Chapter-13: Solid Geometry

Question ▶ 1 The height of a right circular cone is 8cm and the radius of its base is 6 cm. It is melted and formed into several number of solid sphere with diameter 4cm. [D.B.17]

a. Find the area of the curved surface of each sphere. 2

b. Find the whole surface area of the right circular cone. 4

c. Find how many solid spheres can be formed. 4

Solution to the question no. 1

a Given, diameter of the sphere = 4 cm

$\therefore$ Radius of the sphere, $R = \frac{4}{2}$ cm = 2 cm

$\therefore$ Surface area of the sphere = $4\pi R^2$ sq. unit

$= 4 \times 3.1416 \times 2^2$ sq. cm

$= 50.27$ sq. cm (approx.) **(Ans.)**

b Given,

radius of a right circular cone, r = 6 cm

and height, h = 8 cm

$\therefore$ Slant height, $l = \sqrt{h^2 + r^2}$

$= \sqrt{8^2 + 6^2}$ cm

$= \sqrt{100}$ cm

$= 10$ cm

$\therefore$ Whole surface area of the cone

$= \pi r(l + r)$ sq. unit

$= 3.1416 \times 6(10 + 6)$ sq. cm

$= 301.5936$ sq. cm (approx.) **(Ans.)**

c Volume of the cone $= \frac{1}{3}\pi r^2 h$ cubic unit

$= \frac{1}{3} \times \pi \times 6^2 \times 8$ cubic cm

$= 96\pi$ cubic cm

Volume of the solid sphere $= \frac{4}{3}\pi R^3$ cubic unit

$= \frac{4}{3} \times \pi \times 2^3$ cubic cm

$[\because R = 2 \text{ cm}]$

$= \frac{32\pi}{3}$ cubic cm

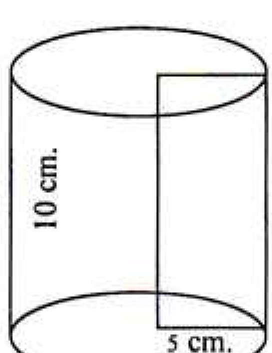
Let, the number of solid sphere can be formed be n.

According to question, $96\pi = n\left(\frac{32\pi}{3}\right)$

Or, $n = \frac{96\pi \times 3}{32\pi}$

$\therefore$ $n = 9$ **(Ans.)**

Question ▶ 2



[Dj.B.17]

a. Find the area of the base of the cylinder. 2

b. Find the area of the curved surface and whole surface of the cylinder. 4

c. A spherical ball exactly fits into the cylinder. Find the volume of the unoccupied portion of the cylinder. 4

Solution to the question no. 2

a Given, the radius of base of the cylinder, r = 5 cm

$\therefore$ The area of base of the cylinder $= \pi r^2$ sq. unit

$= 3.1416 \times 5^2$ sq. cm

$= 78.54$ sq. cm (approx.) **(Ans.)**

b Given, the radius of the cylinder, r = 5 cm

and height, h = 10 cm

$\therefore$ Area of the curved surface of the cylinder

$= 2\pi rh$ sq. unit

$= 2 \times 3.1416 \times 5 \times 10$ sq. cm

$= 314.16$ sq. cm (approx.) **(Ans.)**

Area of the whole surface of the cylinder

$= 2\pi r(h + r)$ sq. unit

$= 2 \times 3.1416 \times 5(10 + 5)$ sq. cm

$= 471.24$ sq. cm (approx.) **(Ans.)**

c Volume of the cylinder $= \pi r^2 h$ cubic unit

$= 3.1416 \times 5^2 \times 10$ cubic cm

$[\because r = 5 \text{ cm and } h = 10 \text{ cm}]$

$= 785.4$ cubic cm (approx.)

Since a spherical ball exactly fits into the cylinder, so the radius of the ball will be equal to the radius of the cylinder.

$\therefore$ Radius of the ball, R = 5 cm.

$\therefore$ Volume of the ball $= \frac{4}{3}\pi R^3$ cubic unit

$= \frac{4}{3} \times 3.1416 \times 5^3$ cubic cm

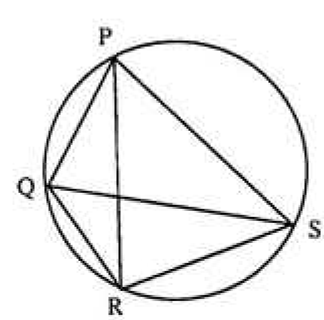
$= 523.6$ cubic cm (approx.)

$\therefore$ Volume of the unoccupied portion of the cylinder

$= (785.4 - 523.6)$ cubic cm

$= 261.8$ cubic cm (approx.) **(Ans.)**

Question ▶ 3



In figure PR = 10cm and QS = 8cm. [C.B.17]

a. Which theorem does support the given figure? Write that theorem. 2

b. Prove that PR.QS = PQ.RS + PS.QR. 4

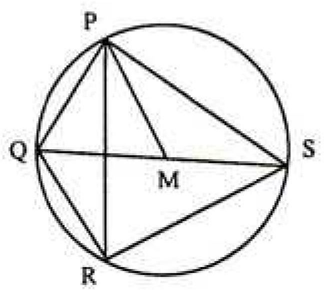
c. If PR and QS are the edges of two cubes, then find the ratio of the area of two cubes. 4

Solution to the question no. 3

a Given figure is supported by Ptolemy's theorem.

Ptolemy's theorem: In any cyclic quadrilateral the area of the rectangle contained by the two diagonal is equal to the sum of the area of the two rectangles contained by the two pairs of opposite sides.

b



Particular enunciation:
Suppose PQRS is a cyclic quadrilateral; PR and QS are its diagonals and PQ, RS and QR, PS are its two pairs of opposite sides. It is required to prove that, PR . QS = PQ . RS + QR . PS.

Construction: Without loss of generality we can assume that ∠QPR is smaller than ∠SPR; we draw ∠SPM making it equal to ∠QPR such that the line PM intersects QS at the point M.

Proof: According to drawing, ∠QPR = ∠SPM

Adding ∠RPM on both sides we get,

∠QPR + ∠RPM = ∠SPM + ∠RPM

That is, ∠QPM = ∠RPS

Now, in ΔPQM and ΔPRS,

∠QPM = ∠RPS

∠PQS = ∠PRS [standing on same arc, the circumference angles are equal]

and remaining ∠PMQ = remaining ∠PSR

∴ ΔPQM and ΔPRS equiangular.

$\frac{QM}{RS} = \frac{PQ}{PR}$

That is, PR . QM = PQ . RS .... (i)

Again, in ΔPQR and ΔPMS,

∠QPR = ∠SPM [by drawing]

∠PSM = ∠PRQ [standing on same arc, the circumference angles are equal]

and remaining ∠PQR = remaining ∠PMS

∴ ΔPQR and ΔPMS equiangular.

∴ $\frac{PS}{PR} = \frac{MS}{QR}$

That is, PR . MS = QR . PS ....(ii)

Now, adding equation (i) and (ii) we get,

PR . QM + PR. MS = PQ . RS + QR . PS

Or, PR (QM + MS) = PQ . RS + QR . PS

∴ PR.QS = PQ . RS + QR . PS [since QM + MS = QS] **(Proved)**

c Given, PR = 10 cm

QS = 8 cm

Area of the cube having edge PR = $6(PR)^2$

$= 6 \times (10)^2$ sq. cm

= 600 sq. cm

Again, area of the cube having edge QS = $6(QS)^2$

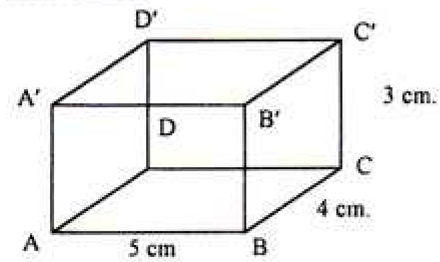
$= 6 \times (8)^2$ sq. cm

= 384 sq. cm

∴ The ratio of the area of two cubes = 600 : 384

= 25 : 16 **(Ans.)**

Question ▶ 4



[Ctg.B.17]

a. Find the volume of the solid mentioned in the diagram. 2

b. The radii of three metal solid sphere is AB, BC and CC′ respectively. A new solid sphere is formed by melting the three spheres. Determine the radius and whole surface area of new sphere. 4

c. A rectangle of size of ABCD surface of the solid is revolved about the greater side. Find the volume and the total surface area of the solid formed. 4

Solution to the question no. 4

a Given, length of the solid = 5 cm

breadth = 4 cm

height = 3 cm

∴ Volume of the solid = length × breadth × height

= 5 × 4 × 3 cubic cm

= 60 cubic cm **(Ans.)**

b Given,

AB = 5 cm, BC = 4 cm and CC′ = 3 cm

Let, radius of the new sphere = r

We know, the volume of the sphere = $\frac{4}{3}\pi\,(\text{radius})^3$ cubic unit

According to question, the sum of the volume of spheres having radii AB, BC and CC′ = volume of the new sphere

Or, $\frac{4}{3}\pi\,(AB)^3 + \frac{4}{3}\pi(BC)^3 + \frac{4}{3}\pi\,(CC')^3 = \frac{4}{3}\pi r^3$

Or, $\frac{4}{3}\pi\,(5^3 + 4^3 + 3^3) = \frac{4}{3}\pi r^3$

Or, $125 + 64 + 27 = r^3$

Or, $r^3 = 216$

∴ r = 6

So radius of the new sphere = 6 cm **(Ans.)**

Again, whole surface area of the new sphere

$= 4\pi r^2$

$= 4 \times 3.1416 \times 6^2$ sq. cm

= 452.39 sq. cm (approx.) **(Ans.)**

c If the surface ABCD is revolved once about the side AB the solid is formed which is cylinder. Whose

radius, r = BC = 4 cm

height, h = AB = 5 cm

∴ Total surface area of the cylinder = $2\pi r(r + h)$

$= 2 \times 3.1416 \times 4\,(4 + 5)$ sq. cm

= 226.20 sq. cm (approx.) **(Ans.)**

Again, volume of the cylinder = $\pi r^2 h$ cubic unit

$= 3.1416 \times 4^2 \times 5$ cubic cm

= 251. 33 cubic cm (approx.)

(Ans.)

Question ▶ 5 A solid sphere of radius 9cm formed by melting three spherical balls of radii 6cm, 8cm and r cm exactly fits into a cylindrical box. [B.B.17]

a. Find the area of surface of the sphere of radius 6cm. 2

b. Find the value of r. 4

c. Find the volume of the unoccupied portion of the box. 4

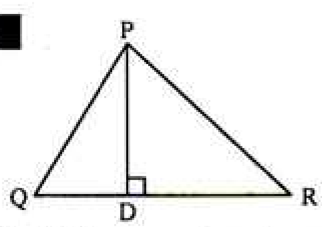
Solution to the question no. 5

a Given, the radius of the sphere R = 6 cm.
We know, if the radius of the sphere R,
the area of surface = $4\pi R^2$ sq. unit
$\therefore$ The area of surface of the given sphere
$= 4\pi \times 6^2$ sq. cm
$= 4\pi \times 36$ sq. cm
= 452.3904 sq. cm (approx.) **(Ans.)**

b We know,
the volume of the sphere $= \frac{4}{3}\pi \times (\text{radius})^3$ cubic unit
Sum of the volume of the spheres having radii 6, 8, r cm
$= \left\{\frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3 + \frac{4}{3}\pi r^3\right\}$ cubic cm
$= \frac{4}{3}\pi(6^3 + 8^3 + r^3)$ cubic cm
Again, the volume of the sphere having radius 9
$= \frac{4}{3}\pi \times 9^3$ cubic cm
According to question,
$\left\{\frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3 + \frac{4}{3}\pi r^3\right\} = \frac{4}{3}\pi \times 9^3$
Or, $\frac{4}{3}\pi(6^3 + 8^3 + r^3) = \frac{4}{3}\pi \times 9^3$
Or, $6^3 + 8^3 + r^3 = 9^3$
Or, $216 + 512 + r^3 = 729$
Or, $r^3 = 729 - 728$
Or, $r^3 = 1$
$\therefore$ r = 1 cm **(Ans.)**

c The volume of the sphere having radius 9 cm $= \frac{4}{3}\pi(9)^3$
$= \frac{4}{3} \times 3.1416 \times 729$
= 3053.6352 cubic cm (approx.)
Since, the sphere having radius 9 cm exactly fits into a cylindrical box.
So the radius of the cylinder will be the radius of the sphere and the height of the cylinder will be the diameter of the sphere.
$\therefore$ Radius of the cylinder, r = 9 cm
and height, h = 9 × 2 cm
= 18 cm
$\therefore$ the volume of the cylinder $= \pi r^2 h$ cubic unit
$= 3.1416 \times 9^2 \times 18$ cubic cm
= 4580.4528 cubic cm (approx.)
$\therefore$ The volume of the unoccupied portion of the box
= (4580.4528 – 3053.6352) cubic cm
= 1526.8176 cubic cm (approx.) **(Ans.)**

Question ▶ 6

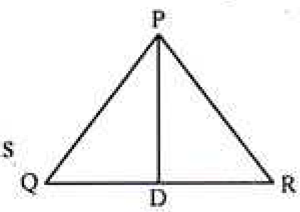


In triangle PQR, ∠R is an acute angle and PD⊥QR. [D.B.16]
a. Define the circumcentre and the centroid of a triangle. 2
b. Considering the stem prove that, $PQ^2 + 2QR.DR = PR^2 + QR^2$. 4
c. Find the area of the total surfaces and volume of the solid formed by revolving one time the rectangle about the side DR where DR and PD are the length and breadth of the rectangle respectively and DR = 6 cm, PD = 4cm. 4

Solution to the question no. 6

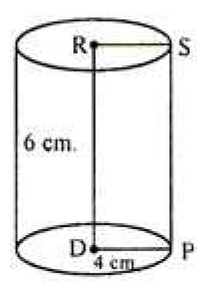
a **Circumcentre of the triangle:** The point of intersection of the perpendicular bisectors of the three sides of a triangle is called circumcentre of the triangle.
Centroid of the triangle: The three medians of a triangle intersect at one point is called centroid of the triangle.

b **Particular Enunciation:**



Suppose, in the triangle, ∠PQR, ∠R. is an acute angle and the opposite side of the acute angle is PQ. The other two sides are PR and QR respectively.
Suppose, PD is a perpendicular on the side QR. So, RD is the orthogonal projection of the side PR on the side QR in the case of both triangle.
Proof: ∠PDQ is right angle of ΔPQR
$\therefore PQ^2 = PD^2 + QD^2$ [the theorem of Pythagoras] ... ... (1)
In the figure QD = QC – DR
$\therefore QD^2 = (QR - DR)^2$
$= QR^2 + DR^2 - 2QR.\,DR$
$= QR^2 + RD^2 - 2QR.\,RD$ [RD = DR]
$\therefore QD^2 = QR^2 + RD^2 - 2QR.\,RD$ ... ... (2)
Now from equation (1) and (2) we get,
$PQ^2 = PD^2 + QR^2 + RD^2 - 2QR.RD$
Or, $PQ^2 = PD^2 + RD^2 + QR^2 - 2QR.\,RD$ ... ... (3)
Again ΔPDR is right angled triangle and ∠D is right angle
$\therefore PR^2 = PD^2 + RD^2$ [the theorem of Pythagoras] ... ... (4)
From the equation (3) and (4), we get
$PQ^2 = PR^2 + QR^2 - 2QR.\,RD$
$\therefore PQ^2 + 2QR.\,DR = PR^2 + QR^2$. **(Proved)**

c Given, DR = 6 cm
PD = 4 cm
Assuming DR and PD as length and breath respectively rotating once around D a right circular cylinder is formed whose radious r = PD = 4 cm and height DR = 6 cm.



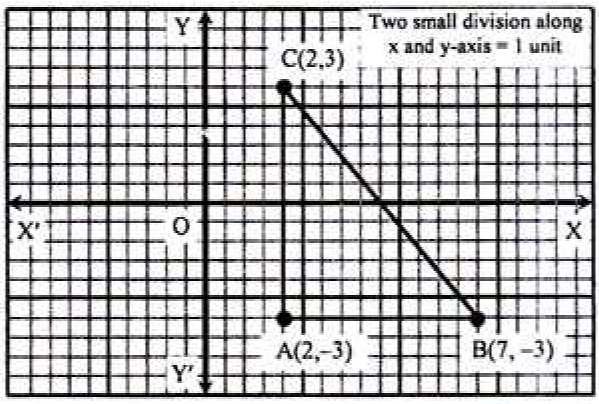
We know,
The whole surface area of the cylinder
$= 2\pi r(r + h)$ square units.
$= 2 \times 3.1416 \times 4(4 + 6)$ square units.
= 251.33 square units (approx.) **(Ans.)**
and volume of the cylinder $= \pi r^2 h$ cubic units
$= 3.1416 \times 4^2 \times 6$ cubic units.
= 301.59 cubic units (approx.) **(Ans.)**

Question ▶ 7 A(2, –3), B(7, –3) and C(2, 3). [Dj.B.16]
a. Find the slope of the straight line BC. 2
b. Plot the three point in the graph and then prove that the points are the vertices of a right angled triangle. 4
c. Find the area of the whole surface of a solid obtained by revolving the triangle ABC about AB. 4

Solution to the question no. 7

a Given, B(7,– 3) and C(2, 3)
$\therefore$ Slope of the line BC $= \frac{-3-3}{7-2} = \frac{-6}{5}$ **(Ans.)**

b



XOX′ and YOY′ are drawn as the x-axis and y-axis respectively in a graph paper. Taking O as origin the coordinates A(2, −3), B(7, −3) and C(2, 3) are ploted. Join AB, AC and BC with pencil. Then a triangle ABC is formed.

Here, $AB = \sqrt{(7-2)^2 + \{-3-(-3)\}^2} = \sqrt{5^2 + 0^2} = 5$

$\therefore AB^2 = 25$

$BC = \sqrt{(7-2)^2 + (-3-3)^2} = \sqrt{5^2 + (-6)^2}$

$= \sqrt{25+36} = \sqrt{61}$

$\therefore BC^2 = 61$

and $AC = \sqrt{(2-2)^2 + (-3-3)^2} = \sqrt{0^2 + (-6)^2}$

$= \sqrt{36} = 6$

$\therefore AC^2 = 36$

$\therefore AB^2 + AC^2 = 25 + 36 = 61 = BC^2$

$\therefore AB^2 + AC^2 = BC^2$

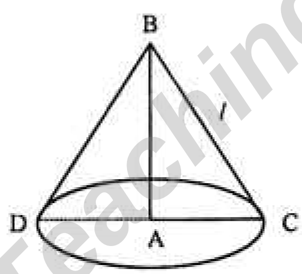
$\therefore$ ΔABC is a right angled triangle whose ∠A = 90°.

The points A(2, – 3), B(7, – 3) and C(2, 3) are the vertices of a right angled triangle. **(Proved)**

c Obtained from 'b' we get AB = 5

$BC = \sqrt{61}$ and AC = 6

In figure if ΔABC is revolved about assuming AB as axis then BCD right circular lone will be formed whose height AB = h = 5 units, radius of the base AC = r = 6 units and slant height $BC = l = \sqrt{61}$



$\therefore$ Area of the whole surface $= \pi r(r + l)$ square units

$= 3.1416 \times 6\,(6 + \sqrt{61})$

= 260.32 square units **(Ans.)**

Question ▶8 The line 2y – 3x + 6 = 0 passing through the point P(t, 2) intersects x-axis at the point A and y-axis at B. [J.B.16]

a. Find the slope of the line. 2

b. Determine the area of the ΔAPB. 4

c. Find the total surface area of the solid formed by revolving ΔOAB once about OB. 4

Solution to the question no. 8

a Given Equation, 2y – 3x + 6 = 0

Or, 2y = 3x – 6

Or, $y = \frac{3}{2}x - \frac{6}{2}$

$\therefore y = \frac{3}{2}x - 3$ .... (i)

Comparing y = mx + c in (i) we get, $m = \frac{3}{2}$

Slope of the straight line $= \frac{3}{2}$ **(Ans.)**

b Give straight line, 2y – 3x + 6 = 0 .... (ii)

The straight line intersects at point A to the x-axis.

$\therefore y = 0$

In (ii), 2.0 – 3x + 6 = 0

Or, – 3x = – 6 $\therefore$ x = 2

$\therefore$ Coordinates of A (2, 0)

Again the straight line intersects at point B to the y-axis.

$\therefore x = 0$

in (ii), 2y – 3.0 + 6 = 0 Or, 2y = – 6 $\therefore$ y = – 3

$\therefore$ The coordinates of B (0, – 3)

Since the straight line passes the point P(t, 2)

2.2 – 3.t + 6 = 0

Or, 4 – 3t + 6 = 0

Or, 10 – 3t = 0

Or, 10 = 3t

$\therefore t = \frac{10}{3}$

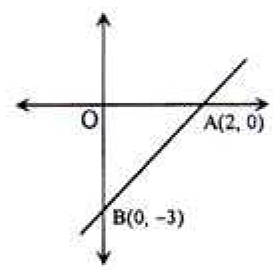
$\therefore$ Coordinates of the point P $\left(\frac{10}{3}, 2\right)$

Area of the $\Delta APB = \frac{1}{2}\begin{vmatrix} 2 & \frac{10}{3} & 0 & 2 \\ 0 & 2 & -3 & 0 \end{vmatrix}$ squire units

$= \frac{1}{2}\{(4 - 10 + 0) - (0 + 0 - 6)\}$

$= \frac{1}{2}(-6 + 6) = \frac{1}{2} \times 0 = 0$ **(Ans.)**

c



If the ΔOAB is revolved once about the side OB that solid is formed which is cone.

radious of the cone,

r = OA

$= \sqrt{(0-2)^2 + (0-0)^2} = \sqrt{4+0} = 2$

Height of the cone, $h = OB = \sqrt{(0-0)^2 + (0+3)^2}$

$= \sqrt{0+9} = 3$

$\therefore$ Slant height, $l = \sqrt{h^2 + r^2} = \sqrt{3^2 + 2^2}$

$= \sqrt{9+4} = \sqrt{13}$

$\therefore$ The whole area of the cone $= \pi r(l + r)$

$= 3.1416 \times 2 \times (\sqrt{13} + 2)$

= 35.221 square units (Approx)

(Ans.)

Question ▶9 In triangle ABC, AB = 12 cm, AC = 5 cm, BC = 13 cm and O is the point of intersection of the three medians. [B.B.16]

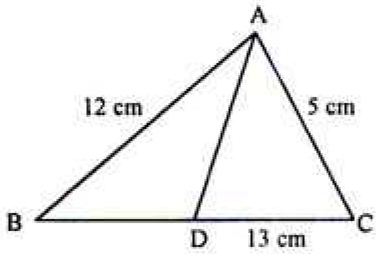
a. Find the length of the median from the vertex A on the opposite side. 2

b. According to the stem, show that the sum of the squares of the three sides of the triangle is equal to the three times of the sum of the squares of the distances from O to the vertices. 4

c. Find the difference of the numerical values of the volume and the whole surface areas of the solid which is formed by the complete revolution of the triangle about its smallest side. 4

Solution to the question no. 9

a Given, ABC triangle
AB = 12 cm, AC = 5 cm
and BC = 13 cm ।
Draw median AD from the vertex A on the opposite side of BC. From theorem of Apollonius.



$AB^2 + AC^2 = 2(AD^2 + BD^2)$

Or, $12^2 + 5^2 = 2AD^2 + 2.\left(\frac{BC}{2}\right)^2$

[$\because$ D is the midpoint of BC]

Or, $144 + 25 = 2AD^2 + 2.\left(\frac{13}{2}\right)^2$

Or, $169 = 2AD^2 + \frac{169}{2}$

Or, $2AD^2 = 169 - \frac{169}{2}$

Or, $2AD^2 = \frac{169}{2}$

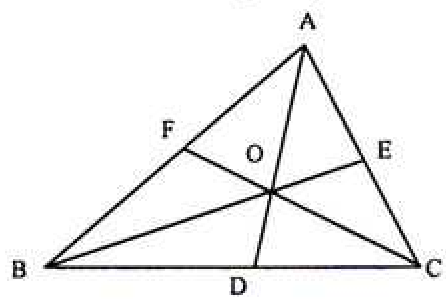
Or, $AD^2 = \frac{169}{4}$

Or, $AD = \sqrt{\frac{169}{4}}$

$\therefore AD = \frac{13}{2} = 6.5$ cm

$\therefore$ Length of median 6.5 cm **(Ans.)**

b



Given, ΔABC , AB = 12 cm, AC = 5 cm
and BC = 13 cm Apollonius theorem we get,
$AC^2 + BC^2 = 2(CF^2 + AF^2)$

Or, $5^2 + 13^2 = 2CF^2 + 2.\left(\frac{12}{2}\right)^2$

Or, $25 + 169 - 72 = 2CF^2$

Or, $CF^2 = \frac{122}{2}$

$\therefore CF^2 = 61$

Again, $AB^2 + BC^2 = 2(BE^2 + AE^2)$

Or, $12^2 + 13^2 = 2BE^2 + 2.\left(\frac{5}{2}\right)^2$

Or, $144 + 169 - 12.5 = 2BE^2$

Or, $BE^2 = \frac{300.5}{2}$

$\therefore BE^2 = 150.25$

and obtained from 'a' we get, $AD^2 = 42.25$

We know, the point of intersection of three medians divide medians in the ratio 2 : 1

$\therefore \frac{OD}{OA} = \frac{1}{2}$

Or, $\frac{OD + OA}{OA} = \frac{1+2}{2}$ [componendo-dividendo]

Or, $\frac{AD}{OA} = \frac{3}{2}$

Or, $OA = \frac{2}{3}AD$

Or, $OA^2 = \frac{4}{9}AD^2$ [squiring]

$\therefore OA^2 = \frac{4}{9} \times 42.25 = \frac{169}{9}$

Similarly, we get, $OB^2 = \frac{4}{9} \times BE^2 = \frac{4}{9} \times 150.25 = \frac{601}{9}$

and $OC^2 = \frac{4}{9} \times CF^2 = \frac{4}{9} \times 61 = \frac{244}{9}$

$\therefore 3(OA^2 + OB^2 + OC^2) = 3\left(\frac{169}{9} + \frac{601}{9} + \frac{244}{9}\right)$

$= 3 \times \frac{1014}{9} = 338$

and $AB^2 + BC^2 + AC^2 = 12^2 + 13^2 + 5^2 = 144 + 169 + 25$
$= 338$

$\therefore$ $AB^2 + BC^2 + AC^2 = 3(OA^2 + OB^2 + OC^2)$ **(Shown)**

c According to the above stem, revolving about the small side of the triangle a right circular cone is produce whose radius of the base r = AE = 12 cm, height h = AC = 5 cm and slant height l = BC = 13 cm.

$\therefore$ Area of the whole cone $= \pi r(r + l)$ square units
$= 3.1416 \times 12 \times (12 + 13)$ square units
$= 37.6992 \times 25$ square units
$= 942.48$ square units (approx.)

and volume of the cone $= \frac{1}{3}\pi r^2 h$ cubic units

$= \frac{1}{3} \times 3.1416 \times 12^2 \times 5$ cubic cm

$= 753.98$ cubic units (approx.)

$\therefore$ The difference of the numeric value of the area of whole coned volume of the cone = 942.48 – 753.98
= 188.5 (approx) **(Ans.)**

Question ▸ 10 $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ are respectively the position vectors of the vertices A, B, C and D of a rectangle ABCD.

[Mymensingh Girls' Cadet College, Mymensingh]

a. Is AB = $\underline{b} - \underline{a}$ true? Write in favor of your answer. 2
b. Show, by the help of vectors, that the diagonals of the rectangle ABCD bisect each other. 4
c. If AB = 6 cm, BC = 4 cm and ABCD is rotated about one of its length, then find the area of the whole surface and volume of the solid so formed. 4

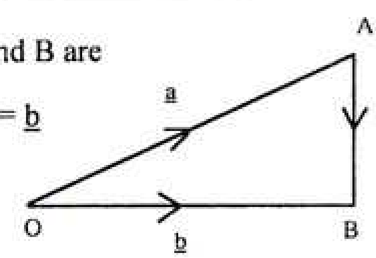
Solution to the question no. 10

a Let, for origin O,
Position vector for A and B are

given, $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$

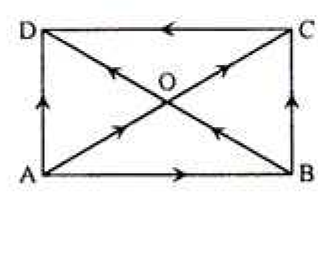
$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \underline{b} - \underline{a}$



$\therefore \overrightarrow{AB} = \underline{b} - \underline{a}$ is true.

b



Let, the diagonals AC and BD of the rectangle intersect at O.

Suppose, $\overrightarrow{AO} = \underline{a}$, $\overrightarrow{BO} = \underline{b}$, $\overrightarrow{OC} = \underline{c}$, $\overrightarrow{OD} = \underline{d}$

If is required to prove that, $|\underline{a}| = |\underline{c}|$, $|\underline{b}| = |\underline{d}|$

Proof : $\overrightarrow{AO} + \overrightarrow{OD} = \overrightarrow{AD}$ and $\overrightarrow{BO} + \overrightarrow{OC} = \overrightarrow{BC}$
Since, the opposite sides of a rectangle are equal and parallel.

$\therefore \overrightarrow{AD} = \overrightarrow{BC}$ i.e, $\overrightarrow{AO} + \overrightarrow{OD} = \overrightarrow{BO} + \overrightarrow{OC}$
Or, $\underline{a} + \underline{d} = \underline{b} + \underline{c}$

Or, $\underline{a} - \underline{c} = \underline{b} - \underline{d}$ [adding $(-\underline{c} - \underline{d})$ to both sides]

But AC is the support line of both $\underline{a}$ and $\underline{c}$

$\therefore$ AC is also the support of $\underline{a} - \underline{c}$

BD is the support line of both $\underline{b}$ and $\underline{d}$

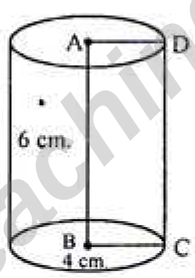
$\therefore$ BD is also the support of $\underline{b} - \underline{d}$

If $\underline{a} - \underline{c}$ and $\underline{b} - \underline{d}$ are two equal and non zero vectors, then their lines of support are same or parallel. But AC and BD are two intersecting straight lines which are not parallel.
Hence $\underline{a} - \underline{c}$ and $\underline{b} - \underline{d}$ are zero vectors

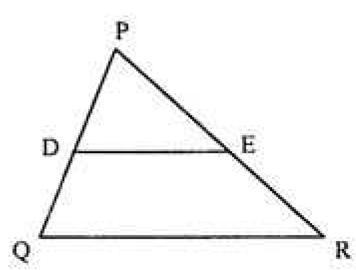
$\therefore \underline{a} - \underline{c} = 0$ or, $\underline{a} = \underline{c}$ Also $\underline{b} - \underline{d} = 0$ or, $\underline{b} = \underline{d}$
$\therefore |\underline{a}| = |\underline{c}|$ and $|\underline{b}| = |\underline{d}|$

$\therefore$ The diagonals of a rectangle bisect each other. **(Proved)**

c Given, AB = 6 cm
BC = 4 cm
Assuming AB and BC as length and breadth respectively rotating once around B a right circular cylinder is formed whose radius r = BC = 4 cm and height AB = 6 cm.
We know,
The whole surface area of the cylinder
$= 2\pi r (r + h)$ square units.
$= 2 \times 3.1416 \times 4(4 + 6)$ square units.
$= 251.33$ square units (approx.) **(Ans.)**
and volume of the cylinder $= \pi r^2 h$ cubic units
$= 3.1416 \times 4^2 \times 6$ cubic units.
$= 301.59$ cubic units (approx.) **(Ans.)**



Question ▸ 11 i.



In ΔPQR, D and E are the middle point of PQ and PR respectively.

ii. Three spherical glass of radii 9, 12 and 5 cm are melted and formed into a single cone with radius of the base 6 cm.

[Rajshahi Cadet College, Rajshahi]

a. In vector method, explain the relation between DE and QR. 2

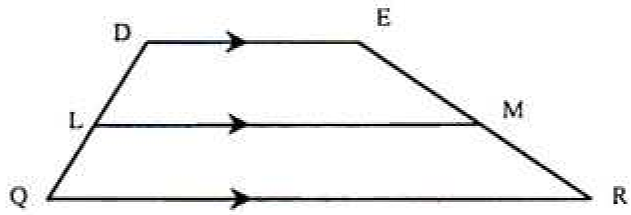
b. If L and M are the middle points of DQ and ER respectively then by the vector method, prove that DE ∥ LM ∥ QR $= \frac{1}{2}$(DE + QR) 4

c. Find the area of whole surface of the cone. 4

Solution to the question no. 11

a Similer to example–3, chapter–12, page–279 of text book.

b



Here, M and N are the midpoints of DQ and ER respectively.

We have to prove that, DE ∥ LM ∥ QR and LM $= \frac{1}{2}$(DE + QR)

Proof:

Let, position vector of D, Q, R, E are $\underline{a}$, $\underline{b}$, $\underline{c}$, and $\underline{d}$ respectively.

$\therefore$ QR $= \underline{c} - \underline{b}$ and DE = d – a.

$\therefore$ Position vector of point L $= \frac{1}{2}(\underline{a} + \underline{b})$

[∵ L is midpoint of DQ]

Position vector of point M $= \frac{1}{2}(\underline{c} + \underline{d})$

[∵ M is midpoint of RE]

Now,

$$\text{LM} = \frac{1}{2}(\underline{c} + \underline{d}) - \frac{1}{2}(\underline{a} - \underline{b}) = \frac{1}{2}(\underline{c} + \underline{d} - \underline{a} - \underline{b})$$

$$= \frac{1}{2}\left\{(\underline{c} - \underline{b}) + (\underline{d} - \underline{a})\right\} = \frac{1}{2}\left(\overrightarrow{QR} + \overrightarrow{DE}\right)$$

But as DE and QR are parallel (From 'a'), so $(\overrightarrow{DE} + \overrightarrow{QR})$ will be parallel to DE and and QR

$\therefore$ $\overrightarrow{LM} = \frac{1}{2}(\overrightarrow{DE} + \overrightarrow{QR})$ will be parallel to DE and QR. So, MN will be parallel to DE and QR

$\therefore \overrightarrow{LM} = \frac{1}{2}(\overrightarrow{DE} + \overrightarrow{QR})$

Or, $|\overrightarrow{LM}| = \frac{1}{2}|\overrightarrow{DE} + \overrightarrow{QR}|$

So, LM $= \frac{1}{2}$(DE + QR)

So, DE ∥ LM ∥ QR and LM $= \frac{1}{2}$(DE + QR) **(Proved)**

c Given,
The radii of three spherical glasses are 9 cm, 12 cm and 5 cm.
These three spheres fromed a cone.
Radius of the cone, r = 6 cm

$\therefore$ Volume of The cone $= \frac{4}{3}\pi(9)^3 + \frac{4}{3}\pi(12)^3 + \frac{4}{3}\pi(5)^3$

$= \frac{4}{3}\pi(9^3 + 12^3 + 5^3)$

$= \frac{10328}{3} \pi cm^3$

We know,

Volume of cone $= \frac{1}{3} \pi r^2 h$

here, h = hight of the cone

$\frac{1}{3} \pi r^2 h = \frac{10328}{3} \pi$

Or, $h = \frac{10328 \times 3}{3 \times 6^2}$

$= 286.89$

∴ Slant height, $l = \sqrt{h^2 + r}$

$= \sqrt{(286.95)^2 + 6^2}$

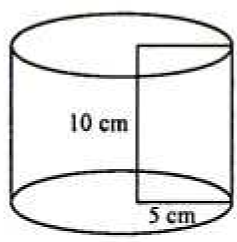
$= 286.95$ cm

∴ Area of the whole surface $= \pi r(l + r)$

$= 3.1416 \times 6 (286.95 + 6)$

$= 5521.99 \text{ cm}^2$ **(Ans.)**

Question ▶ 12



[Pabna Cadet College, Pabna]

a. Find the area of the base of the cylinder? 2

b. Find the area of the curved surface & whole surface of the cylinder? 4

c. A spherical ball exactly fits into the cylinder. Find the volume of the unoccupied portion of the cylinder. 4

Solution to the question no. 12

a Given, the radius of base of the cylinder, r = 5 cm

∴ The area of base of the cylinder $= \pi r^2$ sq. unit

$= 3.1416 \times 5^2$ sq. cm

$= 78.54$ sq. cm (approx.) **(Ans.)**

b Given, the radius of the cylinder, r = 5 cm

and height, h = 10 cm

∴ Area of the curved surface of the cylinder $= 2\pi rh$ sq. unit

$= 2 \times 3.1416 \times 5 \times 10$ sq. cm

$= 314.16$ sq. cm (approx.) **(Ans.)**

Area of the whole surface of the cylinder $= 2\pi r(h + r)$ sq. unit

$= 2 \times 3.1416 \times 5(10 + 5)$ sq. cm

$= 471.24$ sq. cm (approx.) **(Ans.)**

c Volume of the cylinder $= \pi r^2 h$ cubic unit

$= 3.1416 \times 5^2 \times 10$ cubic cm

[∵ r = 5 cm and h = 10 cm]

$= 785.4$ cubic cm (approx.)

Since a spherical ball exactly fits into the cylinder, so the radius of the ball will be equal to the radius of the cylinder.

∴ Radius of the ball, R = 5 cm.

∴ Volume of the ball $= \frac{4}{3} \pi R^3$ cubic unit

$= \frac{4}{3} \times 3.1416 \times 5^3$ cubic cm

$= 523.6$ cubic cm (approx.)

∴ Volume of the unoccupied portion of the cylinder

$= (785.4 - 523.6)$ cubic cm

$= 261.8$ cubic cm (approx.) **(Ans.)**

Question ▶ 13 Given, $3x + 4y = 12$ *[Rangpur Cadet College, Rangpur]*

a. Determine the intersecting point of $y = x - 4$ and $y = x + 4$. 2

b. If P(x, y) is the equidistant from the intersecting point of the straight line with the axes, then prove that $8x - 6y = 7$. 4

c. Find the total surface area of the solid formed, if the perpendicular height is 8 unit whose triangular base is produced by the line with the two axes. 4

Solution to the question no. 13

a Given,

$y = -x - 4$ .... (i)

and, $y = x + 4$ .... (ii)

∴ $x - 4 = -x + 4$

or, $2x = 8$

∴ $x = 4$

Putting x in (i), $y = 4 - 4 = 0$

∴ Intersecting point is (4,0) **(Ans.)**

b Given,

$3x + 4y = 12$ .... (i)

For, y = 0,

$3x + 4.0 = 12$

∴ $x = 4$

∴ Straight line (i) intersects x-axis at (4,0).

For x = 0,

$3.0 + 4.y = 12$

∴ $y = 3$

∴ Straight line (i) intersects y-axis at (0,3).

According to question P(x,y) is equidistant from (4,0) and (0,3)

∴ $\sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(x-0)^2 + (y-3)^2}$

Or, $x^2 - 8x + 16 + y^2 = x^2 + y^2 - 6y + 9$

Or, $16 - 9 = x^2 + y^2 - 6y + 8x - x^2 - y^2$

Or, $7 = 8x - 6y$

∴ $8x - 6y = 7$ **(Proved)**

c From 'b',

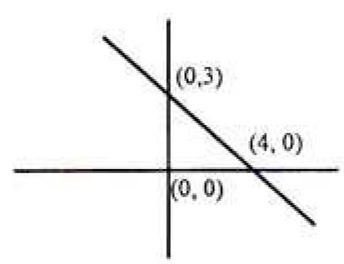
Straight line $3x + 4y = 12$ intersects x-axis and y-axis at (4,0) and (0,3) respectively.

So, the veritices of the triangle produced by the line with both axis are (4,0), (0,3) and (0,0) [∴ Area of the triangle]

$= \frac{1}{2} \begin{vmatrix} 0 & 4 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{vmatrix}$

$= \frac{1}{2}. 12$ sq. unit

$= 6$ sq. unit



Perimeter of the triangle $= 4 + 3 + \sqrt{3^2 + 4^2}$

$= 7 + 5$

$= 12$ unit.

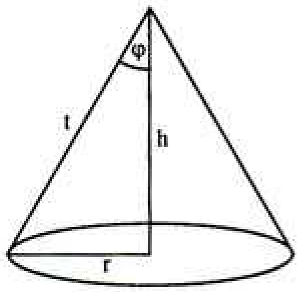
A prism will be produced with, triangle base and height of 8 unit.

∴ Area of the prism = 2 (base) + perimeter of base × height

$= (2 \times 6 + 12 \times 8)$ sq. unit

$= 108$ sq. unit. **(Ans.)**

Question ▶ 14



In the figure, r is the radius of the base, h is height and φ is a semi vertical angle of a right circular cone.

[Cumilla Cadet College, Cumilla]

a. Fine the surface area and volume of a sphere of radius 6cm. 2

b. Show that, $S = \frac{\pi h^2 \tan\varphi}{\cos\varphi} = \pi r^2 \operatorname{cosec}\varphi$ where S is the area of the curved surface. 4

c. If the volume of the cone is 1178cc and h = 12cm, find the semi vertical angle in degree. 4

Solution to the question no. 14

a Here,

Radius of sphere, r = 6 cm

$\therefore$ Surface area $= 4\pi r^2$ sq. unit

$= 4 \times 3.1416 \times (6)^2\ cm^2$

$= 452.39\ cm^2$ (approx.) **(Ans.)**

And volume of sphere $= \frac{4}{3}\pi r^3$ cubic unit

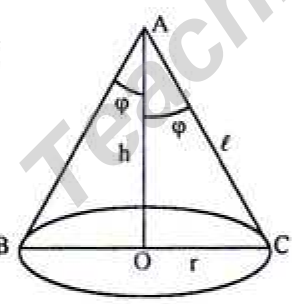
$= \frac{4}{3} \times 3.1416 \times (6)^3\ cm^3$

$= 904.78\ cm^3$ (approx) **(Ans.)**

b Here, vertical angle of the cone, $\angle CAB = 2\varphi$

Half of the vertical angle,

$\angle CAO = \varphi$



It is seen from the figure that, radius of base of the cone, OC = r,

Height of the cone, OA = h and

The slant height, AC = ℓ

Slant height, $\ell = \sqrt{h^2 + r^2}$ ... ... (i)

$\tan\varphi = \frac{r}{h}$

Or, $h = \frac{r}{\tan\varphi} = r\cot\varphi$ ... ... (ii)

Area of the curved surface, $S = \pi r l = \pi r \ell$

$= \pi \times r \times \sqrt{h^2 + r^2}$ [from (i)]

$= \pi r\sqrt{h^2 + h^2\tan^2\varphi}$ [from (ii)]

$= \pi r\, h\sqrt{1 + \tan^2\varphi}$

$= \pi r h \sec\varphi$ [$\because 1 + \tan^2\varphi = \sec^2\varphi$]

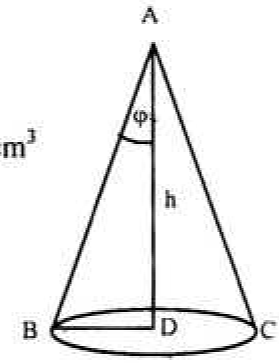
$= \pi.h\tan\varphi.\, h.\frac{1}{\cos\varphi}$ [from (ii)]

$= \frac{\pi h^2 \tan\varphi}{\cos\varphi}$ sq. unit

Again, $S = \frac{\pi h^2 \tan\varphi}{\cos\varphi} = \frac{\pi h^2 \tan^2\varphi}{\cos\varphi.\tan\varphi} = \frac{\pi(h\tan\varphi)^2 \cos\varphi}{\sin\varphi.\cos\varphi}$

$= \frac{\pi r^2}{\sin\varphi}$ sq. unit [$\because$ h tan φ = r] **(Shown)**

c Given, the height of the cone, h = 12 cm



Let, radius of base = r cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h\ cm^3$

According to the question,

$\frac{1}{3}\pi r^2 h = 1178$

Or, $r^2 = \frac{1178 \times 3}{\pi \times h}$

$= \frac{1178}{3.1416 \times 12}$ [$\because$ h = 12]

$= 93.7423$

$\therefore r = 9.682$ cm

Now, from right angled triangle ABD.

$\tan\varphi = \frac{BD}{AD} = \frac{9.682}{12}$

$\therefore \varphi = \tan^{-1}(0.8068)$

$= 38.9°$ (Appr.)

Semi-vertical angle 38.9° **(Ans.)**

Question ▶ 15 Scenario : The length and radius of a capsule is 15cm and 3cm respectively. *[Faujdarhat Cadet College, Chattogram]*

a. Prove that, $m(\underline{a}) = -m\underline{a}$. Where m is a scalar. 2

b. According to scenario – find the volume and total area of surfaces of the capsule. 4

c. If the length, height & breadth of a rectangular parallelopiped are respectively 15 meters, 3 meters and 12 meters. Find the area of its surface and length of diagonal. 4

Solution to the question no. 15

a See your text book of chapter-13, page-278.

b See your text book of chapter-13, example-9, page-297.

c Given, the length, height and breadth of a rectangular parallelepiped are respectively a = 15 meters, c = 3 meters and b = 12 meters.

So, the area of its surface = 2(ab + bc + ca) sq.m

$= 2(15\times12 + 12\times3 + 3\times15)$

$= 2(180 + 36 + 45)$

= 522 sq. meter **(Ans.)**

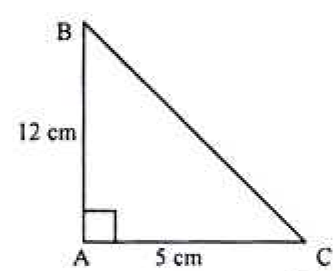
And length of diagonal $= \sqrt{a^2 + b^2 + c^2}$

$= \sqrt{(15)^2 + (12)^2 + (3)^2}$

$= \sqrt{378}$

= 19.44 meter **(Ans.)**

Question ▶ 16



[Sylhet Cadet College, Sylhet]

a. What is the perimeter of the ΔABC? 2

b. Find the area of the whole surface of the solid that will be formed if ΔABC is rotated once around AB? 4

c. If ΔABC is rotated once around AB, then a tent will be formed like a right-circular cone. What will be the cost of the canvas of the tent if its price is tk. 150 per square meter? 4

Solution to the question no. 16

a In ΔABC, $\angle A = 90°$,

$AC = 5$ m, $AB = 12$ m

By pythagoras theorem,

$BC^2 = AB^2 + AC^2$

$= 12^2 + 5^2 = 144+25 = 169 = 13^2$

$\therefore$ BC = 13 m

$\therefore$ Perimeter of $\Delta ABC = AB + BC + CA$

$= (12 + 13 + 5)$ m

= 30 m. **(Ans.)**

b If ΔABC is rotated once around AB a right circular cone will be fromed of which

Radius of base, r = 5m

Height, h = 12 m.

$\therefore$ Slant height, $l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2}$ m

$= \sqrt{169}$ m = 13 m

$\therefore$ Whole surface area of the

cone $= \pi r(r + l)$

$= 3.1416 \times 5\ (5 + 13)$ sq.m.

$= 3.1416 \times 5\ (18)$ sq.m.

$= 3.1416 \times 90$ sq.m.

= 282.744 sq. m. (Approx) **(Ans.)**

c Radius of base of the tent, r = 5 m.

Height, h = 12 m.

From 'b', Slant height. $l = 13$ m

$\therefore$ Curved surface area $= \pi rl$ sq.unit

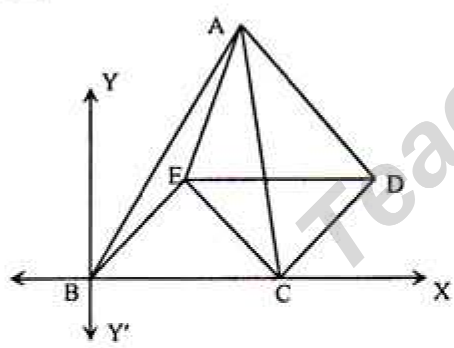
$= 3.1416 \times 5 \times 13$ sq.m.

= 204.204 sq.m.

$\therefore$ Cost for canvas $= 204.204 \times 150$ Tk.

= 30630.6 Tk. **(Ans.)**

Question ▸ 17



Here BCDE is a parallelogram in XY plane, equation of BE and CE are respectively $y = 2x$ and $x + y = 6$

[Jhenidah Cadet College, Jhenidah]

a. What is the slope of CE? 2

b. Find the coordinate of point D in X, Y plane. 4

c. If we consider ABCDE is a solid and height of it is 10 unit, find the volume of it. 4

Solution to the question no. 17

a Here, equation of CE is

$x + y = 6$

Or, $y = -x + 6$

$\therefore$ Slope of CE is -1 **(Ans.)**

b Given, Equation of CE, $x + y = 6$ which intersects x – axis at c

when y = o, then x = 6

$\therefore$ The Co-or dinate of C is (6, 0)

Again, equation of CE, $x + y = 6$ .... (i)

equation of BE, $y = 2x$ .... (ii)

From (i) and (ii) we get.

$x + 2x = 6$

Or, $3x = 6$

Or, $x = 2$

and $y = 4$

$\therefore$ Co-ordinate of E is (2, 4)

Since, ED is parallel to x– axis.

$\therefore$ Let, Co-ordinate of D is (x, 4)

Again, CD||BE and equation of BE is $y = 2x$ or $y - 2x = 0$

$\therefore$ Equation of CD is $y - 2x + k =$ --- (iii) which passes through (6,0)

then, $0 - 2 \times 12 + k = 0$

$\therefore$ Equation of CD is $y - 2x + k =$ ...... (iv) which passes through (6, 0)

then $-2 \times 6 + k = 0$

Or, $-12 + k = 0$

Or, K = 12

Putting the value of k in (iv), we get equation of CD is

$y - 2x + 12 = 0$ ... ... (v)

y = 4 in equ. (v) we get.

$4 - 2x + 12 = 0$

Or, $-2x = -16$

Or, $x = 8$

$\therefore$ Co-ordinate of D is (8, 4)

c If we consider ABCDE as a solid it will be a pyramid and its base is a parallelogram

From 'b', we know the co-ordinate of B, C, D and E are (0,0), (6,0), (8,4) and (2,4) respectively

Now, Area of the base $= \frac{1}{2}\begin{vmatrix} 0 & 6 & 8 & 2 & 0 \\ 0 & 0 & 4 & 4 & 0 \end{vmatrix}$

$= \frac{1}{2}(24 + 32 - 8)$

$= \frac{1}{2} \times 48$

= 24

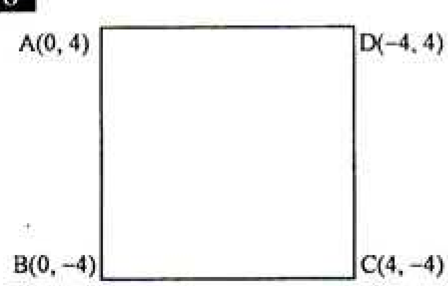
$\therefore$ Volume of pyramid $= \frac{1}{3} \times$ (Area of base) $\times$ height

$= \frac{1}{3} \times$ (Area of base) $\times$ height

$= \frac{1}{3} \times 24 \times 10$ cubic unit [Given, height = 10 unit]

= 80 Cubic unit. **(Ans.)**

Question ▸ 18



In figure, ABCD is a quadrilateral. It is revolved about AB and form a solid. *[Barishal Cadet College, Barishal]*

a. Find the area of triangle BCD. 2

b. Find the total surface area of the solid that was formed by revolving ABCD. 4

c. If a sphere has the same volume of the solid which was formed by revolving ABCD, then calculate the radius of the sphere. 4

Solution to the question no. 18

a The co-ordinates are B(0, – 4), C(4, – 4) and D(–4, 4).

$$\therefore \Delta BCD = \frac{1}{2}\begin{vmatrix} 0 & 4 & -4 & 0 \\ -4 & -4 & 4 & -4 \end{vmatrix}$$

$$= \frac{1}{2}\{(0 + 16 + 16) - (-16 + 16 + 0)\}$$

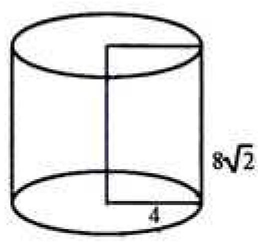
$$= \frac{1}{2}(32 - 0)$$

= 16 sq. unit **(Ans.)**

b Co-ordinate of B, C and D are (0, – 4), (4, – 4) and (– 4, 4).

$$\therefore BC = \sqrt{(0-4)^2 + (-4+4)^2} = \sqrt{16+0} = \sqrt{16} = 4$$

$$CD = \sqrt{(4+4)^2 + (-4-4)^2} = \sqrt{64+64} = 8\sqrt{2}$$



$\therefore AB = CD = 8\sqrt{2}$

If a rectangle of size ABCD surface of the solid is revolved about larger side, then a circular cylinder will be formed, whose height $h = 8\sqrt{2}$ and radius of the base r = 4

∴ The whole surface area of the cylinder

$= 2\pi r(r + h)$ sq.unit

$= 2 \times 3.1416 \times 4(4 + 8\sqrt{2})$ sq.unit

= 1137.38 sq.unit (approx.) **(Ans.)**

c From 'b' we have

radius of cylinder, r = 4

and height of cylinder, $h = 8\sqrt{2}$

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$= 3.1416 \times (4)^2 \times 8\sqrt{2}$

= 568.69 cubic units

Given,

Volume of sphere = Volume of cylinder

= 568.69

Let, radius of sphere be r

$\therefore$ Volume of sphere $= \frac{4}{3}\pi r^3$

$\therefore \frac{4}{3}\pi r^3 = 568.69$

Or, $r^3 = \frac{3 \times 568.69}{4 \times 3.1416}$

Or, $r^3 = 135.76$

$\therefore r = 5.1395$

∴ Radius of the sphere is 5.1395 unit (approx.) **(Ans.)**

Question ▶ 19 The hypotenuse and sum of other two sides of a right angled triangle are 5cm and 7cm respectively.

[RAJUK Uttara Model College, Dhaka]

a. Find the area of the triangle. 2

b. Construct the triangle. [Description and sign of construction are necessary.] 4

c. Find the volume and area of curve surface of the solid formed by greater side other than hypotenuses. 4

Solution to the question no. 19

a Let, one side of triangle = x cm

then other side = 7 – x cm

hypotenuse = 5 cm

From pythagoras theorem,

$$(5)^2 = x^2 + (7 - x)^2$$

Or, $25 = x^2 + 49 - 14x + x^2$

Or, $2x^2 - 14x + 24 = 0$

Or, $x^2 - 7x + 12 = 0$

Or, $x^2 - 3x - 4x + 12 = 0$

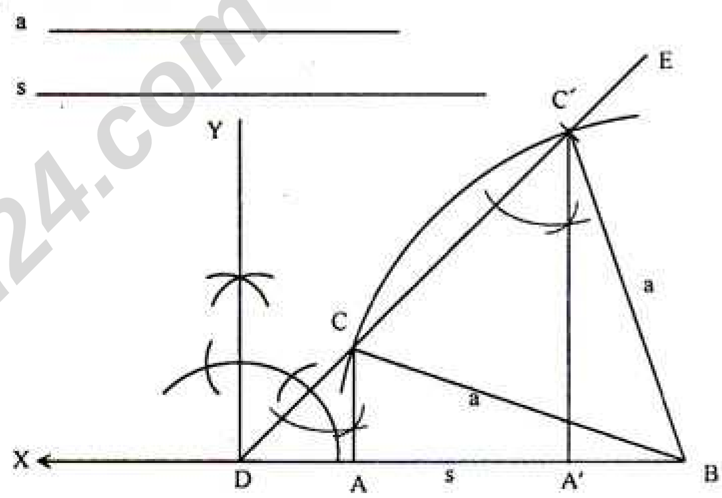
Or, $(x - 3)(x - 4) = 0$

$\therefore$ x = 3 Or, 4

∴ Length of other two sides are 3cm and 4cm.

So, the area of right angle triangle $= \frac{1}{2} \times 3 \times 4 = 6$ sq. cm. **(Ans.)**

b **General enunciation:** The length of the hypotenus and the sum of the other two sides of a right angle are given. The triangle needs to be constructed.



Particular enunciation: Let, length of the hypotenuse a and the sum of the other two sides s of a right angled triangle are given. The triangle needs to be constructed.

Steps of the construction:

Step 1: Cut the part BD = s from any ray BX.

Step 2: Draw DY ⊥ BD at D and ∠BDE = half of ∠BDY.

Step 3: Taking B as centre and the radius = a, draw a segment of circle.

Step 4: The segment intersects the line DE at C and C′.

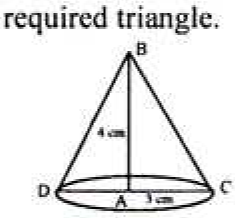
Step 5: Join B, C and B, C′.

Step 6: Now, at C and C′ draw ∠DCA= ∠CDB and ∠DC′A′ = ∠CDB.

Step 7: The lines CA and C′A′ intersect the line DB at A and A′ respectively.

Then, ΔABC or ΔA′BC′ is the required triangle.

c If we revolve the triangle about AB = 4 cm then, it creates a circular cone whose



radius of base, r = 3 cm

height, h = 4 cm

and slant height, $l = \sqrt{h^2 + r^2}$ cm

$= \sqrt{4^2 + 3^2}$ cm

= 5 cm

$\therefore$ Volume of circular cone $= \frac{1}{3}\pi r^2 h$ cubic units

$= \frac{1}{3}\pi . 3^2 . 4 \text{ cm}^3$

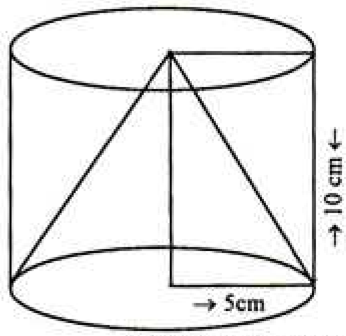
$= 37.699 \text{ cm}^3$ **(Ans.)**

And area of whole surface $= \pi r(r + l)$ sq. units

$= \pi . 3(3 + 5) \text{ cm}^2$

$= 75.398 \text{ cm}^2$ **(Ans.)**

Question ▶ 20



[Viqarunnisa Noon School & College, Dhaka]

a. Find the area of the curved surface of the right circular cone of the given stem. 2

b. Determine the area of the whole surfaces of the cylinder and the right circular cone. 4

c. If a spherical ball exactly fits into the cylinder, find the volume of the unoccupied port portion of the cylinder. 4

Solution to the question no. 20

a Given, the radius r = 5 cm and height h = 10 cm of the right circular cone.

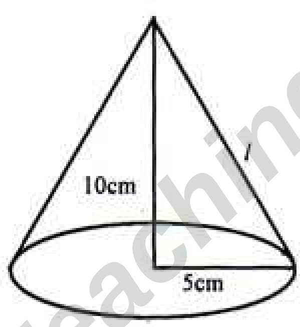
Let, l is the slant height.

Now, $l^2 = h^2 + r^2$

$= (10)^2 + (5)^2$

$= 100 + 25$

$\therefore l = 11.18$ cm



So, the area of the curved surface of the right circular cone

$= \pi r l$

$= 3.1416 \times 5 \times 11.18$

$= 175.6154 \text{ cm}^2$ **(Ans.)**

b From 'a' we get,

$l = 11.18$ cm

Also given for cylinder and right circular cone, r = 5 cm and h = 10 cm.

The whole surface area of cylinder $= 2\pi r(r + h)$

$= 2 \times 3.1416 \times 5(5 + 10)$

$= 471.24 \text{ cm}^2$ **(Ans.)**

The whole area of right circular cone $= \pi r(r + l)$

$= 3.1416 \times 5(5 + 11.18)$

$= 254.1554 \text{ cm}^2$ **(Ans.)**

c Volume of the cylinder $= \pi r^2 h$ cubic unit

$= 3.1416 \times 5^2 \times 10$ cubic cm [$\because$ r = 5 cm and h = 10 cm]

$= 785.4$ cubic cm (approx.)

Since a spherical ball exactly fits into the cylinder, so the radius of the ball will be equal to the radius of the cylinder.

$\therefore$ Radius of the ball, R = 5 cm.

$\therefore$ Volume of the ball $= \frac{4}{3}\pi R^3$ cubic unit

$= \frac{4}{3} \times 3.1416 \times 5^3$ cubic cm

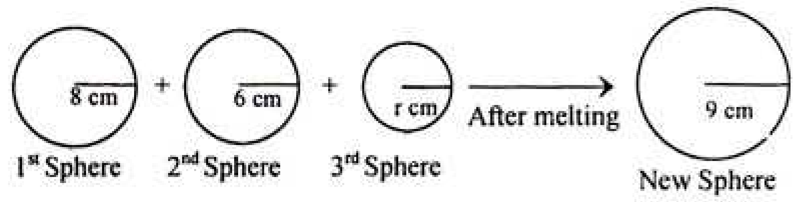
$= 523.6$ cubic cm (approx.)

$\therefore$ Volume of the unoccupied portion of the cylinder

$= (785.4 - 523.6)$ cubic cm

$= 261.8$ cubic cm (approx.) **(Ans.)**

Question ▶ 21 *[Milestone College, Dhaka]*



a. Find the area of the surface of the 2nd sphere. 2

b. Find the diameter of the 3rd sphere. 4

c. The new sphere exactly fits into a cylindrical box. Find the volume of the unoccupied portion of the box. 4

Solution to the question no. 21

a Given, the radius of the sphere R = 6 cm.

We know, if the radius of the sphere, R

the area of surface $= 4\pi R^2$ sq. unit

$\therefore$ The area of surface of the given sphere

$= 4\pi \times 6^2$ sq. cm

$= 4\pi \times 36$ sq. cm

$= 452.3904$ sq. cm (approx.) **(Ans.)**

b We know,

the volume of the sphere $= \frac{4}{3}\pi \times (\text{radius})^3$ cubic unit

Sum of the volume of the spheres having radii 6, 8, r cm

$= \left\{\frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3 + \frac{4}{3}\pi r^3\right\}$ cubic cm

$= \frac{4}{3}\pi(6^3 + 8^3 + r^3)$ cubic cm

Again, the volume of the sphere having radius 9

$= \frac{4}{3}\pi \times 9^3$ cubic cm

According to question,

$\left\{\frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3 + \frac{4}{3}\pi r^3\right\} = \frac{4}{3}\pi \times 9^3$

Or, $\frac{4}{3}\pi(6^3 + 8^3 + r^3) = \frac{4}{3}\pi \times 9^3$

Or, $6^3 + 8^3 + r^3 = 9^3$

Or, $216 + 512 + r^3 = 729$

Or, $r^3 = 729 - 728$

Or, $r^3 = 1$

$\therefore r = 1$ cm

Now, r = 1 cm

$\therefore$ Diameter = 2r

$= 2 \times 1$

$= 2$ cm **(Ans.)**

c The volume of the sphere having radius 9 cm $= \frac{4}{3}\pi(9)^3$

$= \frac{4}{3} \times 3.1416 \times 729$

$= 3053.6352$ cubic cm (approx.)

Since, the sphere having radius 9 cm exactly fits into a cylindrical box.
So the radius of the cylinder will be the radius of the sphere and the height of the cylinder will be the diameter of the sphere.

$\therefore$ Radius of the cylinder, $r = 9$ cm

and height, $h = 9 \times 2$ cm
$= 18$ cm

$\therefore$ the volume of the cylinder $= \pi r^2 h$ cubic unit
$= 3.1416 \times 9^2 \times 18$ cubic cm
$= 4580.4528$ cubic cm (approx.)

$\therefore$ The volume of the unoccupied portion of the box
$= (4580.4528 - 3053.6352)$ cubic cm
$= 1526.8176$ cubic cm (approx.) **(Ans.)**

Question ▸ 22 The circumference of a spherical ball is 44 cm. The ball fits into a cubical box.

[Saint Joseph Higher Secondary School, Dhaka]

a. Find the radius of the sphere. 2
b. Determine the volume and unoccupied portion of the box. 4
c. If the ball fits into a cylindrical box and the unoccupied portion of the box is $89\frac{5}{8}$ cm, then find the circumference of the spherical ball. 4

Solution to the question no. 22

a Let, the radius of the ball is r then circumference is $2\pi r$
According to the question, $2\pi r = 44$ cm
$\therefore r = 7$ cm (appr.)
$\therefore$ Radius of sphere is 7 cm **(Ans.)**

b From 'a' we get,
Radius of ball is 7 cm
$\therefore$ One side of cube $= 2 \times$ radius
$= 2 \times 7$ cm
$= 14$ cm
$\therefore$ Volume of the cube $= (14)^3$ cm^3
$= 2744$ cm^3
and volume of ball $= \frac{4}{3}\pi r^3$ cubic units
$= \frac{4}{3} \times 3.1416 \times (7)^3$ cm^3
$= 1436.755$ cm^3
$\therefore$ Volume of unoccupied portion
$= (2744 - 1436.755)$ cm^3
$= 1307.24$ cm^3 (appr.) **(Ans.)**

c Let, the Radius of new ball is $= R$
Then height of cylindrical box $= 2R$
$\therefore$ Volume of cylindrical box $= \pi.R^2.2R$ cubic unit
$= 2\pi R^3$ cubic unit
and volume of ball $= \frac{4}{3}\pi R^3$ cubic unit
$\therefore$ Volume of unoccupied portion
$= \left(2\pi R^3 - \frac{4}{3}\pi R^3\right)$ cubic unit
$= \frac{2}{3}\pi R^3$ cubic unit

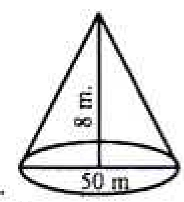
According to the question,
$\frac{2}{3}\pi R^3 = 89\frac{5}{8}$
Or, $R^3 = \frac{717}{8} \times \frac{3}{2\pi}$
Or, $R^3 = 42.792$
$\therefore R = 3.4977$
$\therefore$ Radius of spherical ball, $R = 3.4977$ cm (appr.)
and circumference $= 2\pi R$ unit
$= 2 \times 3.1416 \times 3.4977$ cm
$= 22$ cm (appr.) **(Ans.)**

Question ▸ 23 The radius of the base of a tent like a right circular cone is 30 meters and its height is 8 meters.

[BAF Shaheen College, Tejgaon, Dhaka]

a. Find the slant height of the tent. 2
b. How much land in square meters will be required to construct the tent? Find the volume of the vacuum space inside the tent? 4
c. What will be the cost of the canvas of the tent if its price is tk. 200 per square meters? 4

Solution to the question no. 23



a Given, height of the tent, $h = 8$ m.
and Radius of the base, $r = 30$ m.
We know,
Slant height, $\ell = \sqrt{h^2 + r^2}$ unit $= \sqrt{8^2 + 30^2}$ m.
$= 31.048$ m. (approx.) **(Ans.)**

b To construct the tent the required land will be equal to the area of its base which is a circle.
$\therefore$ Area of base of the tent $= \pi r^2$ sq. unit
$= 3.1416 \times 30^2$ sq. m.
$= 2827.44$ sq. m.
$\therefore$ The required land for constructing the tent is 2827.44 sq. m. **(Ans.)**
Again, the vacuum space in the tent is equal to the volume of the tent.
We know, volume of the tent $= \frac{1}{3}\pi r^2 h$ cubic unit
$= \frac{1}{3} \times 3.1416 \times 30^2 \times 8$ cubic m.
$= 7539.84$ cubic m.
$\therefore$ The vacuum space of the tent 7539.84 cubic m.

c We know, the surface area of the tent
$= \pi r l$ sq. unit
$= 3.1416 \times 30 \times 31.048$ sq.cm [from 'a']
$= 2926.212$ sq. m (approx.)
$\therefore$ If the cost for per square metre is Tk. 150, the total cost
$= (2926.212 \times 150)$ tk
$= 585242.4$ tk (approx) **(Ans.)**

Question ▸ 24 The line $2y - 3x + 6 = 0$ passing through the point P (t, 2) intersect x -axis at the point A and y-axis at B.

[Bangladesh International School & College, Dhaka]

a. Find the slope of the line. 2
b. Determine the area of the triangle APB. 4
c. Find the total surface area of the solid formed by revolving the triangle OPB once about OB. 4

Solution to the question no. 24

a Given Equation, $2y - 3x + 6 = 0$

or, $2y = 3x - 6$

or, $y = \frac{3}{2}x - \frac{6}{2}$

$\therefore y = \frac{3}{2}x - 3$ .... (i)

Comparing $y = mx + c$ with (i) we get, $m = \frac{3}{2}$

Slope of the straight line $= \frac{3}{2}$ **(Ans.)**

b Give straight line, $2y - 3x + 6 = 0$ .... (ii)

The straight line intersects at point A to the x-axis.

$\therefore y = 0$

In (ii), $2.0 - 3x + 6 = 0$

or, $-3x = -6 \;\therefore x = 2$

$\therefore$ Coordinates of A (2, 0)

Again the straight line intersects at point B to the y-axis.

$\therefore x = 0$

in (ii), $2y - 3.0 + 6 = 0$ or, $2y = -6 \;\therefore y = -3$

$\therefore$ The coordinates of B (0, –3)

Since the straight line passes the point P(t, 2)

$2.2 - 3.t + 6 = 0$

or, $4 - 3t + 6 = 0$

or, $10 - 3t = 0$

or, $10 = 3t$

$\therefore t = \frac{10}{3}$

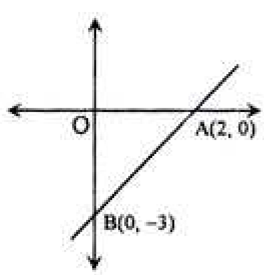
$\therefore$ Coordinates of the point P $\left(\frac{10}{3}, 2\right)$

Area of the $\Delta APB = \frac{1}{2}\begin{vmatrix} 2 & \frac{10}{3} & 0 & 2 \\ 0 & 2 & -3 & 0 \end{vmatrix}$ square units

$= \frac{1}{2}\{(4 - 10 + 0) - (0 + 0 - 6)\}$

$= \frac{1}{2}(-6 + 6) = \frac{1}{2} \times 0 = 0$ **(Ans.)**

c



If the ΔOAB is revolved once about the side OB that solid is formed which is cone.

radius of the cone,

$r = OA$

$= \sqrt{(0-2)^2 + (0-0)^2} = \sqrt{4+0} = 2$

Height of the cone, $h = OB = \sqrt{(0-0)^2 + (0+3)^2}$

$= \sqrt{0+9} = 3$

$\therefore$ Slant height, $l = \sqrt{h^2 + r^2} = \sqrt{3^2 + 2^2}$

$= \sqrt{9+4} = \sqrt{13}$

$\therefore$ The whole area of the cone $= \pi r(l + r)$

$= 3.1416 \times 2 \times (\sqrt{13} + 2)$

$= 35.221$ square units (Approx)

(Ans.)

Question ▸25 The length, breadth and height of a rectangular solid are in the ratio 4 : 3 : 2 and its area of the whole surface is 468 square meters. *[Baridhara Scholars' Institution (BSI), Dhaka]*

a. What is the length of rectangular solid? 2

b. One side of a square is equal to the length of the rectangular solid. Find the circumference of a circle which area is equal to the area of this square. 4

c. The height of a cone is 3.52 metres more than the diagonal of the rectangular solid and the radius of its base is 8 metres. Find the volume and area of the whole of the cone. 4

Solution to the question no. 25

a Let, the length, breadth and height be respectively 4x, 3x, 2x meters

Then, $2(4x.3x + 3x.2x + 2x.4x) = 468$

Or, $52x^2 = 468$

Or, $x^2 = 9$

Or, $x = 3$

$\therefore$ Length of solid is (4×3)m = 12m **(Ans.)**

b Here, side of square = 12m

By condition, Area of square = Area of circle

Let, radius of circle = r

Now, $12^2 = \pi r^2$ Or, $r^2 = \frac{12^2}{\pi}$ Or, $r = \frac{12}{\sqrt{\pi}}$

Now, the circumference of circle

$= 2\pi r = 2\pi \times \frac{12}{\sqrt{\pi}} = 24\sqrt{\pi}$ **(Ans.)**

c Diagonal of the rectangular solid

$= \sqrt{(4x)^2 + (3x^2) + (2x)^2}$

$= \sqrt{12^2 + 9^2 + 6^2}$ [$\because x = 3$]

$= \sqrt{261}$ meter

$\therefore$ Height of cone, $h = (3.52 + \sqrt{261})$ m = 19.68 m

Radius of base, r = 8m

$\therefore$ Slant height, $= \sqrt{h^2 + r^2} = \sqrt{19.68^2 + 8^2} = 21.24$ m

$\therefore$ Area of whole surface $= \pi r(l + r) = 8\pi(21.24 + 8)$

$= 734.98\ m^2$ **(Ans.)**

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8)^2 \times 19.68\ m^3 = 1318.97\ m^3$ **(Ans.)**

Question ▸26 The diameter of the base of a tent like a right circular cone is 12 meters and its height is 8 meters.

[Millennium Scholastic School & College, Bogura]

a. Find the slant height of the tent. 2

b. How much land in square meters will be required to construct the tent? Find the volume of the vacuum space inside the tent. 4

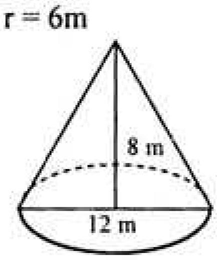
c. What will be the cost of the canvas of the tent if the price is Tk. 25 per square meter? 4

Solution to the question no. 26

a Given, height of the tent, h = 8m

and diameter of the base = 12m

$\therefore$ radius of base, r = 6m



We know, slant height, $l = \sqrt{h^2 + r^2}$ unit

$= \sqrt{8^2 + 6^2}$ m

$= \sqrt{64 + 36}$ m

$= \sqrt{100}$

= 10m **(Ans.)**

b To construct the tent, the required land will be equal to the area of its base

$\therefore$ Area of base of tent $= \pi r^2$ sq. unit

$= 3.1416 \times 6^2\ m^2$

$= 113.098\ m^2$ (appr.) **(Ans.)**

Again, the vacuum space in the tent is equal to the volume of the tent.

We know, volume of the tent $= \frac{1}{3}\pi r^2 h$ cubic unit

$= \frac{1}{3} \times 3.1416 \times (6)^2 . 8\ m^3$

$= 301.59 m^3$ (approx.) **(Ans.)**

c We know, the curved surface area of the tent $= \pi rl$ sq. unit

$= 3.1416 \times 6 \times 10\ m^2$

$= 188.4955\ m^2$ (appr.)

If the cost per square meter is Tk. 25,

then the total cost = surface area of the tent × cost for per square meter

$= (188.4955 \times 25)$

$= 4712.39$

$\therefore$ Total cost Tk. 4712.3 (Appr.) **(Ans.)**

Question ▶ 27 The straight line intersects x-axis and y-axis at the points P and Q respectively whose slope is 3 and passes through the point (–3, – 6). Another line passes through the points R (5, 3) and S(4, 0). *[Cantonment English School & College, Chattogram]*

a. Find the co-ordinates of P and Q. 2

b. Find the equation of QR and area of the quadrilateral PQRS. 4

c. The height of a conical tent is equal to the length of SP in metre. How much canvas will be required if it is desired to enclose a land of 2000 square metres? 4

Solution to the question no. 27

a The point (– 3, – 6) and slope, m = 3

The equation of straight line,

$y + 6 = m(x + 3)$

Or, $y + 6 = 3(x + 3)$

Or, $y + 6 = 3x + 9$

Or, $3x - y + 9 - 6 = 0$

$\therefore$ $3x - y + 3 = 0$ .... (i)

The straight line (i) intersects x-axis and y-axis at the points P and Q respectively.

So, the co-ordinates of P(x, 0) and Q (0, y)

Putting y = 0 in equation (i) we have,

$3x + 3 = 0$

$\therefore$ $x = -1$

So, the co-ordinates P(– 1, 0) **(Ans.)**

Putting x = 0 in equation (i) we have,

$3 \times 0 - y + 3 = 0$

$\therefore$ $y = 3$

So, the co-ordinates Q (0, 3) **(Ans.)**

b Here, Coordinate of Q = (0, 3)

" " R = (5, 3)

$\therefore$ Equation of the line QR is

$\frac{y-3}{x-0} = \frac{3-3}{0-5}$

Or, $\frac{y-3}{x} = \frac{0}{-5}$

Or, $y - 3 = 0$

$\therefore$ $y = 3$ **(Ans.)**

Area of the quadrilateral PQRS

$= \frac{1}{2}\begin{vmatrix} 5 & 0 & -1 & 4 & 5 \\ 3 & 3 & 0 & 0 & 3 \end{vmatrix}$

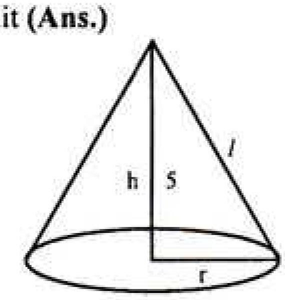
$= \frac{1}{2}(15 + 0 - 0 + 12 + 0 + 3 + 0 + 0)$

$= \frac{1}{2}(27 + 3)$

$= \frac{1}{2} \times 30$

= 15 sq. unit **(Ans.)**

c From 'a'



Coordinate of P is (–1, 0)

Given " " S " (4,0)

Length of SP $= \sqrt{(4+1)^2 + (0-0)^2}$

$= \sqrt{5^2 + 0}$

= 5 units

$\therefore$ Height of the tent, h = 5 m

And area of the land = 2000 sq.m

So, area of the base of the cone is 2000 sq.m

Let, radius of the base, r = x m.

According to the question, $\pi x^2 = 2000$ [$\because$ area of the base of cone $= \pi r^2$]

Or, $x^2 = \frac{2000}{3.1416}$ [$\because \pi = 3.1416$]

Or $x^2 = 636.6183$

$\therefore$ $x = 25.2313$

We know, slant height of the cone, $l = \sqrt{h^2 + r^2}$ units

$= \sqrt{5^2 + (25.2313)^2}$ m.

$= \sqrt{25 + 636.6183}$ m.

$= \sqrt{661.6183}$

= 25.7219 m

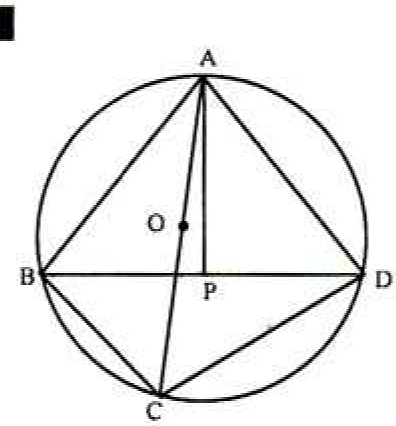
Total required canvas will be equal to the area of the curved surface of the cone.

$\therefore$ Canvas of the tent $= \pi rl$ sq.m

$= (3.1416 \times 25.2313 \times 25.7219)$ sq.m

= 2038.889 sq.m (approx.)

Question ▶ 28



[Navy Anchorage School & College, Chattogram]

a. State the ptolemy's theorem & describe according to above figure. 2

b. In $\triangle ABD$. $AP \perp BD$, AC is the diameter of the circum–circle; prove that AB. AD = AC.AP. 4

c. The solid is formed by revolving the triangle APD about AP. If AP = 8cm, PD = 6cm, what will be the cost of the canvas of the solid if its price is Tk. 110 per square meter? 4

Solution to the question no. 28

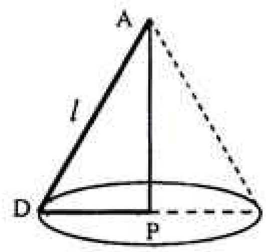
a See theorem 3.12 chap-3.2 page 78 of your text book.

b Similar to theorem 3.11, chap-3.2 page-77 of your text book.

c We know, by revolving a triangle we get a conic.

Given, AP = 8cm

PD = 6cm



So, slant height of the cone

$l = \sqrt{AP^2 + PD^2}$ cm

$= \sqrt{8^2 + 6^2}$ cm

= 10cm

$\therefore$ Curved surface of the cone $= \pi r l$

$= \pi \times PD \times 10$ sq.cm

= 188.4 sq.cm

= 0.0188 sq.m (approx.)

$\therefore$ Total required canvas will be equal to the area of the curved surface of the cone.

$\therefore$ Total cost of canvas = (0.0188 × 110) Tk

= 2.074 Tk(approx.) **(Ans.)**

Question ▶ 29 The height of a right circular cone, a semi-sphere and a cylinder are the same. They stand on the equal basis.

[Navy Anchorage School and College, Chattogram]

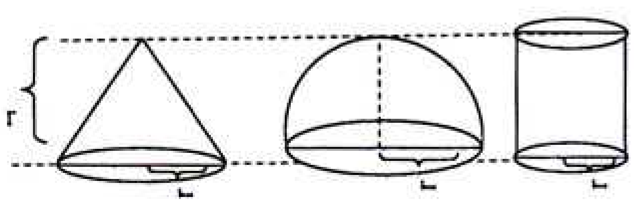
a. Draw a figure according to the given stem. 2

b. Show that, the ratio of the cone, semi- sphere and cylinder is 1 : 2 : 3. 4

c. If the volume of the cone is 150 m^3 then find the volume of the semi- sphere and the cylinder. 4

Solution to the question no. 29

a



b Let, the common height and the radius of the equal bases be h and r units respectively. Since the height of a semi -sphere is equal to its radius.

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$ cubic units.

Volume of the semi-sphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$ cubic units.

Volume of the cylinder $= \pi r^2 h = \pi r^3$

$\therefore$ Required ratio $= \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 = \frac{1}{3} : \frac{2}{3} : 1 = 1 : 2 : 3$

c Here, radius of base of cone = r

and height of cone = r

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2.r$

$= \frac{1}{3}\pi r^3$

According to the question, $\frac{1}{3}\pi r^3 = 150$

r = 5. 232 m

Now, volume of the semi sphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= 300\ m^3$

and volume of cylinder $= \pi r^2.r$

$= \pi r^3$

$= 450\ m^3$

$\therefore$ 300 m^3 ; 450 m^3 **(Ans.)**

Question ▶ 30 The height of a right-circular cone, a semi-sphere and a cylinder are the same. They stand on the equal basis.

[SCHOLARSHOME, Sylhet]

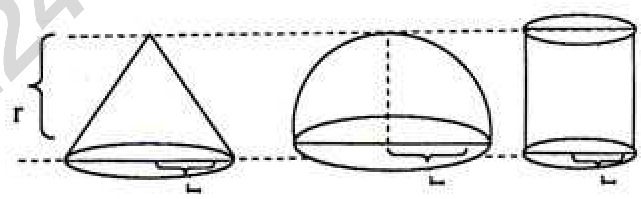
a. Draw the figure according to the given stem. 2

b. Show that, the ratio of the volume of the cone, semi-sphere and cylinder is 1 : 2 : 3. 4

c. If the volume of the cone is 150m^3 then find the volume of the semi-sphere and the cylinder. 4

Solution to the question no. 30

a



b Let the common height and the radius of the equal bases be h and r units respectively. Since the height of a semi -sphere is equal to its radius.

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$ cubic units.

Volume of the semi-sphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$ cubic units.

Volume of the cylinder $= \pi r^2 h = \pi r^3$

$\therefore$ Required ratio $= \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 = \frac{1}{3} : \frac{2}{3} : 1 = 1 : 2 : 3$

c Here, radius of base of cone = r

and height of cone = r

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2.r$

$= \frac{1}{3}\pi r^3$

According to the question, $\frac{1}{3}\pi r^3 = 150$

r = 5. 232 m

Now, volume of the semi sphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= 300\ m^3$

and volume of cylinder

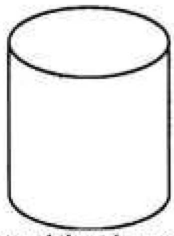
$= \pi r^2.r$

$= \pi r^3$

$= 450\ m^3$

$\therefore$ Volume of the semi sphere and the cylinder are respectively 300 m^3 ; 450 m^3 **(Ans.)**

Question ▶ 31



[The Sylhet Khajanchibari International School & College, Sylhet]

a. Find the area of the base of the cylinder, where radius is 5 cm. and height 10 cm. 2

b. Find the area of the curved surface and whole surface of the cylinder. 4

c. A spherical ball is exactly fits into the cylinder. Find the volume of the unoccupied portion of the cylinder. 4

Solution to the question no. 31

a Given, the radius of base of the cylinder, r = 5 cm

$\therefore$ The area of base of the cylinder = πr^2 sq. unit

$= 3.1416 \times 5^2$ sq. cm

$= 78.54$ sq. cm (approx.) **(Ans.)**

b Given, the radius of the cylinder, r = 5 cm

and height, h = 10 cm

$\therefore$ Area of the curved surface of the cylinder = $2\pi rh$ sq. unit

$= 2 \times 3.1416 \times 5 \times 10$ sq. cm

$= 314.16$ sq. cm **(Ans.)**

Area of the whole surface of the cylinder = $2\pi r(h + r)$ sq. unit

$= 2 \times 3.1416 \times 5(10 + 5)$ sq. cm

$= 471.24$ sq. cm **(Ans.)**

c Volume of the cylinder = $\pi r^2 h$ cubic unit

$= 3.1416 \times 5^2 \times 10$ cubic cm; [$\because$ r = 5 cm and h = 10 cm]

$= 785.4$ cubic cm

Since a spherical ball exactly fits into the cylinder, so the radius of the ball will be equal to the radius of the cylinder.

$\therefore$ Radius of the ball, R = 5 cm.

$\therefore$ Volume of the ball $= \frac{4}{3}\pi R^3$ cubic unit

$= \frac{4}{3} \times 3.1416 \times 5^3$ cubic cm

$= 523.6$ cubic cm

$\therefore$ Volume of the unoccupied portion of the cylinder

$= (785.4 - 523.6)$ cubic cm

$= 261.8$ cubic cm **(Ans.)**

Question ▶ 32 A sphere of glass with 162π cm^3 is fixed in a cylindrical box. Melting the sphere a solid right circular prism is made whose base is in the form of regular pentagon where the length of side is 2.5 cm.

[Secondary & Higher Secondary Education Board, Jashore]

a. Find the height of the cone whose volume is 48π cm^3 and radius of the base is 6 cm. 2

b. Find the volume of unoccupied portion of the cylindrical box. 4

c. Find the height of the right circular prism. 4

Solution to the question no. 32

a Given,

volume of cone = 48π cm^3

radius of cone, r = 6 cm

height, h = ?

We know,

volume of cone $= \frac{1}{3}\pi r^2 h$

Or, $48\pi = \frac{1}{3}\pi \times 6^2 \times h$

Or, $48\pi = \frac{36\pi}{3}h$

Or, $h = \frac{48\pi}{12\pi}$

$\therefore$ h = 4cm **(Ans.)**

b Given,

volume of sphere $= \frac{4}{3}\pi r^3$

Or, $162\pi = \frac{4}{3}\pi r^3$

Or, $R^3 = \frac{3 \times 162\pi}{4\pi}$

Or, $R = \sqrt[3]{121.5}$

$\therefore$ R = 4.953 cm (approx.)

So the radious of the cylinder = 4.953cm and height h = 9.906 cm

$\therefore$ Volume of the cylinder = $\pi r^2 h$ cubic unit

$= \pi \times (4.953)^2 \times 9.906$

$= 243.02\pi cm^3$

Volume of unoccupied portion of the cylinder

$= (243.02\pi - 162\pi)cm^3$

$= 81.02\pi cm^3$ **(Ans.)**

c Given, side of a regular pentagen, a = 2.5cm

$\therefore$ Area of pentagen $= \frac{5 \times (2.5)^2}{4}\cot 36°$

$= 7.8125 \times 1.3764$

$= 10.753 cm^3$

Melting the sphere of volume $162\pi cm^3$ a right circular prism is made.

So, volume of prism = 162π

Or, Area of base × height = 162π

Or, $10.753 \times h = 162\pi$

Or, $h = \frac{162 \times 3.1416}{10.753}$

$= 47.3299$ cm (approx.) **(Ans)**