

$\overrightarrow{AD} + \overrightarrow{CF} + \overrightarrow{BE} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} + \frac{1}{2}\overrightarrow{AB} - \overrightarrow{AC} + \frac{1}{2}\overrightarrow{AC} - \overrightarrow{AB}$

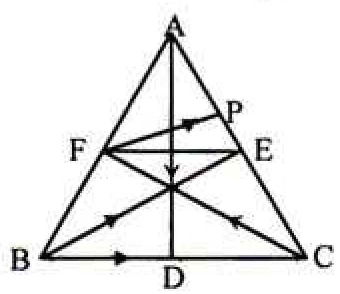
Or, $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} - \frac{1}{2}\overrightarrow{AC}$

$= \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) - \frac{1}{2}\overrightarrow{AC}$

$= \frac{1}{2}\overrightarrow{AC} - \frac{1}{2}\overrightarrow{AC} = \underline{0}$

$\therefore \overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \vec{0}$ **(Proved)**

c Let, F is the middle point of the side AB of the triangle ABC. The line drawn parallel to BC intersects the side AC at the point E. It is required to prove that, E is the middle point of AC.
Let, not E rather P is the middle point of AC.



Then, $\overrightarrow{AF} = \frac{1}{2}\overrightarrow{AB}$; [$\because$ F is the middle point of AB and $\overrightarrow{AP}$ $= \frac{1}{2}\overrightarrow{AC}$; [$\because$ P is the middle point of $\overrightarrow{AC}$]

$\therefore \overrightarrow{FP} = \overrightarrow{FA} + \overrightarrow{AP} = -\overrightarrow{AF} + \overrightarrow{AP}$ [$\therefore \overrightarrow{FA} = -\overrightarrow{AF}$]

$= \overrightarrow{AP} - \overrightarrow{AF} = \frac{1}{2}\overrightarrow{AC} - \frac{1}{2}\overrightarrow{AB}$

$= \frac{1}{2}(\overrightarrow{AC} - \overrightarrow{AB}) = \frac{1}{2}\overrightarrow{BC}$

$\therefore \overrightarrow{FP} = \frac{1}{2}\overrightarrow{BC}$

That is, FP || BC. But FE || BC (given)

So, the both lines $\overrightarrow{FE}$ and $\overrightarrow{FP}$ pass through the point F and parallel to $\overrightarrow{BC}$. So, they (that is, $\overrightarrow{FE}$ and $\overrightarrow{FP}$) must be coincide to each other.

$\therefore$ E and P will be the same point. That is, E is the middle point of AC. **(Proved)**

Question ▸ 31 A(t, 3t), B (t^2, 2t), C(t-2,t) and D (1, 1) are four different points. *[Millennium Scholastic School & College, Bogura]*

a. Find the value of t if the line joining the points A and B has slope $\frac{1}{2}$. 2

b. If the lines AB and CD are parallel, find the admissible value of t. 4

c. If M and N are the middle points of non parallel side AB and CD of the trapezium ABCD, then prove with the help of vectors that MN||AD||BC and MN $= \frac{1}{2}$(AD + BC). 4

Solution to the question no. 31

a Coordinates of points B and C are (t^2, 2t) and (t − 2, t) respectively.

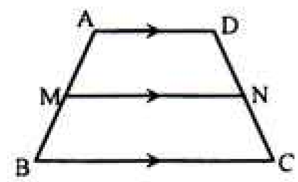
$\therefore$ Slope of the line BC $= \frac{t - 2t}{t - 2 - t^2} = \frac{-t}{t - 2 - t^2}$

According to question, $\frac{-t}{t - 2 - t^2} = \frac{1}{2}$

Or, $-2t = t - 2 - t^2$
Or, $t^2 - 3t + 2 = 0$
Or, $t^2 - 2t - t + 2 = 0$
Or, $t(t - 2) - 1(t - 2) = 0$
Or, $(t - 2)(t - 1) = 0$
$\therefore$ t = 1, 2 **(Ans.)**

b See example-5 of exercise-11.3 from your textbook. Page-257 [N.B. t will be replaced with p]

c Let, ABCD is a trapezium, AB and CD are the non–parallel sides, BC and AD are the parallel sides. M and N are respectively the middle points of AB and CD. Join M, N.



It is required to prove that, MN is parallel to both AD and BC, and

$\overrightarrow{MN} = \frac{1}{2}(\overrightarrow{AD} + \overrightarrow{BC})$ $\overrightarrow{MN}$

Proof: Let, the position vectors of the points A, B, C, D be $\underline{a}$, $\underline{b}$, $\underline{c}$, $\underline{d}$ respectively with respect to the origin.

$\therefore \overrightarrow{BC} = \underline{c} - \underline{b}$, $\overrightarrow{AD} = \underline{d} - \underline{a}$

$\therefore$ The position vector of M $= \frac{1}{2}(\underline{a} + \underline{b})$

[$\because$ M is the middle point of AB]

and the position vector of N $= \frac{1}{2}(\underline{c} + \underline{d})$

[$\because$ N is the middle point of CD]

$\therefore \overrightarrow{MN} = \frac{1}{2}(\underline{c} + \underline{d}) - \frac{1}{2}(\underline{a} + \underline{b} = \frac{1}{2}(\underline{c} + \underline{d} - \underline{a} - \underline{b})$

$= \frac{1}{2}\{(\underline{c} - \underline{b}) + (\underline{d} - \underline{a})\}$

$\therefore \overrightarrow{MN} = \frac{1}{2}(\overrightarrow{BC} + \overrightarrow{AD})$

Therefore the line support $\overrightarrow{MN}$, $\overrightarrow{AD}$. and $\overrightarrow{BC}$ are either same or parallel. But clearly they are not same. Therefore they are parallel,

$\therefore$ MN || AD || BC and

MN $= \frac{1}{2}$(AD + BC) **(Proved)**

Question ▸ 32 A(p, 3p), B(p^2, 2p), C(p–2-p) and D(1, 1) are the four different points. *[Dinajpur Laboratory School & College, Dinajpur]*

a. Find the value of p if the line joining the points B and C has slope $\frac{1}{2}$ 2

b. It the lines AB and CD are parallel, find admissible value of p. 4

c. With negative value of p from 'b' the straight line joining the middle points of the non-parallel side R and S of a trapezium ABCD. Prove with the help of vectors that RS||AB||CD and RS $= \frac{1}{2}$(AB + CD). 4

Solution to the question no. 32

a Co-ordinates of points B and C are (p^2, 2p) and (p − 2, p) respectively.

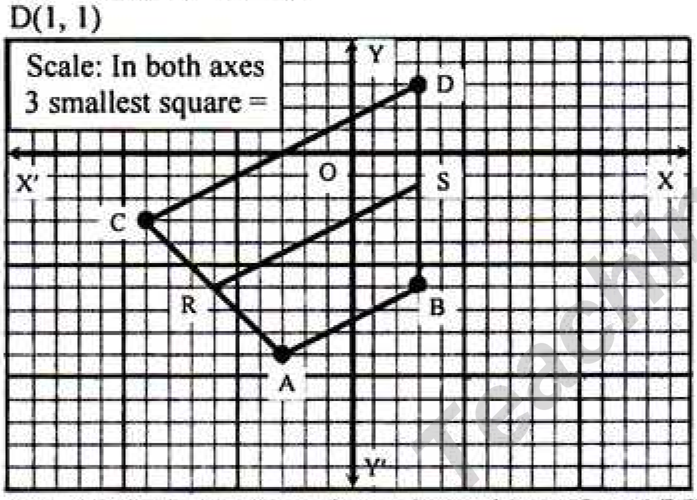
$\therefore$ Slope of the line BC $= \frac{p - 2p}{p - 2 - p^2} = \frac{-p}{p - 2 - p^2}$

According to question, $\frac{-p}{p - 2 - p^2} = \frac{1}{2}$

Or, $-2p = p - 2 - p^2$
Or, $p^2 - 3p + 2 = 0$
Or, $p^2 - 2p - p + 2 = 0$
Or, $p(p - 2) - 1(p - 2) = 0$
Or, $(p - 2)(p - 1) = 0$
$\therefore$ p = 1, 2 **(Ans.)**

b See example-5 of exercise-11.3 from your textbook. Page-257
[N.B. t will be replaced with p]

c From 'b' we get, p = − 1
$\therefore$ A(p, 3p) ≡ (− 1, − 3)
B(p^2, 2p) ≡ (1, − 2)
C(p − 2, p) ≡ (− 3, − 1)
D(1, 1)



Let, ABCD be the trapezium whose sides AC and BD are non-parallel and the sides AB and CD are parallel. The midpoints of sides AC and BD are R and S respectively. R, S are joined. It is required to prove that, RS || AB || CD and RS $= \frac{1}{2}$(AB + CD).

Proof: Let, the position vectors of the points A, B, C and D with respect to origin be $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ respectively.

$\therefore \overrightarrow{AB} = \underline{b} - \underline{a}, \overrightarrow{CD} = \underline{d} - \underline{c}$

$\therefore$ The position vector of point R $= \frac{1}{2}(\underline{a} + \underline{c})$

and the position vector of point S $= \frac{1}{2}(\underline{b} + \underline{d})$

$\therefore \overrightarrow{RS} = \frac{1}{2}(\underline{b} + \underline{d}) - \frac{1}{2}(\underline{a} + \underline{c})$

$= \frac{1}{2}(\underline{b} + \underline{d} - \underline{a} - \underline{c})$

$= \frac{1}{2}\{(\underline{b} - \underline{a}) + (\underline{d} - \underline{c})\}$

$\therefore \overrightarrow{RS} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{CD})$

$\therefore |\overrightarrow{RS}| = \frac{1}{2}|(\overrightarrow{AB} + \overrightarrow{CD})|$

So, RS $= \frac{1}{2}$(AB + CD)

Since $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel so $\overrightarrow{AB} + \overrightarrow{CD}$ will be parallel to $\overrightarrow{AB}$ and $\overrightarrow{CD}$.So the vector $\overrightarrow{RS}$ will be parallel to $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

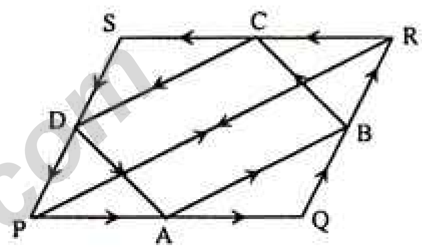
Therefore, RS || AB || CD and RS $= \frac{1}{2}$(AB + CD) **(Proved)**

Question ▶ 33 A, B, C and D are the middle points of quadrilateral PQRS. *[Cantonment Public School & College, Saidpur]*

a. Express $\overrightarrow{AB}$ interms of $\overrightarrow{PQ}$ and $\overrightarrow{QR}$. 2
b. Prove with the help of vectors that, ABCD is a parallelogram. 4
c. Prove with the help of vectors that, AB || PR and AB $= \frac{1}{2}$PR 4

Solution to the question no. 33

a



According to triangle law of vector addition, from ΔABQ,

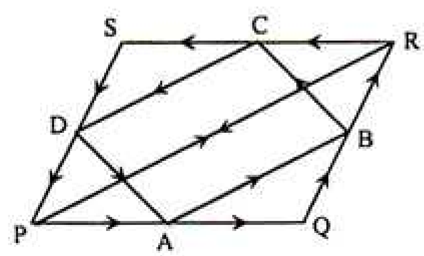
$\overrightarrow{AQ} + \overrightarrow{QB} = \overrightarrow{AB}$

Or, $\frac{1}{2}\overrightarrow{PQ} + \frac{1}{2}\overrightarrow{QR} = \overrightarrow{AB}$

[∵ A and B are the midpoints of PQ and QR respectively]

$\therefore \overrightarrow{AB} = \frac{1}{2}(\overrightarrow{PQ} + \overrightarrow{QR})$ **(Ans.)**

b



Let, the midpoints of the sides of the quadrilateral PQRS be A, B, C and D respectively. It is required to prove that, ABCD is a parallelogram.

Proof: Let, $\overrightarrow{PQ} = \underline{p}$, $\overrightarrow{QR} = \underline{q}$, $\overrightarrow{RS} = \underline{r}$, $\overrightarrow{SP} = \underline{s}$

Thus,

$\overrightarrow{AB} = \overrightarrow{AQ} + \overrightarrow{QB} = \frac{1}{2}(\overrightarrow{PQ} + \overrightarrow{QR}) = \frac{1}{2}(\underline{p} + \underline{q})$

Similarly, $\overrightarrow{BC} = \frac{1}{2}(\underline{q} + \underline{r})$, $\overrightarrow{CD} = \frac{1}{2}(\underline{r} + \underline{s})$,

DA $= \frac{1}{2}(\underline{s} + \underline{p})$

Again, $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \underline{p} + \underline{q}$

and $\overrightarrow{RP} = \overrightarrow{RS} + \overrightarrow{SP} = \underline{r} + \underline{s}$

But $(\underline{p} + \underline{q}) + (\underline{r} + \underline{s}) = \overrightarrow{PR} + \overrightarrow{RP} = \overrightarrow{PR} - \overrightarrow{PR} = 0$

That is $(\underline{p} + \underline{q}) = -(\underline{r} + \underline{s})$

$\therefore \overrightarrow{AB} = \frac{1}{2}(\underset{\sim}{p} + \underset{\sim}{q}) = -\frac{1}{2}(\underset{\sim}{r} + \underset{\sim}{s}) = -\overrightarrow{CD} = \overrightarrow{DC}$

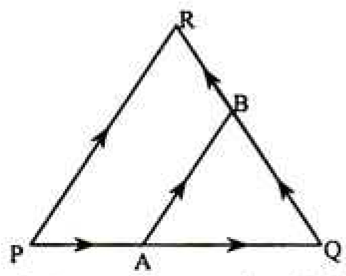
Here, the line of supports of $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are either same or parallel. Here line of supports are not same.

$\therefore$ Line of supports are parallel. $\therefore \overrightarrow{AB} \parallel \overrightarrow{DC}$.

Now, $|\overrightarrow{AB}| = |\overrightarrow{DC}|$ $\therefore$ AB = DC

$\therefore$ AB and DC are equal and parallel.
Similarly, BC and AD are equal and parallel.
$\therefore$ ABCD is a parallelogram. **(Proved)**

c



Proof: According to triangle law of vector addition, from ΔABQ,

$\overrightarrow{AQ} + \overrightarrow{QB} = \overrightarrow{AB}$ ... ... (i)

Again, in ΔPQR, $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

Or, $2\overrightarrow{AQ} + 2\overrightarrow{QB} = \overrightarrow{PR}$ [$\because$ A and B are the midpoints of PQ and QR respectively]

Or, $2\left(\overrightarrow{AQ} + \overrightarrow{QB}\right) = \overrightarrow{PR}$

Or, $2\overrightarrow{AB} = \overrightarrow{PR}$

Or, $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{PR}$

Or, $|\overrightarrow{AB}| = \frac{1}{2}|\overrightarrow{PR}|$

$\therefore AB = \frac{1}{2}PR$.

Again, the line of supports of $\overrightarrow{AB}$ and $\overrightarrow{PR}$ are either same or parallel. Here line of supports are not same.

$\therefore$ Line of supports of $\overrightarrow{AB}$ and $\overrightarrow{PR}$ are parallel.

$\therefore AB \parallel PR$ and $AB = \frac{1}{2}PR$ **(Proved)**

Question ▶ 34 The vertices of ΔABC are A (–5,4), B (–7,2) and C(2,x), where x > 0

[Mainamati International School and College, Cumilla]

a. .Find the slope of AB. 2
b. Find the value of x if AC = BC. 4
c. With the help of vectors, prove that the line segment joining the middle points D, E of two sides AB and AC respectively of a triangle is parallel to and half of the third side. 4

Solution to the question no. 34

a Given, co-ordinates of the points A & B are (– 5, 4) & (– 7, 2)

$\therefore$ Slope of the line, $AB = \frac{2-4}{-7+5}$

$= \frac{-2}{-2} = 1$ **(Ans.)**

b Given, A(– 5, 4), B(– 7, 2) & C(2, x) are three points.

$\therefore AC = \sqrt{(2+5)^2 + (x-4)^2}$

$= \sqrt{49 + (x-4)^2}$

And $BC = \sqrt{(2+7)^2 + (x-2)^2}$

$= \sqrt{81 + (x-2)^2}$

According to question, AC = BC

$\therefore \sqrt{49 + (x-4)^2} = \sqrt{81 + (x-2)^2}$

Or, $49 + x^2 - 8x + 16 = 81 + x^2 - 4x + 4$ [Sequaring both sides]

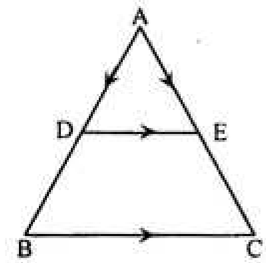
Or, $49 + 16 - 81 - 4 = x^2 - 4x - x^2 + 8x$

Or, $-20 = 4x$

$\therefore x = -5$ **(Ans.)**

c Let, in ΔABC, D and E be the mid points of AB and AC. Join D, E. It is required to prove that $DE \parallel BC$

and $DE = \frac{1}{2}BC$



According to Triangle law of subtruction of vector,

$\overrightarrow{AE} - \overrightarrow{AD} = \overrightarrow{DE}$ .... (i)

and $\overrightarrow{AC} - \overrightarrow{AB} = \overrightarrow{BC}$ .... (ii)

But, $\overrightarrow{AC} = 2\overrightarrow{AE}$, $\overrightarrow{AB} = 2\overrightarrow{AD}$

[$\because$ D and E are the middle points of AB and AC]

From (ii), we get,

$2\overrightarrow{AE} - 2\overrightarrow{AD} = \overrightarrow{BC}$ Or, $2\left(\overrightarrow{AE} - \overrightarrow{AD}\right) = \overrightarrow{BC}$

Or, $2\overrightarrow{DE} = \overrightarrow{BC}$, [From (i)]

$\therefore \overrightarrow{DE} = \frac{1}{2}\overrightarrow{BC}$

Again, $\left|\overrightarrow{DE}\right| = \frac{1}{2}\left|\overrightarrow{BC}\right|$ Or, $DE = \frac{1}{2}BC$

So, DE and BE must lie on the same line or are parallel to each other. But they do not lie on the same line.

$\therefore$ DE and DE are parallel to each other and $DE = \frac{1}{2}BC$. **(Proved)**

Question ▶ 35 A(3,4), B (–4,2), C (6–1), D(k,3)

[Mainamati International School and College, Cumilla]

a. Find the distance of AB. 2
b. Determine the value of 'k' if the area of the quadrilateral ABCD is thrice that of ΔABC. 4
c. Prove by vector method that, the straight lines joining the middle points of the adjacent sides of a quadrilateral form a parallelogram. 4

Solution to the question no. 35

a Given, A(3, 4) and B(– 4, 2) are two points.

$\therefore AB = \sqrt{(3+4)^2 + (4-2)^2}$

$= \sqrt{7^2 + 2^2} = \sqrt{49 + 4}$

$= \sqrt{53}$ unit **(Ans.)**

b Given,
A(3, 4), B (–4, 2), C (6, – 1) and D (k, 3) and the points are arranged in anti– clockwise order.

$\therefore$ Area of the quadrilateral ABCD

$= \frac{1}{2}\begin{vmatrix} 3 & -4 & 6 & k & 3 \\ 4 & 2 & -1 & 3 & 4 \end{vmatrix}$

$= \frac{1}{2}\{6 + 4 + 18 + 4k - (-16) - 12 - (-k) - 9\}$

$= \frac{1}{2}(23 + 5k)$ sq.unit

Again, considering the points A, B and C in the anti-clockwise direction we get the triangle ABC.

∴ Area of the triangle ABC,

$= \frac{1}{2}\begin{vmatrix} 3 & -4 & 6 & 3 \\ 4 & 2 & -1 & 4 \end{vmatrix}$

$= \frac{1}{2}\ \{6 + 4 + 24 - (-16) - 12 - (-3)\}$

$= \frac{1}{2} \times (53 - 12) = \frac{41}{2}$ sq. unit

According to the question,

Area of the quadrilateral ABCD = 3 × area of ABC

$\therefore \frac{1}{2}(23 + 5k) = 3 \times \frac{41}{2}$

Or, $23 + 5k = 123$

Or, $5k = 123 - 23$

Or, $5k = 100$

Or, $k = \frac{100}{5}$

Or, $k = 20$

∴ k = 20 **(Ans.)**

c See, example–5, chapter-12, of your text book, page-281.

Question ▶ 36 M and N are mid point of sides PQ and PR of a ΔPQR. *[Bangladesh Mahila Somitee Girls' High School & College, Chattogram]*

a. Define position vector of point with figure. 2

b. With the help of vector, prove that, $MN = \frac{1}{2}QR$. 4

c. According to the information, if D and E are mid points of two diagonal of trapezium QRNM, then with the help of the vector prove that, $DE = \frac{1}{2}(QR - MN)$ 4

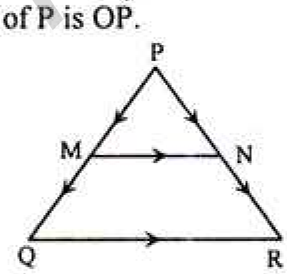
Solution to the question no. 36

a In any plane, the position vector of a point P can be defined by $\overrightarrow{OP}$ with respect to origin O. It is represents the distance and direction of P from origion O

O ——→ P

In the figure, the position vector of P is $\overrightarrow{OP}$.

b Given the middle points of the sides PQ and PR of the triangle PQR are M and N respectively. Join M, N. It is required to prove with the help of vectors that $MN = \frac{1}{2}QR$ and MN || QR



Proof: M and N are the middle points of PQ and PR respectively.

$\therefore \overrightarrow{MQ} = \overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ}$ and $\overrightarrow{PN} = \overrightarrow{NR} = \frac{1}{2}\overrightarrow{PR}$

According to the triangle law we get,

$\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PR}$

Or, $\overrightarrow{QR} = -\overrightarrow{PQ} + \overrightarrow{PR} = \overrightarrow{PR} - \overrightarrow{PQ}$ .... (i)

and $\overrightarrow{MN} = \overrightarrow{MP} + \overrightarrow{PN}$

$= -\overrightarrow{PM} + \overrightarrow{PN}$

$= -\frac{1}{2}\overrightarrow{PQ} + \frac{1}{2}\overrightarrow{PR}$ [$\because \overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ}, \overrightarrow{PN} = \frac{1}{2}\overrightarrow{PR}$]

$= \frac{1}{2}(\overrightarrow{PR} - \overrightarrow{PQ}) = \frac{1}{2}\overrightarrow{QR}$ [from the equation (i)]

So, $|\overrightarrow{MN}| = \frac{1}{2}|\overrightarrow{QR}|$

$\therefore MN = \frac{1}{2}QR$ and the support line of $\overrightarrow{MN}$ and $\overrightarrow{QR}$ will be same or parallel.

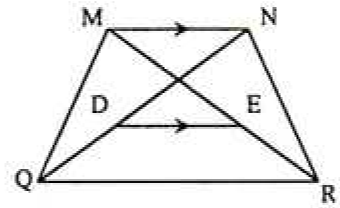
But, M and N are the middle points of PQ and PR respectively.

So, the support line of $\overrightarrow{MN}$ and $\overrightarrow{QR}$ can not be same.

∴ MN | | QR

That is, $MN = \frac{1}{2}QR$ and MN ∥ QR **(Proved)**

c Let, in the trapezium QRNM, MN ∥ QR and the middle points of RM and QN are E and D respectively. Let us join D, E.



It is required to prove that $DE = \frac{1}{2}(QR - MN)$ and MN ∥ DE ∥ QR.

Proof: Let the position vectors of points Q,R,N,M are $\underline{b}$, $\underline{c}$, $\underline{e}$, $\underline{d}$ respectively with respect to the origin.

$\overrightarrow{QR} = \underline{c} - \underline{b}$

$\overrightarrow{MN} = \underline{e} - \underline{d}$

∴ The position vector of $D = \frac{1}{2}(\underline{b} + \underline{e})$

[∵ D is the middle point of QN]

and the position vector of $E = \frac{1}{2}(\underline{c} + \underline{d})$

[∵ E is the middle point of RM]

$\therefore \overrightarrow{DE} = \frac{1}{2}(\underline{c} + \underline{d}) - \frac{1}{2}(\underline{b} + \underline{e}) = \frac{1}{2}(\underline{c} + \underline{d} - \underline{b} - \underline{e})$

$= \frac{1}{2}\{(\underline{c} - \underline{b}) - (\underline{e} - \underline{d})\} = \frac{1}{2}(\overrightarrow{QR} - \overrightarrow{MN})$

Since, MN ∥ QR, So the vector $(\overrightarrow{QR} - \overrightarrow{MN})$ is parallel to both $\overrightarrow{QR}$ and $\overrightarrow{MN}$. So the vector $\overrightarrow{DE}$ will be parallel to both $\overrightarrow{QR}$ and $\overrightarrow{MN}$.

Because, $\overrightarrow{DE} = \frac{1}{2}(\overrightarrow{QR} - \overrightarrow{MN})$

$\therefore |\overrightarrow{DE}| = \frac{1}{2}|(\overrightarrow{QR} - \overrightarrow{MN})| = \frac{1}{2}(|\overrightarrow{QR}| - |\overrightarrow{MN}|)$

$\therefore DE = \frac{1}{2}(QR - MN)$

That is, DE ∥ MN ∥ QR

and $DE = \frac{1}{2}(QR - MN)$ **(Proved)**

Question ▶ 37 A(0, – 1), B (–2, 3), C (6,7) and D (8, 3) are vertices of a quadrilateral. *[Navy Anchorage School and College, Chattogram]*

a. Find the equation of the straight line AC. 2

b. What type of quadrilateral ABCD is? Explain it. 4

c. If P, Q, R and S are the mid– point of the side AB, BC, CD and DA respectively with the help of vector show that PQRS is a parallelogram. 4

Solution to the question no. 37

a Given, A (0, –1), C (6, 7)

∴ The equation of the straight line AC

$\frac{x - 0}{y - (-1)} = \frac{0 - 6}{-1 - 7}$

Or, $\frac{x}{y + 1} = \frac{6}{8}$

Or, $\frac{x}{y+1} = \frac{3}{4}$

Or, $4x = 3y + 3$

Or, $4x - 3y - 3 = 0$ **(Ans.)**

b Let, A (0, –1), B (8, 3), C (6, 7) and D (–2, 3)

Now,

Length of the side AB $= \sqrt{(0-8)^2 + (-1-3)^2}$

$= \sqrt{64 + 16} = \sqrt{80}$ unit

Length of the side BC $= \sqrt{(8-6)^2 + (3-7)^2}$

$= \sqrt{2^2 + (-4)^2}$

$= \sqrt{4 + 16} = \sqrt{20}$ unit

Length of the side CD $= \sqrt{(6+2)^2 + (7-3)^2}$

$= \sqrt{64 + 16} = \sqrt{80}$ unit

And length of the side AD $= \sqrt{(0+2)^2 + (-1-3)^2}$

$= \sqrt{4 + 16} = \sqrt{20}$ unit **(Ans.)**

Again, length of the diagonal AC

$= \sqrt{(0-6)^2 + (-1-7)^2} = \sqrt{36 + 64}$

$= \sqrt{100} = 10$ unit **(Ans.)**

and length of the diagonal BD $= \sqrt{(8+2)^2 + (3-3)^2}$

$= \sqrt{10^2} = 10$ unit

It is seen that, AB = CD, BC = AD and diagonal AC = diagonal BD.

$\therefore$ The points A, B, C, D are the vertices of a rectangle.

c See example 5, ch-12 of your textbook page-281

Question ▶ 38 If $\underline{a}$, $\underline{b}$, $\underline{c}$, $\underline{d}$ are the position vectors respectively of the points A, B, C, D.

[Jalalabad Cantonment Public School & College, Sylhet]

a. State parallelogram law of addition of vectors. 2

b. Show that, ABCD will be a parallelogram if and only if $\underline{b} - \underline{a} = \underline{c} - \underline{d}$. 4

c. If ABCD is a trapezium then prove with the help of vectors that the straight line joining the middle points of the diagonals of that trapezium is parallel to and half of the difference of the parallel sides. 4

Solution to the question no. 38

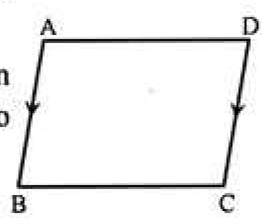
a See your text book of chapter–12, page– 271.

b Given, the position vectors of the points A, B, C, D are $\underline{a}$, $\underline{b}$, $\underline{c}$, $\underline{d}$ respectively.

It is required to show that, ABCD will be a parallelogram if and only if $\underline{b} - \underline{a} = \underline{c} - \underline{d}$.

The position vectors of A, B, C and D are $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ respectively with respect to the origin.

$\therefore \overrightarrow{AB} = \underline{b} - \underline{a}$ and $\overrightarrow{DC} = \underline{c} - \underline{d}$



Let, ABCD be a parallelogram. Then AB and DC are parallel and equal to each other.

$\therefore \overrightarrow{AB} = \overrightarrow{DC}$

$\therefore \underline{b} - \underline{a} = \underline{c} - \underline{d}$

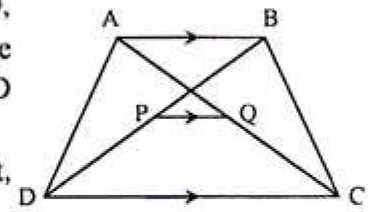
Conversely, let, $\underline{b} - \underline{a} = \underline{c} - \underline{d}$

$\therefore \overrightarrow{AB} = \overrightarrow{DC}$

So, AB and DC are parallel and equal to each other. That is, ABCD is a parallelogram.

$\therefore$ ABCD will be a parallelogram if and only if $\underline{b} - \underline{a} = \underline{c} - \underline{d}$. **(Shown)**

c Let, in the trapezium ABCD, AB || CD and Q and P are the middle points of AC and BD respectively. Join P, Q.



It is required to prove that,

$PQ = \frac{1}{2}(DC - AB)$

And PQ || AB || CD.

Proof: Let, the position vectors of the points A, B, C, D be $\underline{a}$, $\underline{b}$, $\underline{c}$, $\underline{d}$ respectively with respect to the origin.

$\overrightarrow{AB} = \underline{b} - \underline{a}$

$\overrightarrow{DC} = \underline{c} - \underline{d}$

$\therefore$ The position vector of P $= \frac{1}{2}(\underline{b} + \underline{d})$

[$\because$ P is the middle point of BD]

And the position vectors of Q $= \frac{1}{2}(\underline{a} + \underline{c})$

[$\because$ Q is the middle point of AC]

$\therefore \overrightarrow{PQ} = \frac{1}{2}(\underline{a} + \underline{c}) - \frac{1}{2}(\underline{b} + \underline{d}) = \frac{1}{2}(\underline{a} + \underline{c} - \underline{b} - \underline{d})$

Or, $\overrightarrow{PQ} = \frac{1}{2}\{(\underline{c} - \underline{d}) - (\underline{b} - \underline{a})\}$

$\therefore \overrightarrow{PQ} = \frac{1}{2}(\overrightarrow{DC} - \overrightarrow{AB})$

Since, AB || CD, so the vector $\overrightarrow{DC} - \overrightarrow{AB}$ is also parallel to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$. So, the vector $\overrightarrow{PQ}$ will be parallel to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

Because, $\overrightarrow{PQ} = \frac{1}{2}(\overrightarrow{DC} - \overrightarrow{AB})$

$\therefore |\overrightarrow{PQ}| = \frac{1}{2}|(\overrightarrow{DC} - \overrightarrow{AB})| = \frac{1}{2}(|\overrightarrow{DC}| - |\overrightarrow{AB}|)$

$\therefore PQ = \frac{1}{2}(DC - AB)$

That is, PQ || AB || DC

$\therefore PQ = \frac{1}{2}(DC - AB)$ **(Proved)**

Question ▶ 39 The vertices of the quadrilateral ABCD arranged in anti-clockwise order are A(6, –4), B(2, 2) C(– 2, 2), D(– 6, –4).

[The Sylhet Khajanchibari International School & College, Sylhet]

a. Find the length of BD. 2

b. Determine the length of the diagonal of the square where the area of the square is equal to the area of the quadrilateral ABCD. 4

c. If ABCD is a trapezium and P and Q are the middle points of AB and CD respectively, then prove with the help of vectors that PQ ||AD||BC and $PQ = \frac{1}{2}(AD + BC)$. 4

Solution to the question no. 39

a Distance between the two points B(2, 2) and D(–6, –4) that is, length of BD $= \sqrt{(-6-2)^2 + (-4-2)^2}$ unit

$= \sqrt{64 + 36}$ unit $= \sqrt{100}$ unit

$= 10$ unit**(Ans)**

b The vertices A(6, –4), B(2, 2), C(–2, 2) and D(–6, –4) are taken in anti-clockwise order. The area of the quadrilateral

$ABCD = \frac{1}{2}\begin{vmatrix} 6 & 2 & -2 & -6 & 6 \\ -4 & 2 & 2 & -4 & -4 \end{vmatrix}$ square unit

$= \frac{1}{2}(12 + 4 + 8 + 24 + 8 + 4 + 12 + 24)$ square unit

$= \frac{1}{2} \times 96$ square unit = 48 square unit

According to the question,
area of square = area of the quadrilateral ABCD = 48 square unit

∴ Length of one side of square $= \sqrt{48}$ unit $= \sqrt{16 \times 3}$
$= 4\sqrt{3}$ unit

∴ Length of diagonal of square $= \sqrt{2} \times$ Length of one side
$= \sqrt{2} \times 4\sqrt{3}$ unit
$= 4\sqrt{6}$ unit **(Ans.)**

c Here, P and Q are midpoints of non-parallel sides AB and CD of a trapezium ABCD. It is required to prove that,

PQ || AD || BC and $PQ = \frac{1}{2}(AD + BC)$

Proof: let, position vector ofA, B, C and D are $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ respectively

∴ $\overrightarrow{BC} = \underline{c} - \underline{b}$ and $\overrightarrow{AD} = \underline{d} - \underline{a}$

∴ Position vector of point $P = \frac{1}{2}(\underline{a} + \underline{b})$;

[∵ P is the midpoint of AB]

Position vector of point $Q = \frac{1}{2}(\underline{c} + \underline{d})$;

[∵ Q is the midpoint of CD]

Now, $\overrightarrow{PQ} = \frac{1}{2}(\underline{c} + \underline{d}) - \frac{1}{2}(\underline{a} + \underline{b}) = \frac{1}{2}(\underline{c} + \underline{d} - \underline{a} - \underline{b})$

$= \frac{1}{2}\{(\underline{c} - \underline{b}) + (\underline{d} - \underline{a})\} = \frac{1}{2}(\overrightarrow{BC} + \overrightarrow{AD})$

But as BC and AD are parallel so $(\overrightarrow{BC} + \overrightarrow{AD})$ will be parallel to BC and AD. So PQ vector will be parallel to BC and AD.

$\overrightarrow{PQ} = \frac{1}{2}(\overrightarrow{AD} + \overrightarrow{BC})$ Or, $|\overrightarrow{PQ}| = \frac{1}{2}|\overrightarrow{BC} + \overrightarrow{AD}|$

So, $PQ = \frac{1}{2}(AD + BC)$

So, PQ || AD || BC and $PQ = \frac{1}{2}(AD + BC)$ **(Proved)**

Question ▶40 A(6, 5), B(– 4, 3) are the co-ordinates of A and B of the parallelogram where diagonals AC and BD intersect at O.

[Secondary & Higher Secondary Education Board, Jashore]

a. Find the slope of 2x – 3y = – 1 — 2
b. Find the area of the triangle formed by AB with axes. — 4
c. Prove by vector method BD = 2BO — 4

Solution to the question no. 40

a Given,
$2x - 3y = -1$
Or, $-3y = -2x - 1$
Or, $y = \frac{-2x - 1}{-3}$
∴ $y = \frac{2}{3}x + \frac{1}{3}$
∴ the slope of the equation is $\frac{2}{3}$ **(Ans.)**

b Given, A (6, 5), B (– 4, 3)

The equation of AB, $\frac{y - 5}{x - 6} = \frac{5 - 3}{6 + 4}$

Or, $\frac{y - 5}{x - 6} = \frac{2}{10}$

Or, $10y - 50 = 2x - 12$
Or, $2x - 10y + 38 = 0$
∴ $x - 5y + 19 = 0$ .... (i)

When y = 0, x = – 19

When x = 0, $y = \frac{19}{5}$

So the equation (i) cuts the x-axis at (– 19, 0) and y-axis at $\left(0, \frac{19}{5}\right)$

∴ The area of the desired triangle

$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 19 & 0 \\ 0 & \frac{19}{5} & 0 & 0 \end{vmatrix}$

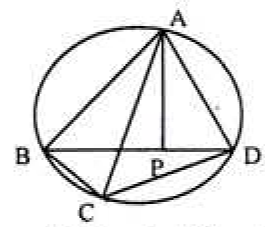
$= \frac{1}{2}\left(0 + 0 + 0 - 0 + \frac{19}{5} \times 19 - 0\right)$

$= \frac{1}{2} \times \frac{19}{5} \times 19$

= 36.1 sq. unit. **(Ans.)**

c See the Example-4, chapter-12 of your Higher Mathematics textbook.

Question ▶41 Observe the following figure:



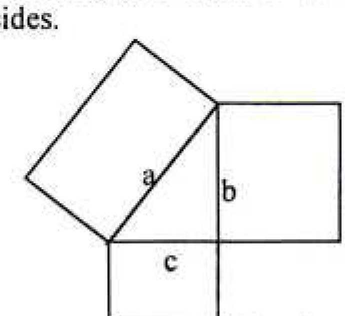
If ABCD is a cyclic quadrilateral; AC and BD are its diagonals and AB, CD and BC, AD are its two pair of opposite sides, then —

[Jashore English School and College (JESC), Jashore]

a. Write down the Pythagoras theorem with figure. — 2
b. Prove that, AC.BD = AB. CD + AD.BC — 4
c. Prove that by vector methods that the line segment joining the middle points of two sides of a triangle is parallel to and half of the third side. — 4

Solution to the question no. 41

a **Pythagoras theorem :** The square of the hypotenuse (the opposite side of right angle) is equal to the sum of the squares of the other two sides.



According to pythagoras theorem : $a^2 = b^2 + c^2$

b See theorem 12, chapter-3.12 of your text book. page-78

c See example–3, chapter–12 of your text book, page-279