Chapter-1: Sets and Functions

Question ▶ 1 The functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = \frac{2x+2}{x-1}$ and $g(x) = \frac{x-3}{2x+1}$ [D.B.17]

a. Find the domain of f. 2
b. Show that g is one-one and onto function. 4
c. If $3f^{-1}(x) = x$, then find the value of x. 4

Solution to the question no. 1

a Given, $f(x) = \frac{2x+2}{x-1}$

Now, the function $f(x)$ will be defined if and only if
$x - 1 \neq 0$ or, $x \neq 1$
$\therefore$ Domain $= \mathbb{R} - \{1\}$ **(Ans.)**

b Given, $g(x) = \frac{x-3}{2x+1}$

Now, $g(x) \in \mathbb{R}$ is valid if and only if $2x + 1 \neq 0$
Or, $x \neq -\frac{1}{2}$.
$\therefore$ Domain, $g = \mathbb{R} - \left\{-\frac{1}{2}\right\}$

g(x) will be one-one if and only if for any a, b $\in$ Domain g, g(a) = g(b) implies a = b.
Let, g(a) = g(b)
Or, $\frac{a-3}{2a+1} = \frac{b-3}{2b+1}$
Or, $2ab - 6b + a - 3 = 2ab - 6a + b - 3$
Or, $a + 6a = b + 6b$
Or, $7a = 7b$ $\therefore a = b$
Therefore, the function g(x) is one-one. **(Shown)**

Let, $y = g(x) = \frac{x-3}{2x+1}$
Or, $2xy + y = x - 3$
Or, $y + 3 = x - 2xy$
Or, $y + 3 = x(1 - 2y)$
$\therefore x = \frac{y+3}{1-2y} \in \mathbb{R}$ is valid if and only if $1 - 2y \neq 0$
Or, $y \neq \frac{1}{2}$.
$\therefore$ Range of $g(x) = \mathbb{R} - \left\{\frac{1}{2}\right\}$ = co-domain
$\therefore$ The function g(x) is onto.
[**N.B.** In case of $g : \mathbb{R} \to \mathbb{R}$ the question is not correct. The function g(x) will be defined, one-one and onto in the case of $g : \mathbb{R} - \left\{-\frac{1}{2}\right\} \to \mathbb{R} - \left\{\frac{1}{2}\right\}$ and considering this case the solution of the question is given.]

c Given, $f(x) = \frac{2x+2}{x-1}$

Let, $f^{-1}(x) = a$
Or, $x = f(a)$
Or, $x = \frac{2a+2}{a-1}$
Or, $ax - x = 2a + 2$
Or, $ax - 2a = x + 2$
Or, $a(x - 2) = x + 2$
Or, $a = \frac{x+2}{x-2}$ $\therefore f^{-1}(x) = \frac{x+2}{x-2}$
According to question, $3f^{-1}(x) = x$
Or, $3\left(\frac{x+2}{x-2}\right) = x$
Or, $3x + 6 = x^2 - 2x$
Or, $x^2 - 5x - 6 = 0$
Or, $x^2 - 6x + x - 6 = 0$
Or, $x(x - 6) + 1(x - 6) = 0$
Or, $(x - 6)(x + 1) = 0$
$\therefore x = -1, 6$ **(Ans.)**

Question ▶ 2 $E = \{x : x \in \mathbb{R}$ and $x^2 - (a+b)x + ab = 0, a, b \in \mathbb{R}\}$. $F = \{3, 4\}$ and $G = \{4, 5, 6\}$ [B.B.17]

a. Find the elements of the set E. 2
b. Prove that, $P(F \cap G) = P(F) \cap P(G)$. 4
c. Show that, $E \times (F \cup G) = (E \times F) \cup (E \times G)$. 4

Solution to the question no. 2

a Given,
$E = \{x : x \in \mathbb{R}$ and $x^2 - (a+b)x + ab = 0, a, b \in \mathbb{R}\}$
Here, $x^2 - (a+b)x + ab = 0$
Or, $x^2 - ax - bx + ab = 0$
Or, $x(x - a) - b(x - a) = 0$
Or, $(x - a)(x - b) = 0$
$\therefore x = a, b$
$\therefore E = \{a, b\}$
The elements of the set E are a and b. **(Ans.)**

b Given, $F = \{3, 4\}$ and $G = \{4, 5, 6\}$
Here, $F \cap G = \{3, 4\} \cap \{4, 5, 6\} = \{4\}$
Now, $P(F) = \{\{3, 4\}, \{3\}, \{4\}, \varnothing\}$
and $P(G) = \{\{4, 5, 6\}, \{4, 5\}, \{4, 6\}, \{5, 6\}, \{4\}, \{5\}, \{6\}, \varnothing\}$
L.H.S. $= P(F \cap G) = \{\{4\}, \varnothing\}$
and R.H.S. $= P(F) \cap P(G)$
$= \{\{3, 4\}, \{3\}, \{4\}, \varnothing\} \cap \{\{4, 5, 6\}, \{4, 5\}, \{4, 6\}, \{5, 6\}, \{4\}, \{5\}, \{6\}, \varnothing\}$
$= \{\{4\}, \varnothing\}$
$\therefore P(F \cap G) = P(F) \cap P(G)$ **(Proved)**

c Given, F = {3, 4} and G = {4, 5, 6}
From 'a' we get, E = {a, b}
Here, $F \cup G = \{3, 4\} \cup \{4, 5, 6\} = \{3, 4, 5, 6\}$
$E \times F = \{a, b\} \times \{3, 4\}$
$= \{(a, 3), (a, 4), (b, 3), (b, 4)\}$
and $E \times G = \{a, b\} \times \{4, 5, 6\}$
$= \{(a, 4), (a, 5), (a, 6), (b, 4), (b, 5), (b, 6)\}$
Now, L.H.S. $= E \times (F \cup G)$
$= \{a, b\} \times \{3, 4, 5, 6\}$
$= \{(a, 3), (a, 4), (a, 5), (a, 6), (b, 3), (b, 4), (b, 5), (b, 6)\}$
And R.H.S. $= (E \times F) \cup (E \times G)$
$= \{(a, 3), (a, 4), (b, 3), (b, 4)\} \cup \{(a, 4), (a, 5), (a, 6), (b, 4), (b, 5), (b, 6)\}$
$= \{(a, 3), (a, 4), (a, 5), (a, 6), (b, 3), (b, 4), (b, 5), (b, 6)\}$
$\therefore E \times (F \cup G) = (E \times F) \cup (E \times G)$ **(Shown)**

Question ▶ 3 $A = \{x : x \in \mathbb{R} \text{ and } x^2 - (p + q)x + pq = 0;\ p, q \in \mathbb{R}\}$, B = {2, 3} and C = {3, 4, 5}. [D.B.16]

a. Define subset and complementary set. 2
b. Show that, $P(B \cap C) = P(B) \cap P(C)$. 4
c. Prove that, $A \times (B \cup C) = (A \times B) \cup (A \times C)$. 4

Solution to the question no. 3

a **Sub-set :** If P and Q are two sets such that each of the element of Q belongs to P, then it is said that Q is a sub-set of P and it can be denoted as $Q \subset P$.
Complementary set : If U is a universal set and P be any sub-set of U, then complement set of P is defined as a set which is a sub-set of U containing all the elements of U other than the elements of P and it can be denoted as : $P' = \{x : x \notin P \text{ and } x \in U\}$.

b Given that, B = {2, 3}, C = {3, 4, 5}
$\therefore B \cap C = \{2, 3\} \cap \{3, 4, 5\} = \{3\}$
L.H.S $= P(B \cap C) = \{\{3\}, \varnothing\}$
Again, $P(B) = \{\{2\}, \{3\}, \{2, 3\}, \varnothing\}$
and $P(C) = \{\{3\}, \{4\}, \{5\}, \{3, 4\}, \{4, 5\}, \{3, 5\}, \{3, 4, 5\}, \varnothing\}$
R.H.S $= P(B) \cap P(C)$
$= \{\{2\}, \{3\}, \{2, 3\}, \varnothing\} \cap \{\{3\}, \{4\}, \{5\}, \{3, 4\}, \{4, 5\}, \{3, 5\}, \{3, 4, 5\}, \varnothing\}$
$= \{\{3\}, \varnothing\}$
$\therefore P(B \cap C) = P(B) \cap P(C)$ **(Shown)**

c Given that,
$A = \{x : x \in \mathbb{R} \text{ and } x^2 - (p + q)x + pq = 0;\ p, q \in \mathbb{R}\}$
B = {2, 3}
C = {3, 4, 5}
Now, $x^2 - (p + q)x + pq = 0$
Or, $x^2 - px - qx + pq = 0$
Or, $x(x - p) - q(x - p) = 0$
$\therefore (x - p)(x - q) = 0$
Either, $x - p = 0$ or, $x - q = 0$
$\therefore x = p$ $\therefore x = q$
$\therefore A = \{p, q\}$
Again, $B \cup C = \{2, 3\} \cup \{3, 4, 5\} = \{2, 3, 4, 5\}$
$A \times B = \{p, q\} \times \{2, 3\}$
$= \{(p, 2), (p, 3), (q, 2), (q, 3)\}$
$A \times C = \{p, q\} \times \{3, 4, 5\}$
$= \{(p, 3), (p, 4), (p, 5), (q, 3), (q, 4), (q, 5)\}$
L.H.S $= A \times (B \cup C)$
$= \{p, q\} \times \{2, 3, 4, 5\}$
$= \{(p, 2), (p, 3), (p, 4), (p, 5), (q, 2), (q, 3), (q, 4), (q, 5)\}$
R.H.S $= (A \times B) \cup (A \times C)$
$= \{(p, 2), (p, 3), (q, 2), (q, 3)\} \cup \{(p, 3), (p, 4), (p, 5), (q, 3), (q, 4), (q, 5)\}$
$= \{(p, 2), (p, 3), (p, 4), (p, 5), (q, 2), (q, 3), (q, 4), (q, 5)\}$
$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$ **(Proved)**

Question ▶ 4 $f(x) = \frac{2}{x-3}$ [R.B.17]

a. Find the domain of $f(x)$ 2
b. Determine $f^{-1}(5)$. 4
c. Draw the graph of the given function. 4

Solution to the question no. 4

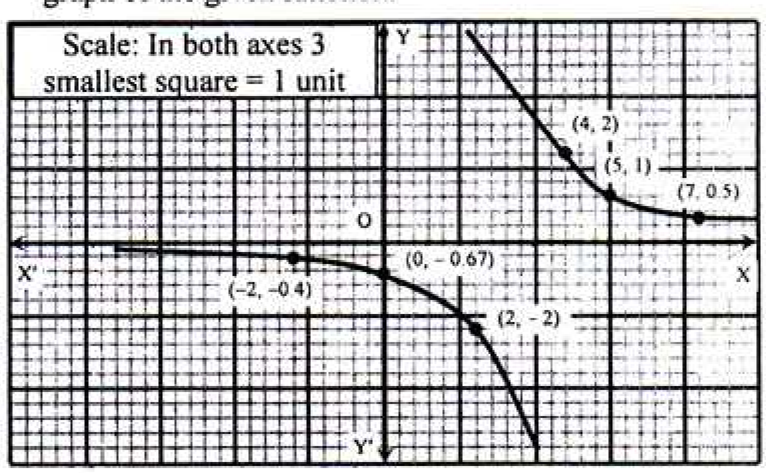
a Given, $f(x) = \frac{2}{x - 3}$
The function $f(x)$ will be defined if and only if,
$x - 3 \neq 0$ Or, $x \neq 3$.
$\therefore$ Domain $= \mathbb{R} - \{3\}$ **(Ans.)**

b Let, $f^{-1}(x) = a$
$\therefore x = f(a)$
Or, $x = \frac{2}{a - 3}$
Or, $x(a - 3) = 2$
Or, $ax - 3x = 2$
Or, $ax = 3x + 2$
Or, $a = \frac{3x + 2}{x}$
$\therefore f^{-1}(x) = \frac{3x + 2}{x}$
$\therefore f^{-1}(5) = \frac{3.5 + 2}{5} = \frac{17}{5}$ **(Ans.)**

c Let, $y = f(x) = \frac{2}{x - 3}$
Now, determine the corresponding values of y for some values of x:

x	– 2	0	2	4	5	7
y = f(x)	– 0.4	– 0.67	– 2	2	1	0.5

x-axis and y-axis have been drawn on the graph paper. Here 3 units of graph paper along x-axis and y-axis has been taken to represent 1 unit. Then plot the points (x, y). Adding these points smoothly, we have the graph of the graph of the given function.



Question ▶ 5 $f(x) = \frac{2x + 3}{x - 3}$; $x \neq 3$ is a function. [C.B.17]

a. Find the value of $f(a-1)$. 2

b. Find the inverse function of given function. 4

c. Show that the given function is one-one and onto. 4

Solution to the question no. 5

a Given, $f(x) = \frac{2x+3}{x-3}$

$\therefore f(a-1) = \frac{2(a-1)+3}{(a-1)-3}$

$= \frac{2a-2+3}{a-1-3}$

$= \frac{2a+1}{a-4}$ **(Ans.)**

b Let, $f^{-1}(x) = a$

$\therefore x = f(a)$

Or, $x = \frac{2a+3}{a-3}$

Or, $ax - 3x = 2a + 3$

Or, $ax - 2a = 3x + 3$

Or, $a(x-2) = 3x + 3$

Or, $a = \frac{3x+3}{x-2}$

$\therefore f^{-1}(x) = \frac{3x+3}{x-2}; x \neq 2$ **(Ans.)**

c $f(x)$ will be one-one if and only if for any $x_1, x_2 \in$ Domain f, $x_1 \neq x_2$; $f(x_1) = f(x_2)$ implies $x_1 = x_2$

Let, $f(x_1) = f(x_2)$

Or, $\frac{2x_1+3}{x_1-3} = \frac{2x_2+3}{x_2-3}$

Or, $2x_1x_2 + 3x_2 - 6x_1 - 9 = 2x_1x_2 - 6x_2 + 3x_1 - 9$

Or, $-6x_1 - 3x_1 = -6x_2 - 3x_2$

Or, $-9x_1 = -9x_2$

$\therefore x_1 = x_2$

$\therefore$ The function f is one-one.

Again, let, $y = f(x)$

$\therefore y = \frac{2x+3}{x-3}$

Or, $xy - 3y = 2x + 3$

Or, $xy - 2x = 3y + 3$

Or, $x(y-2) = 3y + 3$

$\therefore x = \frac{3y+3}{y-2}$

Now, $f\left(\frac{3y+3}{y-2}\right) = \frac{2 \cdot \frac{3y+3}{y-2} + 3}{\frac{3y+3}{y-2} - 3}$

$= \frac{6y+6+3y-6}{y-2} \times \frac{y-2}{3y+3-3y+6}$

$= \frac{9y}{9}$

$= y = f(x)$

$\therefore$ The function f is onto.

Therefore, the function f is one-one and onto. **(Shown)**

Question ▶ 6 $S = \{(x, y): x^2 + y^2 + 6x + 8y + 9 = 0\}$ be a relation and $A = \{x : x \in \mathbb{N}, x$ is prime number and $x < 7\}$, $B = \{x : x$ is positive integer and $\sqrt{x} < 2\}$ are two sets. [Ctg.B.17]

a. Express the set B in tabular method. 2

b. Show that, $P(A) \cap P(B) = P(A \cap B)$ 4

c. Draw the graph of the relation "S" and ascertain from the graph whether 'S' is a function or not. 4

Solution to the question no. 6

a Given, $B = \{x : x$ positive integer and $\sqrt{x} < 2\}$

Now, if $x = 1, \sqrt{x} = \sqrt{1} = 1 < 2$

if $x = 2, \sqrt{x} = \sqrt{2} = 1.4142 < 2$

if $x = 3, \sqrt{x} = \sqrt{3} = 1.7320 < 2$

if $x = 4, \sqrt{x} = \sqrt{4} = 2$

$\therefore B = \{1, 2, 3\}$ **(Ans.)**

b Given, $A = \{x : x \in \mathbb{N}, x$ prime number and $x < 7\}$

$= \{2, 3, 5\}$

and $B = \{1, 2, 3\}$ [from 'a']

$\therefore P(A) = \{\{2\}, \{3\}, \{5\}, \{2, 3\}, \{3, 5\}, \{2, 5\}, \{2, 3, 5\}, \varnothing\}$

$P(B) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}, \varnothing\}$

$A \cap B = \{2, 3, 5\} \cap \{1, 2, 3\}$

$= \{2, 3\}$

L.H.S. $= P(A) \cap P(B)$

$= \{\{2\}, \{3\}, \{2, 3\}, \phi\}$

R.H.S. $= P(A \cap B) = \{\{2\}, \{3\}, \{2, 3\}, \phi\}$

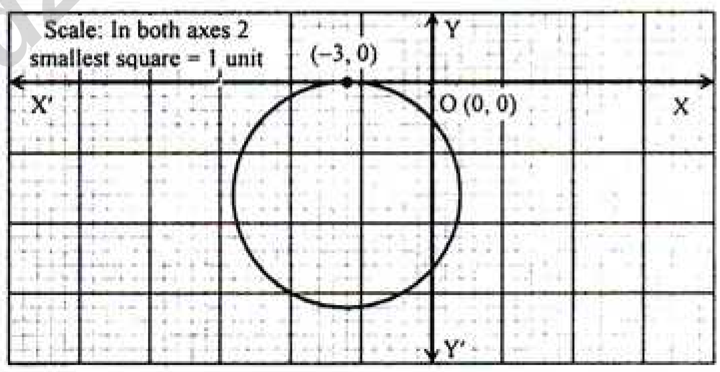
$\therefore P(A) \cap P(B) = P(A \cap B)$ **(Shown)**

c Given, $S = \{(x, y) : x^2 + y^2 + 6x + 8y + 9 = 0\}$

$= \{(x, y) : x^2 + 2.x.3 + 3^2 + y^2 + 2.y.4 + 4^2 - 16 = 0\}$

$= \{(x, y) : (x+3)^2 + (y+4)^2 = 4^2\}$

So, the graph of S is a circle whose centre is $(-3, -4)$ and radius is 4. Plotting the point on the graph paper and drawing a circle by taking 4 units and centre at $(-3, -4)$, we get the graph of S which is shown below:



In the graph we see that, the circle intersects at two points on the y-axis. So, S is not a function.

Question ▶ 7 $f(x) = \sqrt{2x-3}$ is a function. [S.B.17]

a. If $f(x) = 1$, then determine the value of x. 2

b. Determine the Domain of $f(x)$ and show the function is one-one. 4

c. Determine the range of $f^{-1}(x)$. 4

Solution to the question no. 7

a Given, $f(x) = \sqrt{2x-3}$

According to question, $f(x) = 1$

Or, $\sqrt{2x-3} = 1$

Or, $2x - 3 = 1$ [by squaring]

Or, $2x = 1 + 3$

Or, $x = \frac{4}{2}$

$\therefore x = 2$ **(Ans.)**

b $f(x)$ will be defined if and only if $2x - 3 \geq 0$.

Or, $2x - 3 + 3 \geq 3$

Or, $2x \geq 3 \quad \therefore x \geq \frac{3}{2}$

$\therefore$ Domain = $\left\{x \in \mathbb{R} : x \geq \frac{3}{2}\right\}$ **(Ans.)**

$f(x)$ will be one-one if and only if for any $x_1, x_2 \in$ Domain f,

$x_1 \neq x_2$ if $f(x_1) = f(x_2)$ then $x_1 = x_2$.

Let, $f(x_1) = f(x_2)$

$\therefore \sqrt{2x_1 - 3} = \sqrt{2x_2 - 3}$

Or, $2x_1 - 3 = 2x_2 - 3$ [by squaring]

Or, $2x_1 = 2x_2$ $\therefore x_1 = x_2$

$\therefore$ The function f is one-one. **(Shown)**

c Let, $f^{-1}(x) = a$

Or, $x = f(a)$ Or, $x = \sqrt{2a - 3}$

Or, $x^2 = 2a - 3$

Or, $2a = x^2 + 3$

Or, $a = \frac{x^2 + 3}{2}$

$\therefore f^{-1}(x) = \frac{x^2 + 3}{2}$

Again, let, $y = \frac{x^2 + 3}{2}$

Or, $x^2 + 3 = 2y$

Or, $x^2 = 2y - 3$

$\therefore x = \sqrt{2y - 3}$

Now, x will be defined if and only if $2y - 3 \geq 0$ or, $y \geq \frac{3}{2}$.

$\therefore$ Range of $f^{-1}(x) = \left\{y \in \mathbb{R} : y \geq \frac{3}{2}\right\}$ **(Ans.)**

Question ▶8 $f(x) = \frac{4x + 3}{2x + 5}$ [J.B.17]

a. Find the domain of $f(x)$. 2

b. Show that, $f(x)$ is one-one function. 4

c. If $f^{-1}(-2) = p.f^{-1}(-3)$, find the value of p. 4

Solution to the question no. 8

a Given, $f(x) = \frac{4x + 3}{2x + 5}$

$f(x)$ will be defined if and only if $2x + 5 \neq 0$.

or, $2x \neq -5$

$\therefore x \neq -\frac{5}{2}$

$\therefore$ Domain = $\mathbb{R} - \left\{-\frac{5}{2}\right\}$ **(Ans.)**

b For any $x_1, x_2 \in$ Domain f, $f(x)$ will be one-one if and only if $f(x_1) = f(x_2)$ then, $x_1 = x_2$.

Let, $f(x_1) = f(x_2)$

Or, $\frac{4x_1 + 3}{2x_1 + 5} = \frac{4x_2 + 3}{2x_2 + 5}$

Or, $8x_1x_2 + 6x_2 + 20x_1 + 15 = 8x_1x_2 + 6x_1 + 20x_2 + 15$

Or, $20x_1 - 6x_1 = 20x_2 - 6x_2$

Or, $14x_1 = 14x_2$

$\therefore x_1 = x_2$

$\therefore$ the function f is one-one. **(Shown)**

c Let $f^{-1}(x) = a$

Or, $x = f(a)$

Or, $x = \frac{4a + 3}{2a + 5}$

Or, $2ax + 5x = 4a + 3$

Or, $2ax - 4a = 3 - 5x$

Or, $a(2x - 4) = 3 - 5x$

Or, $a = \frac{3 - 5x}{2x - 4}$

$\therefore f^{-1}(x) = \frac{3 - 5x}{2x - 4}$

According to question, $f^{-1}(-2) = p f^{-1}(-3)$

Or, $\frac{3 - 5.(-2)}{2.(-2) - 4} = p\left\{\frac{3 - 5.(-3)}{2.(-3) - 4}\right\}$

Or, $\frac{3 + 10}{-4 - 4} = p\left(\frac{3 + 15}{-6 - 4}\right)$

Or, $\frac{13}{-8} = p\left(\frac{18}{-10}\right)$

Or, $p = \frac{13 \times 10}{18 \times 8}$

$\therefore p = \frac{65}{72}$ **(Ans.)**

Question ▶9 $F(x) = \sqrt{2 - 4x}$ is a function. [C.B.16]

a. Find the domain of the function stated by F(x). 2

b. Determine whether the function F is one-one or not. 4

c. Find the value of $F^{-1}(-3)$. 4

Solution to the question no. 9

a Given,

$F(x) = \sqrt{2 - 4x}$

$F(x) \in \mathbb{R}$ if and only if

$2 - 4x \geq 0$.

Or, $-4x \geq -2$

Or, $4x \leq 2$

$\therefore x \leq \frac{1}{2}$

$\therefore$ Domain F = $\{x \in \mathbb{R} : x \leq \frac{1}{2}\}$ **(Ans.)**

b $F(x) = \sqrt{2 - 4x}$

Let, $x_1, x_2 \in$ domain F

The function will be one-one if and only if $F(x_1) = F(x_2)$ then $x_1 = x_2$.

$\therefore F(x_1) = F(x_2)$

Or, $\sqrt{2 - 4x_1} = \sqrt{2 - 4x_2}$

Or, $2 - 4x_1 = 2 - 4x_2$

Or, $-4x_1 = -4x_2$

$\therefore x_1 = x_2$ [Dividing by -4]

$\therefore$ F(x) is a one-one function **(Ans.)**

c Let, $y = F(x) = \sqrt{2 - 4x}$

Now, $F(x) = y$

$\therefore x = F^{-1}(y)$

Again, $y = \sqrt{2 - 4x}$

Or, $y^2 = 2 - 4x$

Or, $4x = 2 - y^2$

Or, $x = \frac{2 - y^2}{4}$

Or, $F^{-1}(y) = \frac{2 - y^2}{4}$

$\therefore F^{-1}(x) = \frac{2 - x^2}{4}$

$\therefore F^{-1}(-3) = \frac{2 - (-3)^2}{4} = \frac{2 - 9}{4} = \frac{-7}{4}$ **(Ans.)**

Question ▶ 10 Given, $f(a) = \frac{a+y}{a-y}$ and $g(x) = \frac{1}{2x+1}$.

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a. Find the discriminant of the equation $2x^2 - 7x + 1 = 0$. 2

b. Solve: $f(x) + \frac{1}{f(x)} = \frac{5}{2}$ and $x^2 + y^2 = 90$. 4

c. Find the domain and range of g(x) and show that g(x) is one-one function 4

Solution to the question no. 10

a We know, the discriminant of $ax^2 + bx + c = 0$ is $b^2 - 4ac$

$\therefore$ Discriminant of $2x^2 - 7x + 1 = 0$

is $(-7)^2 - 4.2.1$

$= 49 - 8$

$= 41$ **(Ans.)**

b Given, $f(a) = \frac{a+y}{a-y}$

$\therefore f(x) = \frac{x+y}{x-y}$

and $\frac{1}{f(x)} = \frac{x-y}{x+y}$

$\therefore f(x) + \frac{1}{f(x)} = \frac{5}{2}$

Or, $\frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{5}{2}$ .... (i)

and $x^2 + y^2 = 90$ .... (ii)

From (i) we get,

$\frac{(x+y)^2 + (x-y)^2}{(x+y)(x-y)} = \frac{5}{2}$

Or, $\frac{2(x^2+y^2)}{x^2-y^2} = \frac{5}{2}$

Or, $\frac{2 \times 90}{x^2-y^2} = \frac{5}{2}$ [putting $x^2 + y^2 = 90$ from (ii)]

$\therefore x^2 - y^2 = 72$ .... (iii)

(ii) + (iii) implies, $2x^2 = 162$

Or, $x^2 = 81$

Or, $x = \pm 9$

and (ii) – (iii) implies, $2y^2 = 18$

Or, $y^2 = 9$

Or, $y = \pm 3$

$\therefore$ Required solution: (x, y) = (9, 3), (9, –3), (–9, 3), (–9, –3) **(Ans.)**

c Given, $g(x) = \frac{1}{2x+1}$

Now, the function g(x) will not be defined if and only if $2x + 1 = 0$

or, $x = -\frac{1}{2}$

$\therefore$ Dom g $= \mathbb{R} - \left\{-\frac{1}{2}\right\}$ **(Ans.)**

Let, $g(x) = y$

Or, $\frac{1}{2x+1} = y$

Or, $2xy + y = 1$

Or, $2xy = 1 - y$

Or, $x = \frac{1-y}{2y}$

Now, $x = \frac{1-y}{2y} \in \mathbb{R}$ if and only if $y \neq 0$

$\therefore$ Range g $= \mathbb{R} - \{0\}$ **(Ans.)**

Again, Let $x_1, x_2 \in$ Dom g

and $g(x_1) = g(x_2)$

Or, $\frac{1}{2x_1+1} = \frac{1}{2x_2+1}$

Or, $2x_1 + 1 = 2x_2 + 1$

Or, $2x_1 = 2x_2$

Or, $x_1 = x_2$

$\therefore$ g(x) is a one-one function. **(Shown)**

Question ▶ 11 The functions : $f : R \to R$ and $g : R \to R$ are defined by $f(x) = x^7 + 5$ and $g(x) = (x-5)^{\frac{1}{7}}$ respectively.

[Mymensingh Girls' Cadet College, Mymensingh]

a. Find the value of $g^{-1}(-1)$ 2

b. Ascertain whether f(x) is onto function. 4

c. Show that $f = g^{-1}$. 4

Solution to the question no. 11

a Given,

$g(x) = (x-5)^{\frac{1}{7}}$

Let, $g^{-1}(x) = a$

$\therefore$ $x = g(a)$

Or, $x = (a-5)^{\frac{1}{7}}$

Or, $x^7 = a - 5$

Or, $a = x^7 + 5$

$\therefore$ $g^{-1}(x) = x^7 + 5$

$\therefore$ $g^{-1}(-1) = (-1)^7 + 5 = -1 + 5 = 4$ **(Ans.)**

b Given, $f(x) = x^7 + 5$

Let, $y = f(x) = x^7 + 5$

Or, $y = x^7 + 5$

Or, $x^7 = y - 5$

$\therefore x = (y-5)^{\frac{1}{7}}$

Let, $h(y) = (y-5)^{\frac{1}{7}}$

For any real value of y, the function h(y) is defined.

$\therefore$ Range of f $= \mathbb{R}$ = co-domain of h(y)

$\therefore$ The function f(x) is onto function.

c Given, $f(x) = x^7 + 5$

$g(x) = (x-5)^{\frac{1}{7}}$

From, 'a' $g^{-1}(x) = x^7 + 5$

$\therefore$ $f(x) = g^{-1}(x)$

$\therefore f = g^{-1}$ **(Shown)**

Question ▶ 12 $f(x) = \frac{x-2}{x-3}$ where $x \neq 3$ and $g(x) = \frac{x-3}{2x+1}$ where $x \neq -\frac{1}{2}$. *[Rajshahi Cadet College, Rajshahi]*

a. Explain the range of $p(x) = 2x - 3$ 2
b. Justify $f(x)$ is onto function or not. 4
c. Find the value of $g^{-1}(5)$. 4

Solution to the question no. 12

a Given,

$p(x) = 2x - 3$

Let, $p(x) = y = 2x - 3$

Or, $2x = y + 3$

Or, $x = \frac{3}{2} + \frac{y}{2}$

$\therefore p^{-1}(x) = \frac{3}{2} + \frac{x}{2}$

So, $\frac{3}{2} + \frac{x}{2}$ is defined for all possible values of x.

$\therefore$ Range of $p(x) = (-\infty, \infty)$ **(Ans.)**

b Given,

$f(x) = \frac{x-2}{x-3}$

Let, $f(x) = y = \frac{x-2}{x-3}$

$xy - 3y = x - 2$

$xy - x = 3y - 2$

$x(y-1) = 3y-2$

$x = \frac{3y-2}{y-1}$

$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3}$

$= \frac{\frac{3y-2-2y+2}{y-1}}{\frac{3y-2-3y+3}{y-1}} = \frac{\frac{y}{y-1}}{\frac{1}{y-1}}$

$= \frac{y}{(y-1)} \times (y-1) = y = f(x)$

$\therefore$ $f(x)$ is onto function (Justified)

c Let, $y = g(x)$

$\therefore x = g^{-1}(y)$

Now, $g(x) = \frac{x-3}{2x+1}$

Or, $y = \frac{x-3}{2x+1}$

Or, $2xy + y = x - 3$

Or, $2xy - x = -y - 3$

Or, $x(2y-1) = -y - 3$

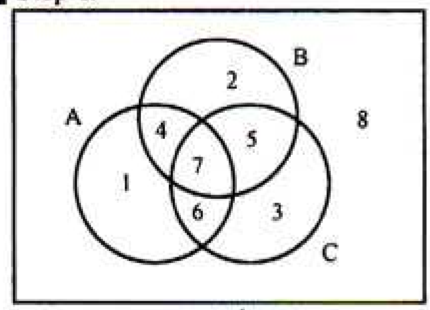
Or, $x = \frac{-y-3}{2y-1} = \frac{y+3}{1-2y}$

$\therefore g^{-1}(y) = \frac{y+3}{1-2y}$

Or, $g^{-1}(x) = \frac{x+3}{1-2x}$ [Interchanging the variables]

$\therefore g^{-1}(5) = \frac{5+3}{1-(2\times5)} = -\frac{8}{9}$ **(Ans.)**

Question ▶ 13 Step-I.



Step-II. The function, $F(x) = \frac{ax+b}{cx+d}$, where a, b, c ∈ R. *[Joypurhat Girls' Cadet College, Joypurhat]*

a. Represent the set area no. –7 by the sets A, B, C in the step-I. 2
b. State and prove "DeMorgans Law" by the sets A, B, C in the step-I. 4
c. Find the domain, range, inverse function and justify the function one-one or onto in step-II. 4

Solution to the question no. 13

a From the figure,

$A = \{1, 4, 6, 7\}$
$B = \{2, 4, 5, 7\}$
$C = \{3, 5, 6, 7\}$

$A \cap B = \{1,4,6,7\} \cap \{2,4,5,7\}$
$= \{4,7\}$

$\therefore (A \cap B) \cap C = \{2,7\} \cap \{3,5,6,7\}$
$= \{7\}$

$\therefore$ The set area is $(A \cap B) \cap C$. **(Ans.)**

b DeMorgans law: For any subsets A and B of a universal set U, we have

i) $(A \cup B)' = A' \cap B'$

ii) $(A \cap B)' = A' \cup B'$.

Proof :

From the figure,

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

From 'a,

$A = \{1, 4, 6, 7\}$
$B = \{2, 4, 5, 7\}$
$C = \{3, 5, 6, 7\}$

(i) $A \cup B = \{1, 4, 6, 7\} \cup \{2, 4, 5, 7\}$
$= \{1, 2, 4, 5, 6, 7\}$

$(A \cup B)' = U - (A \cup B)$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} - \{1, 2, 4, 5, 6, 7\}$
$= \{3, 8\}$

$A' = U - A$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} - \{1, 4, 6, 7\}$
$= \{2, 3, 5, 8\}$

$B' = U - B$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 4, 5, 7\}$
$= \{1, 3, 6, 8\}$

$\therefore A' \cap B' = \{2, 3, 5, 8\} \cap \{1,3, 6, 8\}$
$= \{3, 8\}$

$\therefore (A \cup B)' = A' \cap B'$ **(Proved)**

(ii) $(A \cap B) = \{1, 4, 6, 7\} \cap \{2, 4, 5, 7\}$
$= \{4, 7\}$

$(A \cap B)' = U - (A \cap B)$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} - \{4, 7\}$
$= \{1, 2, 3, 5, 6, 8\}$

$A' = \{2, 3, 5, 8\}$

$B' = \{1, 3, 6, 8\}$

$\therefore \quad A' \cup B' = \{2, 3, 5, 8\} \cup \{1, 3, 6, 8\}$

$= \{1, 2, 3, 5, 6, 8\}$

$\therefore \quad (A \cap B)' = A' \cup B'$ **(Proved)**

c Given function,

$$F(x) = \frac{ax + b}{cx + d}$$

The function $f(x)$ is defined if and only if $cx + d \neq 0$

Or, $cx \neq -d$

$\therefore x \neq \frac{-d}{c}$

$\therefore$ Domain of the function $= \mathbb{R} - \left\{-\frac{d}{c}\right\}$ **(Ans.)**

Let, $y = f(x) \quad \therefore x = f^{-1}(y)$

Or, $y = \frac{ax + b}{cx + d}$

Or, $cxy + yd = ax + b$

Or, $cxy - ax = b - yd$

Or, $x(cy - a) = b - yd$

$\therefore x = \frac{b - yd}{cy - a}$

Now, $x = \frac{b - yd}{cy - a} \in R$ if and only if

$cy - a \neq 0$

$\therefore y = \frac{a}{c}$

$\therefore$ Range of the function $= \mathbb{R} - \left\{\frac{a}{c}\right\}$ **(Ans.)**

Here, $x = \frac{b - yd}{cy - a}$

$\therefore f^{-1}(y) = \frac{b - yd}{cy - a}$

Or, $f^{-1}(x) = \frac{b - xd}{cx - a}$ **(Ans.)**

Let, $x_1, x_2 \in$ domain F

F(x) will be one one if and only if $F(x_1) = F(x_2)$ for which $x_1 = x_2$.

$\therefore f(x_1) = F(x_2)$

Or, $\frac{ax_1 + b}{cx_1 + d} = \frac{ax_2 + b}{cx_2 + d}$

Or, $acx_1x_2 + bcx_1 + adx_2 + bd = acx_1x_2 + adx_1 + bcx_2 + bd$

Or, $bcx_1 - adx_1 = bcx_2 - adx_2$

Or, $x_1(bc - ad) = x_2(bc - ad)$

$\therefore x_1 = x_2$

$\therefore$ F(x) is one one function. **(Ans.)**

Let, $y \in$ Range F,

$$\therefore F\left(\frac{b - yd}{cy - a}\right) = \frac{a\frac{b-yd}{cy-a} + b}{c\frac{b-yd}{cy-a} + d}$$

$$= \frac{ab - ady + bcy - ab}{cy - a} \div \frac{bc - cdy + cdy - ad}{cy - a}$$

$$= \frac{y(bc - ad)}{cy - a} \times \frac{cy - a}{bc - ad} = y$$

$\therefore$ The function is on to. Thus the function is one-one and onto. **(Justified)**

Question ▸ 14 A relation is described by $x^2 + y^2 = 4$.

[Pabna Cadet College, Pabna]

a. Express the relation in the form of $y = f(x)$ & find the domain of f(x). 2

b. If $y \geq 0$ then verify whether the relation is function & also prove that it is not one-one function. 4

c. Draw the graph of the relation & also write down the geometric name of it? 4

Solution to the question no. 14

a Given, $x^2 + y^2 = 4$

Or, $y^2 = 4 - x^2$

$\therefore y = \pm\sqrt{4 - x^2}$

$\therefore y = f(x) = \pm\sqrt{4 - x^2}$

Now, $f(x)$ will be defined if

$4 - x^2 \geq 0$

Or, $4 \geq x^2$

Or, $|x| \leq 2$

$\therefore -2 \leq x \leq 2$

$\therefore$ Domain, $f = \{x : x \in \mathbb{R}$ and $-2 \leq x \leq 2\}$ **(Ans.)**

b From 'a'

$y = \sqrt{4 - x^2} \quad [\because y \geq 0]$

Let, $f(x) = \sqrt{4 - x^2}$

For any value of x there is no more than one value of y.

So, the relation is a function.

Now, the function will be one-one if for any a, b, $\in$ Domain f, $f(a) = f(b)$ implies that $a = b$.

Now, $f(a) = f(b)$

Or, $\sqrt{4 - a^2} = \sqrt{4 - b^2}$

Or, $a^2 = b^2$

Or, $4 - a^2 = 4 - b^2$

Or, $-a^2 = -b^2$

$\therefore a = \pm b$

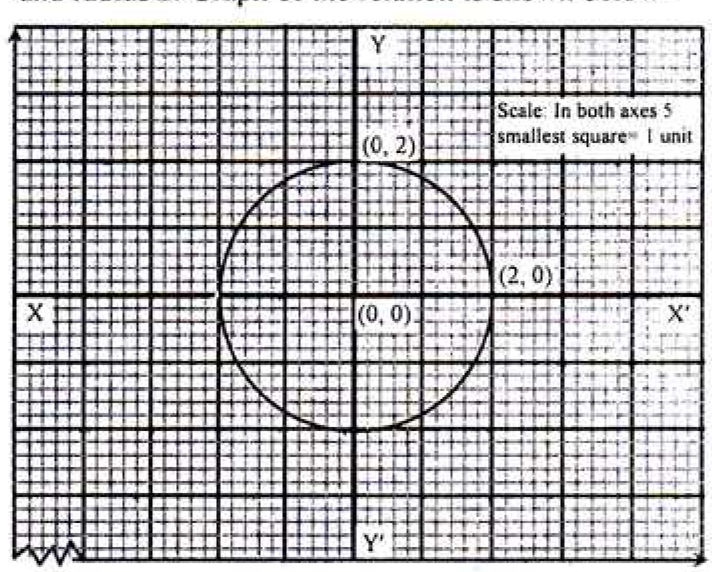
So, the function is not one-one.

c Given,

$x^2 + y^2 = 4$

$\therefore (x - 0)^2 + (y - 0)^2 = 2^2$

The graph of the given relation is a circle with centre (0,0) and radius 2. Graph of the relation is shown below–



Question ▸ 15 Let $f : \mathbb{R} \to \mathbb{R}$ then function is defined as $f(x) = \frac{4x + 3}{2x + 5}$.

[Rangpur Cadet College, Rangpur]

a. Determine whether the relation $x^2 + y^2 = 25$ is function or not? 2

b. Find whether the function f is one-one or not. 4

c. If $f^{-1}(-6) = m.f^{-1}(-2)$, find the value of m. 4

Solution to the question no. 15

a $x^2 + y^2 = 25$

Or, $y^2 = 25 - x^2$

$\therefore y = \pm\sqrt{25 - x^2}$

For a single value of x we are getting two different value of y.

So, $x^2 + y^2 = 25$ is not a function.

b For any $x_1, x_2 \in$ Domain f, $f(x)$ will be one-one if and only if $f(x_1) = f(x_2)$ then, $x_1 = x_2$.

Let, $f(x_1) = f(x_2)$

Or, $\frac{4x_1 + 3}{2x_1 + 5} = \frac{4x_2 + 3}{2x_2 + 5}$

Or, $8x_1x_2 + 6x_2 + 20x_1 + 15 = 8x_1x_2 + 6x_1 + 20x_2 + 15$

Or, $20x_1 - 6x_1 = 20x_2 - 6x_2$

Or, $14x_1 = 14x_2$

$\therefore x_1 = x_2$

$\therefore$ The function f is one-one. **(Shown)**

c Given, $f(x) = \frac{4x + 3}{2x + 5}$

Let, $y = f(x)$

$\therefore x = f^{-1}(y)$

Now, $y = \frac{4x + 3}{2x + 5}$

Or, $2xy + 5y = 4x + 3$

Or, $2xy - 4x = 3 - 5y$

Or, $x(2y - 4) = 3 - 5y$

Or, $x = \frac{3 - 5y}{2y - y}$

$\therefore f^{-1}(y) = \frac{3 - 5y}{2y - 4}$

$\therefore f^{-1}(x) = \frac{3 - 5x}{2x - 4}$

$\therefore f^{-1}(-6) = \frac{3 - 5(-6)}{2(-6) - 4} = \frac{3 + 30}{-12 - 4}$

$= \frac{-33}{16}$

$\therefore f^{-1}(-2) = \frac{3 - 5(-2)}{2(-2) - 4} = \frac{3 + 10}{-4 - 4} = \frac{-13}{8}$

Given,

$f^{-1}(-6) = mf^{-1}(-2)$

Or, $\frac{-33}{16} = m.\left(\frac{-13}{8}\right)$

Or, $\frac{-33}{16} \times \frac{8}{-13} = m$

$\therefore m = \frac{33}{26}$ **(Ans.)**

Question ▸ 16 Scenario : $f(x) = ln\,\frac{3 + x}{3 - x}$ and P, Q are any finite set. *[Faujdarhat Cadet College, Chattogram]*

a. Find the domain of f(x). 2

b. According to scenario prove that f(x) has inverse function then find $f^{-1}(x)$. 4

c. Prove that $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$. 4

Solution to the question no. 16

a Let, $y = f(x) = \ell n\,\frac{3 + x}{3 - x}$

Since, logarithm is defined only for the positive real number,

$\therefore \frac{3 + x}{3 - x} > 0$ if (i) $3 + x > 0$ and $3 - x > 0$,

Or, (ii) $3 + x < 0$ and $3 - x < 0$,

From no.(i), we get, $x > -3$ and $-x > -3$

Or, $x > -3$ and $x < 3$

$\therefore$ Domain $= (x : -3 < x\} \cap \{x : x < 3\}$

$= (-3, \infty) \cap (-\infty, 3)$

$= (-3, 3)$

From no.(ii), we get, $x < -3$ and $-x < -3$

or, $x < -3$ and $x > 3$

$\therefore$ Domain $= \{x : x < -3\} \cap \{x : x > 3\} = \varnothing$

$\therefore$ Domain of the given function

D_f = union of the domain obtained in (i) and (ii)

$= (-3, 3) \cup \varnothing = (-3, 3)$

Ans: Domain of the given function $D_f = (-3, 3)$

b We know that, if a function is one-one and onto then the function has an inverse.

Now, $f(x) = \ln\left(\frac{3 + x}{3 - x}\right)$

f(x) will be one-one if and only if for any $x_1, x_2 \in$ Domain f, $x_1 \neq x_2 : f(x_1) = f(x_2)$ implies $x_1 = x_2$

Let, $f(x_1) = f(x_2)$

Or, $\ln\left(\frac{3 + x_1}{3 - x_1}\right) = \ln\left(\frac{3 + x_2}{3 - x_2}\right)$

Or, $\frac{3 + x_1}{3 - x_1} = \frac{3 + x_2}{3 - x_2}$

Or, $9 - 3x_2 + 3x_1 - x_1x_2 = 9 + 3x_2 - 3x_1 - x_1x_2$

Or, $3x_1 + 3x_1 = 3x_2 + 3x_2$

Or, $6x_1 = 6x_2$

$\therefore x_1 = x_2$

$\therefore$ The function f(x) is one-one.

Let, $y = f(x)$

$\therefore y = \ln\left(\frac{3 + x}{3 - x}\right)$

Or, $e^y = \frac{3 + x}{3 - x}$

Or, $3 + x = 3e^y - xe^y$

Or, $x(1 + e^y) = 3(e^y - 1)$

$\therefore x = \frac{3(e^y - 1)}{e^y + 1}$

Now, $f\left(3\frac{e^y - 1}{e^y + 1}\right) = \ln\left(\frac{3 + \frac{3(e^y - 1)}{e^y + 1}}{3 - \frac{3(e^y - 1)}{e^y + 1}}\right) = \ln\left(\frac{\frac{3e^y + 3 + 3e^y - 3}{e^y + 1}}{\frac{3e^y + 3 - 3e^y + 3}{e^y + 1}}\right)$

$= \ln\left(3.\frac{e^y - 1}{e^y + 1}\right) = \ln e^y = y$

$\therefore$ The function is onto.

So, the function f(x) has in inverse.

Now, let $y = f(x) = \ln\left(\frac{3 + x}{3 - x}\right)$

$\therefore y = \ln\left(\frac{3 + x}{3 - x}\right)$ and $x = f^{-1}(y)$

Or, $e^y = \frac{3 + x}{3 - x}$

Or, $3 + x = 3e^y - xe^y$

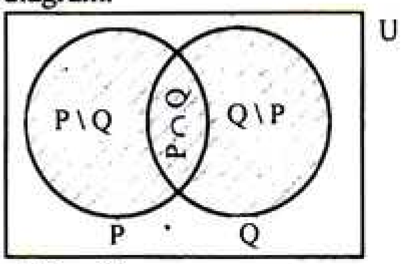
Or, $x(1 + e^y) = 3(e^y - 1)$

$$\text{Or, } x = \frac{3(e^y - 1)}{1 + e^y}$$

$$\text{Or, } f^{-1}(y) = \frac{3(e^y - 1)}{e^y + 1}$$

$$\therefore f^{-1}(x) = \frac{3(e^x - 1)}{e^x + 1}$$ **(Ans.)**

c Here, P\Q, $P \cap Q$ and Q\P are disjoint sets to each other shown in Venn diagram.



and, $P = (P\backslash Q) \cup (P \cap Q)$

$Q = (Q\backslash P) \cup (P \cap Q)$

$P \cup Q = (P\backslash Q) \cup (P \cap Q) \cup (Q\backslash P)$

$\therefore n(P) = n(P\backslash Q) + n(P \cap Q)$ ....(i)

$n(Q) = n(Q\backslash P) + n(P \cap Q)$ ....(ii)

$n(P \cup Q) = n(P\backslash Q) + n(P \cap Q) + n(Q\backslash P)$ ....(iii)

Therefore, from (i) we get,

$n(P\backslash Q) = n(P) - n(P \cap Q)$

From (ii), we get, $n(Q\backslash P) = n(Q) - n(P \cap Q)$

Now, $n(P\backslash Q)$ and $n(Q\backslash P)$ putting in (iii)

We get,

$n(P \cup Q)$

$= n(P) - n(P \cap Q) + n(Q) - n(P \cap Q) + n(P \cap Q)$

$\therefore n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$ **(Shown)**

Question ▶ 17 $A = \{x : x \in R \text{ and } x^2 - (e + f)x + ef = 0\}$; $B = \{1, 2\}$ and $C = \{2, 4, 5\}$ *[Sylhet Cadet College, Sylhet]*

a. Find n(A) 2

b. Show that $P(B \cap C) = P(B) \cap P(C)$ 4

c. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ 4

Solution to the question no. 17

a Given, $A = \{x : x \in R \text{ and } x^2 - (e + f)x + ef = 0\}$

Now,

Or, $x^2 - (e + f)x + ef = 0$

Or, $x^2 - ex - fx + ef = 0$

Or, $x(x - e) - f(x - e) = 0$

Or, $(x - e)(x - f) = 0$

$\therefore$ $x = e, f$

$\therefore$ $A = \{e, f\}$

$\therefore$ $n(A) = 2$ **(Ans.)**

b Given, $B = \{1, 2\}$, $C = \{2, 4, 5\}$

$P(B) = \{\{1\}, \{2\}, \{1, 2\}, \varnothing\}$

$P(C) = \{\{2\}, \{4\}, \{5\}, \{2, 4\}, \{2, 5\}, \{4, 5\}, \{2, 4, 5\}, \varnothing\}$

$\therefore P(B) \cap P(C) = \{\{1\}, \{2\}, \{1, 2\}, \varnothing\} \cap \{\{2\}, \{4\}, \{5\}, \{2, 4\}, \{2, 5\}, \{4, 5\}, \{2, 4, 5\}, \varnothing\}$

$= \{\{2\}, \varnothing\}$

Again, $B \cap C = \{1, 2\} \cap \{2, 4, 5\} = \{2\}$

$\therefore P(B \cap C) = \{\{2\}, \varnothing\}$

$\therefore P(B \cap C) = P(B) \cap P(C)$. **(Shown)**

c Given, $B = \{1, 2\}$, $C = \{2, 4, 5\}$

and $A = \{e, f\}$ [From 'a']

$B \cup C = \{1, 2\} \cup \{2, 4, 5\} = \{1, 2, 4, 5\}$

L.S. $= A \times (B \cup C) = \{e, f\} \times \{1, 2, 4, 5\}$

$= \{(e, 1), (e, 2), (e, 4), (e, 5), (f, 1), (f, 2), (f, 4), (f, 5)\}$

Again, $A \times B = \{e, f\} \times \{1, 2\} = \{(e, 1), (e, 2), (f, 1), (f, 2)\}$

and $A \times C = \{e, f\} \times \{2, 4, 5\}$

$= \{(e, 2), (e, 4), (e, 5), (f, 2), (f, 4), (f, 5)\}$

R.S. $= (A \times B) \cup (A \times C)$

$= \{(e, 1), (e, 2), (f, 1), (f, 2)\} \cup \{(e, 2), (e, 4), (e, 5), (f, 2), (f, 4), (f, 5)\}$

$= \{(e, 1), (e, 2), (e, 4), (e, 5), (f, 1), (f, 2), (f, 4), (f, 5)\}$

$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$ **(Proved)**

Question ▶ 18 $f : R\backslash\{1\} \to R\{2\}$, $f(x) = \frac{2x + 2}{x - 1}$ and $g(x) = \sqrt{x - 2}$ *[Barishal Cadet College, Barishal]*

a. Determine the domain of g(x). 2

b. Show that f(x) is one-one and on-to function. 4

c. If $5 f^{-1}(x) = g^{-1}(3)$, then find the value of x. 4

Solution to the question no. 18

a Given, $g(x) = \sqrt{x - 2}$

g(x) is defined if and if only $x - 2 \geq 0$

or, $x \geq 2$

$\therefore$ Dom, $g = \{x \in R : x \geq 2\}$ **(Ans.)**

b Given that, $f(x) = \frac{2x + 2}{x - 1}$

Assume that, a, b, $\in$ R then,

$$f(a) = \frac{2a + 2}{a - 1}$$

$$f(b) = \frac{2b + 2}{b - 1}$$

So, $f(a) = f(b)$ implies.

Now, $f(a) = f(b)$

Or, $\frac{2a + 2}{a - 1} = \frac{2b + 2}{b - 1}$

Or, $2ab + 2b - 2a - 2 = 2ab + 2a - 2b - 2$

Or, $2ab - 2ab + 2b + 2b - 2a - 2 + 2 = 2a$

Or, $4b = 4a$

$\therefore$ $a = b$

Therefore, $f(x)$ is one-one function.

Again, for any $y \in$ Range of f, let

$y = f(x)$

Or, $y = \frac{2x + 2}{x - 1}$

Or, $xy - y = 2x + 2$

Or, $x(y - 2) = y + 2$

Or, $x = \frac{y + 2}{y - 2}$

$\therefore$ $x = \frac{y + 2}{y - 2}$

$$\therefore f\left(\frac{y + 2}{y - 2}\right) = \frac{2.\frac{y + 2}{y - 2} + 2}{\frac{y + 2}{y - 2} - 1}$$

$$= \frac{2y + 4 + 2y - 4}{y + 2 - y + 2} = \frac{4y}{4} = y = f(x)$$

$\therefore f(x)$ is onto function

Therefore, $f(x)$ is one-one and onto function. **(Shown)**

c The given relation, $5 f^{-1}(x) = g^{-1}(3)$

Therefore, we find $f^{-1}(x)$ and $g^{-1}(x)$

Let, $y = f(x)$

Or, $y = \frac{2x+2}{x-1}$

Or, $xy - y = 2x + 2$

Or, $x(y-2) = 2 + y$

$\therefore \quad x = \frac{2+y}{y-2}$

$\therefore \quad y = f(x)$ or, $x = f^{-1}(y)$

$\therefore \quad f^{-1}(y) = \frac{2+y}{y-2}$

Or, $f^{-1}(x) = \frac{2+x}{x-2}$

Again let, $y = g(x) = \sqrt{x-2}$

$\therefore y = \sqrt{x-2}$

Or, $y^2 = x - 2$

Or, $x = y^2 + 2$

$\therefore \quad g^{-1}(y) = y^2 + 2$

Or, $g^{-1}(x) = x^2 + 2$

$\therefore \quad g^{-1}(3) = 3^2 + 2 = 11$

Now, according to question,

$$5f^{-1}(x) = g^{-1}(3)$$

$$\text{Or, } 5\frac{2+x}{x-2} = 11$$

$$\text{Or, } \frac{10+5x}{x-2} = 11$$

$$\text{Or, } 11x - 22 = 10 + 5x$$

$$\text{Or, } 11x - 5x = 32$$

$$\therefore \quad x = \frac{32}{6} = \frac{16}{3} \quad \textbf{(Ans.)}$$

Question ▶ 19 (i) $\frac{1}{(3x-1)} + \frac{1}{(3x-1)^2} + \frac{1}{(3x-1)^3} + \ldots\ldots\ldots$

is a geometrical series.

(ii) $F(x) = \frac{1}{\sqrt{5x-1}}$

[Viqarunnisa Noon School & College, Dhaka]

a. Show that, $F(x, y, z) = (x + y + z)(xy + yz + zx)$ is a cyclic expression. 2

b. If $x = 1$, then determine the 5^{th} term and sum upto the 15^{th} term of the series. 4

c. With the help of (ii) verified $F(x)$ is one–one and find $F^{-1}(1)$. 4

Solution to the question no. 19

a $F(x, y, z) = (x + y + z)(xy + yz + zx)$

$\therefore F(y, z, x) = (y + z + x)(yz + zx + xy)$

$= (x + y + z)(xy + yz + zx)$

$= F(x, y, z)$

So, $F(x, y, z)$ is a cyclic expression **(Shown)**

b Given,

$$\frac{1}{3x-1} + \frac{1}{(3x-1)^2} + \frac{1}{(3x-1)^3} + \ldots\ldots\ldots\ldots$$

When $x = 1$ then, we have

$$\frac{1}{3x-1} = \frac{1}{3.1-1} = \frac{1}{3-1} = \frac{1}{2}$$

The geometric series is

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \ldots\ldots\ldots\ldots\ldots\ldots$$

1^{st} term of the series, $a = \frac{1}{2}$

Common ratio, $r = \frac{\frac{1}{2^2}}{\frac{1}{2}} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2} < 1$

$\therefore 5^{th}$ term $= ar^{5-1}$

$$= \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{5-1}$$

$$= \frac{1}{2} \cdot \frac{1}{2^4}$$

$$= \frac{1}{2^5}$$

$$= \frac{1}{32} \quad \textbf{(Ans.)}$$

Sum up to 15^{th} term

$$S_{15} = \frac{a(1-r^{15})}{1-r}$$

$$= \frac{\frac{1}{2}\left\{1 - \left(\frac{1}{2}\right)^{15}\right\}}{1 - \frac{1}{2}}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}} \cdot \left(1 - \frac{1}{32768}\right)$$

$$= 1 - \frac{1}{32768}$$

$$= \frac{32767}{32768} \quad \textbf{(Ans.)}$$

c Given, $F(x) = \frac{1}{\sqrt{5x-1}}$

$F(x)$ will be one-one function if and only if $F(x_1) = F(x_2)$ for which $x_1 = x_2$

Now, $F(x_1) = F(x_2)$

Or, $\frac{1}{\sqrt{5x_1-1}} = \frac{1}{\sqrt{5x_2-1}}$

Or, $\sqrt{5x_1-1} = \sqrt{5x_2-1}$

Or, $5x_1 - 1 = 5x_2 - 1$

Or, $5x_1 = 5x_2$

$\therefore x_1 = x_2$

$\therefore F(x)$ is one-one function.

Let, $F^{-1}(1) = x$

Or, $1 = F(x)$

Or, $\frac{1}{\sqrt{5x-1}} = 1$

Or, $\sqrt{5x-1} = 1$

Or, $5x - 1 = 1$

Or, $5x = 1 + 1$

Or, $5x = 2$

$\therefore x = \frac{2}{5}$

$\therefore F^{-1}(1) = \frac{2}{5}$ **(Ans.)**

Question ▶20 $E = \{x : x \in R \text{ and } x^2 - (a+b)x + ab = 0, a, b \in R\}$
$F = \{3, 4\}$ and $G = \{4, 5, 6\}$
[Dhaka Residential Model School and College, Dhaka]

a. Find the elements of the set E. 2
b. Prove that $P(F \cap G) = P(F) \cap P(G)$ 4
c. Show that $E \times (F \cap G) = (E \times F) \cap (E \times G)$ 4

Solution to the question no. 20

a Given,
$E = \{x : x \in \mathbb{R} \text{ and } x^2 - (a+b)x + ab = 0, a, b \in \mathbb{R}\}$
Here, $x^2 - (a+b)x + ab = 0$
or, $x^2 - ax - bx + ab = 0$
or, $x(x-a) - b(x-a) = 0$
or, $(x-a)(x-b) = 0$
$\therefore x = a, b$
$\therefore E = \{a, b\}$
The elements of the set E are a and b. **(Ans.)**

b Given, $F = \{3, 4\}$ and $G = \{4, 5, 6\}$
Here, $F \cap G = \{3, 4\} \cap \{4, 5, 6\}$
$= \{4\}$
Now, $P(F) = \{\{3, 4\}, \{3\}, \{4\}, \varnothing\}$
and $P(G) = \{\{4, 5, 6\}, \{4, 5\}, \{4, 6\}, \{5, 6\}, \{4\}, \{5\}, \{6\}, \varnothing\}$
L.H.S. $= P(F \cap G) = \{\{4\}, \varnothing\}$
and R.H.S. $= P(F) \cap P(G)$
$= \{\{3, 4\}, \{3\}, \{4\}, \varnothing\} \cap \{\{4, 5, 6\}, \{4, 5\}, \{4, 6\}, \{5, 6\}, \{4\}, \{5\}, \{6\}, \varnothing\}$
$= \{\{4\}, \varnothing\}$
$\therefore P(F \cap G) = P(F) \cap P(G)$ **(Proved)**

c Given,
$F = \{3, 4\}, G = \{4, 5, 6\}$
$\therefore F \cap G = \{3, 4\} \cap \{4, 5, 6\}$
$= \{4\}$
From 'a',
$E = \{a, b\}$
$\therefore$ L.H.S $= E \times (F \cap G)$
$= \{a, b\} \times \{4\}$
$= \{(a, 4), (b, 4)\}$
Now,
$E \times F = \{a, b\} \times \{3, 4\}$
$= \{(a, 3), (a, 4), (b, 3), (b, 4)\}$
and $E \times G = \{a, b\} \times \{4, 5, 6\}$
$= \{(a, 4), (a, 5), (a, 6), (b, 4), (b, 5), (b, 6)\}$
$\therefore$ R.H.S $= (E \times F) \cap (E \times G)$
$= \{(a, 3), (a, 4), (b, 3), (b, 4)\} \cap \{(a, 4), (a, 5), (a, 6), (b, 4), (b, 5), (b, 6)\}$
$= \{(a, 4), (b, 4)\}$
$\therefore$ L.H.S = R.H.S **(Shown)**

Question ▶21 A Function $f : R \to R$ is defined by $f(x) = \dfrac{2x + k}{x - 3}$ and that $f(6) = 5$. *[Saint Joseph Higher Secondary School, Dhaka]*

a. Find the value of k. 2
b. Find the inverse function f^{-1}, and state the domain of f^{-1}. 4
c. Determine whether f is one-one, onto function or not. 4

Solution to the question no. 21

a Given, $f : \mathbb{R} \to \mathbb{R}$ and $f(x) = \dfrac{2x + k}{x - 3}$
and $f(6) = 5$
Or, $\dfrac{2.6 + k}{6 - 3} = 5$
Or, $\dfrac{12 + k}{3} = 5$
Or, $12 + k = 15$
Or, $k = 3$ **(Ans.)**

b From 'a' we get, $k = 3$
$\therefore f(x) = \dfrac{2x + 3}{x - 3}$
Let, $f^{-1}(x) = a$
$\therefore x = f(a)$
Or, $x = \dfrac{2a + 3}{a - 3}$
Or, $ax - 3x = 2a + 3$
Or, $ax - 2a = 3x + 3$
Or, $a(x - 2) = 3x + 3$
Or, $a = \dfrac{3x + 3}{x - 2}$
$\therefore f^{-1}(x) = \dfrac{3x + 2}{x - 2}, x \neq 2$ **(Ans.)**
Now, $f^{-1}(x) \in \mathbb{R}$, is invalid if and only if $x - 2 = 0$ or $x = 2$
$\therefore$ Domain $f^{-1} = \mathbb{R} - \{2\}$ **(Ans.)**

c $f(x)$ will be one-one if and only if for any $x_1, x_2 \in$ Domain f, $x_1 \neq x_2$; $f(x_1) = f(x_2)$ implies $x_1 = x_2$
Let, $f(x_1) = f(x_2)$
Or, $\dfrac{2x_1 + 3}{x_1 - 3} = \dfrac{2x_2 + 3}{x_2 - 3}$
Or, $2x_1x_2 + 3x_2 - 6x_1 - 9 = 2x_1x_2 - 6x_2 + 3x_1 - 9$
Or, $-6x_1 - 3x_1 = -6x_2 - 3x_2$
Or, $-9x_1 = -9x_2$
$\therefore x_1 = x_2$
$\therefore$ The function f is one-one.
Again, let, $y = f(x)$
$\therefore y = \dfrac{2x + 3}{x - 3}$
Or, $xy - 3y = 2x + 3$
Or, $xy - 2x = 3y + 3$
Or, $x(y - 2) = 3y + 3$
$\therefore x = \dfrac{3y + 3}{y - 2}$

Now, $f\left(\dfrac{3y + 3}{y - 2}\right) = \dfrac{2 \cdot \dfrac{3y + 3}{y - 2} + 3}{\dfrac{3y + 3}{y - 2} - 3}$
$= \dfrac{6y + 6 + 3y - 6}{y - 2} \times \dfrac{y - 2}{3y + 3 - 3y + 6}$
$= \dfrac{9y}{9}$
$= y$
$= f(x)$
$\therefore$ The function f is onto.
Therefore, the function f is one-one and onto. **(Shown)**

Question ▶22 $F(x) = \sqrt{(3-4x)}$ is a function.

[BAF Shaheen College, Tejgaon, Dhaka]

a. Find the domain of the function stated by F(x). 2

b. Determine whether the function F is one-one or not. 4

c. Find the value of $F^{-1}(-5)$. Also, determine the range of $F^{-1}(x)$. 4

Solution to the question no. 22

a Given, $F(x) = \sqrt{3-4x}$

Here, $F(x) \in \mathbb{R}$, if and only if

$3 - 4x \geq 0$

Or, $-4x \geq -3$

Or, $4x \leq 3$

$\therefore x \leq \frac{3}{4}$

$\therefore$ Domain $F = \{x \in \mathbb{R} : x \leq \frac{3}{4}\}$ **(Ans.)**

b $F(x) = \sqrt{3-4x}$

Let, $x_1, x_2 \in$ domain F

The function will be one-one if and only if

$F(x_1) = F(x_2)$ then $x_1 = x_2$

$\therefore F(x_1) = F(x_2)$

Or, $\sqrt{3-4x_1} = \sqrt{3-4x_2}$

Or, $3 - 4x_1 = 3 - 4x_2$ [Squaring both sides]

Or, $-4x_1 = -4x_2$

$\therefore x_1 = x_2$

$\therefore$ F(x) is a one-one function . **(Ans.)**

c Let, $y = F(x) = \sqrt{3-4x}$

Now, $F(x) = y$

$\therefore x = F^{-1}(y)$

Again, $y = \sqrt{3-4x}$

Or, $y^2 = 3 - 4x$

Or, $4x = 3 - y^2$

Or, $x = \frac{3-y^2}{4}$

$\therefore F^{-1}(y) = \frac{3-y^2}{4}$

$\therefore F^{-1}(x) = \frac{3-x^2}{4}$

$\therefore F^{-1}(-5) = \frac{2-(-5)^2}{4}$

$= \frac{2-25}{4}$

$= \frac{-23}{4}$ **(Ans.)**

Again let, $a = \frac{3-x^2}{4}$

Or, $4a = 3 - x^2$

Or, $x^2 = 3 - 4a$

$\therefore x = \sqrt{3-4a}$

Now, x will be defined if and only if

$3 - 4a \geq 0$

Or, $-4a \geq -3$

Or, $4a \leq 3$

$\therefore a \leq \frac{3}{4}$

$\therefore$ Range of $F^{-1}(x) = \{a \in \mathbb{R} : a \leq \frac{3}{4}\}$ **(Ans.)**

Question ▶23 $f(x) = \frac{4x-9}{x-2}, x \neq 2$

[BAF Shaheen College, Kurmitola, Dhaka]

a. Find $f^{-1}(-1)$ and $f^{-1}(1)$. 2

b. Find the value of x when $4f^{-1}(x) = x$. 4

c. If $f : \{1, 2, 3, 4\} \rightarrow \mathbb{R}$ is a function which is defined by $f(x) = 2x + 1$ then show that, f is a one-one function but not an onto function. 4

Solution to the question no. 23

a Let, $y = f(x) = \frac{4x-9}{x-2}, x \neq 2$

Or, $y = \frac{4x-9}{x-2}$

Or, $xy - 2y = 4x - 9$

Or, $xy - 4x = 2y - 9$

Or, $x(y-4) = 2y - 9$

Or, $x = \frac{2y-9}{y-4}$

Or, $f^{-1}(y) = \frac{2y-9}{y-4}$ [Since, $y = f(x) \therefore f^{-1}(y) = x$]

Or, $f^{-1}(x) = \frac{2x-9}{x-4}$ .... (i)

Or, $f^{-1}(-1) = \frac{2(-1)-9}{-1-4}$

$= \frac{-2-9}{-5} = \frac{-11}{-5} = \frac{11}{5} = 2\frac{1}{5}$

and $f^{-1}(1) = \frac{2.1-9}{1-4} = \frac{-7}{-3} = \frac{7}{3} = 2\frac{1}{3}$

$\therefore f^{-1}(-1) = 2\frac{1}{5}$ and $f^{-1}(1) = 2\frac{1}{3}$ **(Ans.)**

b Given, $4f^{-1}(x) = x$

Or, $4\left(\frac{2x-9}{x-4}\right) = x$ [By equation (i)]

Or, $8x - 36 = x^2 - 4x$

Or, $x^2 - 4x - 8x + 36 = 0$

Or, $x^2 - 12x + 36 = 0$

Or, $x^2 - 2.x.6 + (6)^2 = 0$

Or, $(x-6)^2 = 0$

$x - 6 = 0$

$\therefore x = 6$ **(Ans.)**

c $f : \{1, 2, 3, 4\} \rightarrow \mathbb{R}$

Let, $A = \{1, 2, 3, 4\}$

Given, $f(x) = 2x + 1$

$f(1) = 2.1 + 1 = 3$

$f(2) = 2.2 + 1 = 5$

$f(3) = 2.3 + 1 = 7$

$f(4) = 2.4 + 1 = 9$

So, it is observed that, for distinct values of x distinct images are obtained.

$\therefore f$ is one-one function but it is not onto function.

Because $f(A) \neq \mathbb{R}$ and $f(A) \subset \mathbb{R}$.

So, f is one-one function but it is not onto function.

(Shown)

Question ▶ 24 Functions f: R–{1}→ R and g:R→R are defined by

$f(x) = \frac{2x+2}{x-1}$ and $g(x) = \sqrt{x-2}$.

[Rajshahi Cantonment Public School & College, Rajshahi]

a. Find the domain of f(x) and g(x). 2

b. Show that, f(x) is an one-one function and an onto function. 4

c. Determine the value of x when $3f^{-1}(x) = g^{-1}(2)$ 4

Solution to the question no. 24

a Given that,

$f(x) = \frac{2x+2}{x-1}$.

and $g(x) = \sqrt{x-2}$

Now, the function $f(x)$ will be defined if and only if

$x - 1 \neq 0$ or, $x \neq 1$

$\therefore$ Domain of $f = \mathbb{R} - \{1\}$. **(Ans.)**

And, the function g (x) will be defined if and only if

$x - 2 \geq 0$ or, $x \geq 2$

$\therefore$ Domain of $g = \{x \in \mathbb{R} : x \geq 2\}$. **(Ans.)**

b Given that, $f(x) = \frac{2x+2}{x-1}$

Assume that, a, b, $\in \mathbb{R}$ then,

$f(a) = \frac{2a+2}{a-1}$

$f(b) = \frac{2b+2}{b-1}$

So, $f(a) = f(b)$ implies.

Now, $f(a) = f(b)$

Or, $\frac{2a+2}{a-1} = \frac{2b+2}{b-1}$

Or, $2ab + 2b - 2a - 2 = 2ab + 2a - 2a - 2$

Or, $2ab - 2ab + 2b - 2a + 2a - 2 + 2 = 2a$

Or, $2b = 2a$

$\therefore$ $a = b$

Therefore, $f(x)$ is one-one function.

Again, for any y $\in$ Range of f,

Let,

$y = f(x)$

Or, $y = \frac{2x+2}{x-1}$

Or, $xy - y = 2x + 2$

Or, $x(y - 2) = y + 2$

Or, $x = \frac{y+2}{y-2}$

$\therefore$ $x = \frac{y+2}{y-2}$

$\therefore f\left(\frac{y+2}{y-2}\right) = \frac{2.\frac{y+2}{y-2}+2}{\frac{y+2}{y-2}-1}$

$= \frac{2y+4+2y-4}{y+2-y+2}$

$= \frac{4y}{4} = y = f(x)$

$\therefore f(x)$ is onto function

Therefore, $f(x)$ is one-one and onto function. **(Shown)**

c Let $f^{-1}(x) = a$

$\therefore x = f(a)$

Or, $x = \frac{2a+2}{a-1}$

Or, $ax - x = 2a + 2$

Or, $ax - 2a = x + 2$

Or, $a(x - 2) = x + 2$

Or, $a = \frac{x+2}{x-2}$

$\therefore f^{-1}(x) = \frac{x+2}{x-2}$

Again, let, $b = g^{-1}(x)$

$\therefore g(b) = x$

Or, $\sqrt{b-2} = x$

Or, $b - 2 = x^2$

Or, $b = x^2 + 2$

$\therefore g^{-1}(x) = x^2 + 2$

$\therefore g^{-1}(2) = 2^2 + 2 = 4 + 2 = 6$

Now, According to question,

$3f^{-1}(x) = g^{-1}(2)$

Or, $3.\frac{x+2}{x-2} = 6$

Or, $\frac{x+2}{x-2} = 2$

Or, $x + 2 = 2x - 4$

Or, $2x - x = 2 + 4$

$\therefore x = 6$ **(Ans.)**

Question ▶ 25 $f(x) = \sqrt{2x-3}$ is a function.

[Dinajpur Laboratory School & College, Dinajpur]

a. If $f(x) = 1$, then determine the value of x. 2

b. Determine the domain of $f(x)$ and show that the function is one-one. 4

c. Determine the range of $f^{-1}(x)$. 4

Solution to the question no. 25

a Given, $f(x) = \sqrt{2x-3}$

According to question, $f(x) = 1$

Or, $\sqrt{2x-3} = 1$

Or, $2x - 3 = 1$ [by squaring]

Or, $2x = 1 + 3$

Or, $x = \frac{4}{2}$

$\therefore$ $x = 2$ **(Ans.)**

b $f(x)$ will be defined if and only if $2x - 3 \geq 0$.

Or, $2x - 3 + 3 \geq 3$

Or, $2x \geq 3$ $\therefore x \geq \frac{3}{2}$

$\therefore$ Domain = $\left\{x \in \mathbb{R} : x \geq \frac{3}{2}\right\}$ **(Ans.)**

$f(x)$ will be one-one if and only if for any x_1, x_2 $\in$ Domain f,

$x_1 \neq x_2$ if $f(x_1) = f(x_2)$ then $x_1 = x_2$.

Let, $f(x_1) = f(x_2)$

$\therefore \sqrt{2x_1 - 3} = \sqrt{2x_2 - 3}$

Or, $2x_1 - 3 = 2x_2 - 3$ [by squaring]

Or, $2x_1 = 2x_2$ $\therefore x_1 = x_2$

$\therefore$ The function f is one-one. **(Shown)**

c Let, $f^{-1}(x) = a$

Or, $x = f(a)$ Or, $x = \sqrt{2a-3}$

Or, $x^2 = 2a - 3$

Or, $2a = x^2 + 3$

Or, $a = \frac{x^2+3}{2}$

$\therefore f^{-1}(x) = \frac{x^2+3}{2}$

Again, let, $y = \frac{x^2+3}{2}$

Or, $x^2 + 3 = 2y$

Or, $x^2 = 2y - 3$

$\therefore x = \sqrt{2y-3}$

Now, x will be defined if and only if $2y - 3 \geq 0$ Or, $y \geq \frac{3}{2}$.

$\therefore$ Range of $f^{-1}(x) = \left\{y \in \mathbb{R} : y \geq \frac{3}{2}\right\}$ **(Ans.)**

Question ▸ 26 Given that, $F(x) = \frac{1}{x-5}$

[Mainamati International School and College, Cumilla]

a. If F(x) = 4, find the value of x. 2

b. Determine whether the function is one-one or not. 4

c. Determine $F^{-1}(5)$. 4

Solution to the question no. 26

a Given, $F(x) = \frac{1}{x-5}$

Again, F(x) = 4

$\therefore \frac{1}{x-5} = 4$

Or, $1 = 4x - 20$

Or, $1 + 20 = 4x$

Or, $4x = 21$

$\therefore x = \frac{21}{4}$ **(Ans.)**

b $F(x) = \frac{1}{x-5}$

Let, $x_1, x_2 \in$ dom F then the function F(x) will be one-one for dom $F(x_1)$ = dom $F(x_2)$ if and only if, $x_1 = x_2$.

$\therefore \frac{1}{x_1-5} = \frac{1}{x_2-5}$

Or, $x_1 - 5 = x_2 - 5$

$\therefore x_1 = x_2$

F is a one-one function. **(Ans.)**

c Let, $y = F(x) = \frac{1}{x-5}$

Or, $y = \frac{1}{x-5}$

Or, $xy - 5y = 1$

Or, $xy = 1 + 5y$

Or, $x = \frac{1+5y}{y}$

Or, $F^{-1}(y) = \frac{1+5y}{y}$ $[\because y = F(x) \therefore x = F^{-1}(y)]$

Or, $F^{-1}(x) = \frac{1+5x}{x}$ [Substituting y by x]

$\therefore F^{-1}(5) = \frac{1+5.5}{5} + \frac{1+25}{5} = \frac{26}{5}$ **(Ans.)**

Question ▸ 27 $f(x) = \frac{2x+5}{x+1}$ where $f : R \to R$

[Jalalabad Cantonment Public School & College, Sylhet]

a. Find domain and range of f(x). 2

b. Prove that f(x) is one-one but not onto. 4

c. If $3f^{-1}(x) = 2x$ then find the value of x. 4

Solution to the question no. 27

a Let, $f(x) = y = \frac{2x+5}{x+1}$

$\therefore f^{-1}(x) = \frac{5-x}{x-2}$

Now $\frac{2x+5}{x+1}$ is undefined if $x + 1 = 0$, or, $x = -1$

$\therefore$ Domain of the function $= \{x : x \in R, x \neq -1\}$ **(Ans.)**

Again, $f^{-1}(x) = \frac{5-x}{x-2}$

Here $\frac{5-x}{x-2}$ is undefined for $x - 2 = 0$, or, $x = 2$

$\therefore$ Range of $f = \{x : x \in R$ and $x \neq 2\}$ **(Ans.)**

b $f(x) = \frac{2x+5}{x+1}$

For $x_1, x_2 \in$ Dom f

Let, $f(x_1) = f(x_2)$

Or, $\frac{2x_1+5}{x_1+1} = \frac{2x_2+5}{x_2+1}$

Or, $2x_1x_2 + 5x_2 + 2x_1 + 5 = 2x_1x_2 + 5x_1 + 2x_2 + 5$

Or, $3x_2 = 3x_1$

Or, $x_1 = x_2$

Since, $f(x_1) = f(x_2)$ Or, $x_1 = x_2$, So $f(x)$ is one-one.

Again, for any $y \in$ range of f

Let, $y = f(x)$

Or, $y = \frac{2x+5}{x+1}$

Or, $xy + y = 2x + 5$

Or, $xy - 2x = 5 - y$

Or, $x(y-2) = 5 - y$

$\therefore x = \frac{5-y}{y-2}$

$$\therefore f\left(\frac{5-y}{y-2}\right) = \frac{2.\frac{5-y}{y-2} + 5}{\frac{5-y}{y-2} + 1}$$

$$= \frac{10 - 2y + 5y - 10}{5 - y + y - 2}$$

$$= \frac{3y}{3} = y = f(x)$$

$\therefore f(x)$ is onto function.

Therefore, $f(x)$ is one-one and onto. **(Proved)**

c Here, $f(x) = \frac{2x+5}{x+1}$

Let, $y = f(x)$

Or, $y = \frac{2x+5}{x+1}$

Or, $xy + y = 2x + 5$

Or, $xy - 2x = 5 - y$

Or, $x(y-2) = 5 - y$

Or, $x = \frac{5-y}{y-2}$

$\therefore f^{-1}(y) = \frac{5-y}{y-2}$

Or, $f^{-1}(x) = \frac{5-x}{x-2}$

Given, $3f^{-1}(x) = 2x$

Or, $3.\frac{5-x}{x-2} = 2x$

Or, $\frac{15-3x}{x-2} = 2x$

Or, $2x^2 - 4x = 15 - 3x$

Or, $2x^2 - 4x + 3x - 15 = 0$

Or, $2x^2 - x - 15 = 0$

Or, $2x^2 - 6x + 5x - 15 = 0$

Or, $2x(x-3) + 5(x-3) = 0$

Or, $(x-3)(2x+5) = 0$

Either, $x - 3 = 0$ $\therefore x = 3$

Or, $2x + 5 = 0$

Or, $2x = -5$

$\therefore x = -\frac{5}{2}$

So, the value of x is 3, or, $-\frac{5}{2}$ **(Ans.)**

Question ▶ 28 Given, $f(x) = \frac{2x+5}{x+1}$

[The Sylhet Khajanchibari International School & College, Sylhet]

a. Find domain and range of $f(x)$. 2

b. Prove that $f(x)$ is one-one and onto. 4

c. Determine the inverse function of $f(x)$. 4

Solution to the question no. 28

a Let, $f(x) = y = \frac{2x+5}{x+1}$ $\therefore x = f^{-1}(y)$

$\therefore y = \frac{2x+5}{x+1}$

Or, $y(x+1) = 2x + 5$

Or, $xy - 2x = 5 - y$

Or, $x(y-2) = 5 - y$

Or, $x = \frac{5-y}{y-2}$

Or, $f^{-1}(y) = \frac{5-y}{y-2}$

$\therefore f^{-1}(x) = \frac{5-x}{x-2}$

Now, $\frac{2x+5}{x+1}$ is undefined if $x + 1 = 0$, Or, $x = -1$

$\therefore$ Domain of the function $= \{x : x \in R, x \neq -1\}$

Again, $f^{-1}(x) = \frac{5-x}{x-2}$

Here, $\frac{5-x}{x-2}$ is undefined for $x - 2 = 0$ or $x = 2$

$\therefore$ Range of $f = \{x : x \in R \text{ and } x \neq 2\}$

b $f(x) = \frac{2x+5}{x+1}$

For $x_1, x_2 \in$ Dom f

Let, $f(x_1) = f(x_2)$

Or, $\frac{2x_1+5}{x_1+1} = \frac{2x_2+5}{x_2+1}$

Or, $2x_1x_2 + 5x_2 + 2x_1 + 5 = 2x_1x_2 + 5x_1 + 2x_2 + 5$

Or, $3x_2 = 3x_1$

Or, $x_1 = x_2$

Since, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$, So $f(x)$ is one-one.

Again, for any $y \in$ Range of f,

Let, $y = f(x)$

Or, $y = \frac{2x+5}{x+1}$

Or, $xy + y = 2x + 5$

Or, $xy - 2x = 5 - y$

Or, $x(y-2) = 5 - y$

$\therefore x = \frac{5-y}{y-2}$

$$\therefore f\left(\frac{5-y}{y-2}\right) = \frac{2.\frac{5-y}{y-2} + 5}{\frac{5-y}{y-2} + 1}$$

$$= \frac{10 - 2y + 5y - 10}{5 - y + y - 2}$$

$$= \frac{3y}{3} = y = f(x)$$

$\therefore f(x)$ is onto function.

Therefore, $f(x)$ is one-one and onto. **(Proved)**

c From 'a', we get,

$\therefore f^{-1}(x) = \frac{5-x}{x-2}$

So, inverse function of f(x) is $\frac{5-x}{x-2}$ **(Ans.)**