Chapter-5: Equations

Question ▶ 1 i) $2+\sqrt{2}+1+\frac{1}{\sqrt{2}}+\frac{1}{2}+\frac{1}{2\sqrt{2}}+$ ....

ii) $\sqrt[3]{(1+x)}+\sqrt[3]{(1-x)}=\sqrt[3]{2}$ [R.B.17]

a. Express $0.\dot{1}\dot{2}$ as a rational fraction. 2
b. Find the 7^{th} term and the sum of the given series (i) (up to infinity) if exist. 4
c. Solve the equation (ii). 4

Solution to the question no. 1

a $0.\dot{1}\dot{2}=0.12121212\ldots\ldots\ldots$
$=0.12+0.0012+0.000012+\ldots\ldots\ldots$
Which is an infinite geometric series.
1^{st} term of the series, $a=0.12$
common ratio, $r=\frac{0.0012}{0.12}=0.01$
$\therefore$ Sum to infinity of the series, $S_\infty=\frac{a}{1-r}=\frac{0.12}{1-0.01}$
$=\frac{0.12}{0.99}=\frac{12}{99}=\frac{4}{33}$
$\therefore 0.\dot{1}\dot{2}=\frac{4}{33}$ **(Ans.)**

b Given the series, $2+\sqrt{2}+1+\frac{1}{\sqrt{2}}+\frac{1}{2}+\frac{1}{2\sqrt{2}}+\ldots\ldots\ldots$
1^{st} term of the series, $a=2$
Common ratio, $r=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$
We know,
n^{th} term of a geometric series $=ar^{n-1}$
$\therefore 7(n=7)$ th term $=2\left(\frac{1}{\sqrt{2}}\right)^{7-1}$
$=2\times\frac{1}{8}$
$=\frac{1}{4}$ **(Ans.)**

Here, $r=\frac{1}{\sqrt{2}}<1$,
So, there exist sum to infinity of the series.
$\therefore$ Sum to infinity, $S_\infty=\frac{a}{1-r}$
$=\frac{2}{1-\frac{1}{\sqrt{2}}}$
$=\frac{2}{\frac{\sqrt{2}-1}{\sqrt{2}}}=\frac{2\sqrt{2}}{\sqrt{2}-1}$
$=\frac{2\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}+1)(\sqrt{2}-1)}$
$=\frac{2\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2})^2-1^2}$
$=\frac{4+2\sqrt{2}}{2-1}$
$=4+2\sqrt{2}$ **(Ans.)**

c See example-6 of exercise-5.2 from your textbook. Page-100

Question ▶ 2 $ax^2+bx+c=0$ .... (i)
$5-8x-x^2=0$ .... (ii)
are two quadratic equations with one variable. [Dj.B.17]

a. Find the value of y, when $5^{y+2}=625$. 2
b. Find the roots of the equation (i). 4
c. Solve the equation (ii) and determine the characteristic of roots. 4

Solution to the question no. 2

a Given, $5^{y+2}=625$
Or, $5^{y+2}=5^4$
Or, $y+2=4$ [$\because$ if $a^x=a^y$, $x=y$]
Or, $y=4-2$
$\therefore y=2$ **(Ans.)**

b See article-5.1 of exercise-5.1 from your textbook. Page-95.

c Given quadratic equation,
$5-8x-x^2=0$
Or, $-(x^2+8x-5)=0$
$\therefore x^2+8x-5=0\ldots\ldots\ldots$ (i)
Comparing equation (i) with standard quadratic equation $ax^2+bx+c=0$ we get, $a=1$, $b=8$ and $c=-5$
We know, the roots of standard quadratic equation,
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$=\frac{-8\pm\sqrt{8^2-4.1.(-5)}}{2.1}$ [putting the values]
$=\frac{-8\pm\sqrt{64+20}}{2}$
$=\frac{-8\pm\sqrt{84}}{2}$
$=\frac{-8\pm2\sqrt{21}}{2}$
$=-4\pm\sqrt{21}$
$\therefore x=-4+\sqrt{21}, -4-\sqrt{21}$
$\therefore$ Roots are real, unequal and irrational. **(Ans.)**

Question ▶ 3 $\frac{1}{3x+2}+\frac{1}{(3x+2)^2}+\frac{1}{(3x+2)^3}+$ .... be a infinite geometric series and $px^2+qx+r=0$ be a quadratic equation in one variable, here p, q, r are the real numbers; $p\neq0$ [Ctg.B.17]

a. Find the series when x = 1 and what is the common ratio of the obtain series? 2
b. Impose a condition on 'x' under which the given series will have a sum up to infinity and find the sum. 4
c. If 'a' and 'b' be the roots of this given equation, then show that $a+b=\frac{-q}{p}$ and $ab=\frac{r}{p}$ 4

Solution to the question no. 3

a Given, series, $\frac{1}{3x+2}+\frac{1}{(3x+2)^2}+\frac{1}{(3x+2)^3}+\ldots\ldots$

If x = 1, the series, $\frac{1}{3.1+2}+\frac{1}{(3.1+2)^2}+\frac{1}{(3.1+2)^3}+\ldots$

$=\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\ldots\ldots$ **(Ans.)**

And common ratio $=\frac{1}{5^2}\div\frac{1}{5}=\frac{1}{5}$ **(Ans.)**

b Given, common ratio of the series, $r=\frac{\frac{1}{(3x+2)^2}}{\frac{1}{3x+2}}=\frac{1}{3x+2}$

Now, the sum to infinity of the series will be exist if and only if

$|r|<1$ or, $\left|\frac{1}{3x+2}\right|<1$ that is if $-1<\frac{1}{3x+2}<1$

$\therefore -1<\frac{1}{3x+2}$ or, $\frac{1}{3x+2}<1$

Or, $-1>3x+2$ or, $3x+2>1$

Or, $-1-2>3x+2-2$ or, $3x+2-2>1-2$

Or, $-3>3x$ or, $3x>-1$

$\therefore x<-1$ $\therefore x>-\frac{1}{3}$

$\therefore$ Required conditions: $x<-1$ or, $x>-\frac{1}{3}$ **(Ans.)**

Again, sum to infinity of the series, $S_\infty=\frac{a}{1-r}$

$$=\frac{\frac{1}{3x+2}}{1-\frac{1}{3x+2}}\quad\left[\because a=\frac{1}{3x+2}\right]$$

$$=\frac{1}{3x+2}\times\frac{3x+2}{3x+1}$$

$$=\frac{1}{3x+1}\ \textbf{(Ans.)}$$

c Given, $px^2+qx+r=0;\ p\neq 0$

$$\therefore x=\frac{-q\pm\sqrt{q^2-4pr}}{2p}$$

$$=\frac{-q+\sqrt{q^2-4pr}}{2p},\ \frac{-q-\sqrt{q^2-4pr}}{2p}$$

Again, since the two roots of the equation are a and b.

$\therefore a=\frac{-q+\sqrt{q^2-4pr}}{2p}$ and $b=\frac{-q-\sqrt{q^2-4pr}}{2p}$

Now,

$$a+b=\frac{-q+\sqrt{q^2-4pr}}{2p}+\frac{-q-\sqrt{q^2-4pr}}{2p}$$

$$=\frac{-q+\sqrt{q^2-4pr}-q-\sqrt{q^2-4pr}}{2p}$$

$$=\frac{-2q}{2p}$$

$$\therefore a+b=-\frac{q}{p}$$

Again, $ab=\frac{-q+\sqrt{q^2-4pr}}{2p}.\frac{-q-\sqrt{q^2-4pr}}{2p}$

$$=\frac{(-q)^2-(\sqrt{q^2-4pr})^2}{4p^2}$$

$$=\frac{q^2-(q^2-4pr)}{4p^2}$$

$$=\frac{q^2-q^2+4pr}{4p^2}$$

$$=\frac{4pr}{4p.p}$$

$$\therefore ab=\frac{r}{p}$$

Therefore, $a+b=-\frac{q}{p}$ and $ab=\frac{r}{p}$ **(Proved)**

Question ▶ 4 $K=y^2-y-1$, $L=\frac{2m}{m-1}$, $M=\left(1-\frac{x}{2}\right)^n$, where n is positive integer. [Dj.B.16]

a. If K = 0, then find the discriminant of the equation. 2

b. If in the expansion of M co-efficient of x^2 is $\frac{6}{8}$, then find the value of n. 4

c. If $6\sqrt{L}+\frac{5}{\sqrt{L}}-13=0$, then find the value of m. 4

Solution to the question no. 4

a Given, $K=y^2-y-1$

$\therefore$ If K = 0 then, $y^2-y-1=0$

After comparing this equation with the standard quadratic equation $ay^2+by+c=0$, we can write a = 1, b = – 1, and c = –1

$\therefore$ Discriminant $=b^2-4ac$

$$=(-1)^2-4.1(-1)$$

$$=1+4$$

$$=5\ \textbf{(Ans.)}$$

b Given, $M=\left(1-\frac{x}{2}\right)^n$, Where n is positive integer

By binomial expansion,

$$\left(1-\frac{x}{2}\right)^n=1+\binom{n}{1}\left(-\frac{x}{2}\right)+\binom{n}{2}\left(-\frac{x}{2}\right)^2+\binom{n}{3}\left(-\frac{x}{2}\right)^3+\ldots\ldots$$

$$=1-n.\frac{x}{2}+\frac{n(n-1)}{1.2}.\frac{x^2}{4}-\frac{n(n-1)(n-2)}{1.2.3}.\frac{x^3}{8}+\ldots\ldots$$

According to question,

$$\frac{n(n-1)}{1.2}.\frac{1}{4}=\frac{6}{8}$$

Or, $n(n-1)=6$

Or, $n^2-n-6=0$

Or, $n^2-3n+2n-6=0$

Or, $n(n-3)+2(n-3)=0$

$\therefore (n-3)(n+2)=0$

Either or

$n-3=0$ $n+2=0$

$\therefore n=3$ $\therefore n=-2$

Where n is positive integer

$\therefore n=3$ **(Ans.)**

c Given, $L=\frac{2m}{m-1}$

Given equation, $6\sqrt{L}+\frac{5}{\sqrt{L}}-13=0$

Or, $6\sqrt{\frac{2m}{m-1}}+\frac{5}{\sqrt{\frac{2m}{m-1}}}-13=0$ .... (i)

Let, $\sqrt{\frac{2m}{m-1}} = a$

Or, $6a + \frac{5}{a} - 13 = 0$

Or, $6a + \frac{5}{a} = 13$

Or, $6a^2 + 5 = 13a$

Or, $6a^2 - 13a + 5 = 0$

Or, $6a^2 - 10a - 3a + 5 = 0$

Or, $2a(3a - 5) - 1(3a - 5) = 0$

$\therefore (3a - 5)(2a - 1) = 0$

Either $3a - 5 = 0$

Or, $3a = 5$

Or, $a = \frac{5}{3}$

Or, $\sqrt{\frac{2m}{m-1}} = \frac{5}{3}$

Or, $\frac{2m}{m-1} = \frac{25}{9}$

Or, $25m - 25 = 18m$

Or, $25m - 18m = 25$

Or, $7m = 25$

$\therefore m = \frac{25}{7}$

Or, $2a - 1 = 0$

Or, $2a = 1$

Or, $a = \frac{1}{2}$

Or, $\sqrt{\frac{2m}{m-1}} = \frac{1}{2}$

Or, $\frac{2m}{m-1} = \frac{1}{4}$

Or, $8m = m - 1$

Or, $7m = -1$

$\therefore m = -\frac{1}{7}$

Verification:

If, $m = \frac{25}{7}$ from equation (i)

$$\text{L. H. S} = 6\sqrt{\frac{2.\frac{25}{7}}{\frac{25}{7}-1}} + \frac{5}{\sqrt{\frac{2.\frac{25}{7}}{\frac{25}{7}-1}}} - 13$$

$$= 6\sqrt{\frac{50}{18}} + \frac{5}{\sqrt{\frac{50}{18}}} - 13$$

$$= 6\sqrt{\frac{25}{9}} + \frac{5}{\sqrt{\frac{25}{9}}} - 13$$

$$= \frac{6.5}{3} + \frac{5.3}{5} - 13$$

$$= 10 + 3 - 13$$

$$= 0$$

$$= \text{R. H. S}$$

$\therefore m = \frac{25}{7}$ is root of given equation.

Again, If $m = -\frac{1}{7}$ from equation (i)

$$\text{L. H. S} = 6\sqrt{\frac{2\left(-\frac{1}{7}\right)}{-\frac{1}{7}-1}} + \frac{5}{\sqrt{\frac{2\left(-\frac{1}{7}\right)}{-\frac{1}{7}-1}}} - 13$$

$$= 6\sqrt{\frac{-\frac{2}{7}}{\frac{-8}{7}}} + \frac{5}{\sqrt{\frac{-\frac{2}{7}}{\frac{-8}{7}}}} - 13$$

$$= 6\sqrt{\frac{1}{4}} + \frac{5}{\sqrt{\frac{1}{4}}} - 13$$

$$= 6.\frac{1}{2} + 5.2 - 13$$

$$= 3 + 10 - 13$$

$$= 0 = \text{R. H. S}$$

$\therefore m = -\frac{1}{7}$ is root of given equation.

$\therefore$ The required solution : $m = \frac{25}{7}, -\frac{1}{7}$

Question ▶5 $\frac{2y}{y-1} = q$ and $y \neq 0, y \neq 1$

[Rajshahi Cadet College, Rajshahi]

a. If $q = \frac{8}{y}$, find the value of y. 2

b. If $\left\{\frac{2(q+y)}{q}\right\}^{\frac{1}{3}} + \left(-\frac{2y}{q}\right)^{\frac{1}{3}} = 2^{\frac{1}{3}}$, find the value of y. 4

c. If $6\sqrt{q} + 5\sqrt{\frac{1}{q}} = 13$, what is the value of $(y + 4)$? 4

Solution to the question no. 5

a Given,

$$q = \frac{2y}{y-1}$$

Now, $q = \frac{8}{y}$

Or, $\frac{2y}{y-1} = \frac{8}{y}$

Or, $2y^2 = 8y - 8$

Or, $y^2 = 4y - 4$

Or, $y^2 - 4y + 4$

Or, $(y - 2)^2 = 0$

Or, $y - 2 = 0$

$\therefore$ $y = 2$ **(Ans.)**

b Given, $q = \frac{2y}{y-1}$

Here, $\left\{\frac{2(q+y)}{q}\right\}^{1/3} + \left(-\frac{2y}{q}\right)^{1/3} = 2^{1/3}$

Or, $\frac{2(q+y)}{q} - \frac{2y}{q} + 3\left\{\frac{2(q+y)}{q}\left(\frac{-2y}{q}\right)\right\}^{1/3}.2^{1/3} = 2$

Or, $\frac{2q + 2y - 2y}{q} - 3\left\{\frac{(2q+2y)2y}{q^2}\right\}^{1/3}.2^{1/3} = 2$

Or, $2 - 3\left(\frac{4qy + 4y^2}{q^2}\right)^{1/3}.2^{1/3} = 2$

Or, $-3\left(\frac{4qy + 4y^2}{q^2}\right)^{1/3}.2^{1/3} = 0$

Or, $\frac{4qy + 4y^2}{q^2} = 0$

Or, $4qy + 4y^2 = 0$ [$\because q^2 \neq 0$]

Or, $4y(q-y) = 0$

$\therefore 4y = 0$ Or, $q + y = 0$

Or, $y = 0$ Or, $y = -q$

Or, $y = \frac{-2y}{y-1}$

Or, $y^2 - y = 2y$

Or, $y^2 - y = 0$

Or, $y(y+1) = 0$

Or, $y = 0, \ y = -1$

But $y = 0$ is not acceptable.

$\therefore y = -1$ **(Ans.)**

c Given, $6\sqrt{q} + 5\sqrt{\frac{1}{q}} = 13$

Or, $6\sqrt{q} + \frac{5}{\sqrt{q}} - 13 = 0$

Or, $6(\sqrt{q})^2 - 13\sqrt{q} + 5 = 0$ [multipling by $\sqrt{q}$]

Or, $6(\sqrt{q})^2 - 10\sqrt{q} - 3\sqrt{q} + 5 = 0$

Or, $2\sqrt{q}(3\sqrt{q} - 5) - (3\sqrt{q} - 5) = 0$

Or, $(3\sqrt{q} - 5)(2\sqrt{q} - 1) = 0$

Either,

$3\sqrt{q} - 5 = 0$ | Or, $2\sqrt{q} - 1 = 0$

$\therefore \sqrt{q} = \frac{5}{3}$ | Or, $\sqrt{q} = \frac{1}{2}$

Or, $\sqrt{\frac{2y}{y-1}} = \frac{5}{3}$ | Or, $\sqrt{\frac{2y}{y-1}} = \frac{1}{2}$

Or, $\frac{2y}{y-1} = \frac{25}{9}$ [squaring] | Or, $\frac{2y}{y-1} = \frac{1}{4}$ [squaring]

Or, $18y = 25y - 25$ | Or, $8y = y - 1$

Or $7y = 25$ | Or, $7y = -1$

$\therefore y = \frac{25}{7}$ | $\therefore y = -\frac{1}{7}$

$\therefore$ Required solution ,$y = \frac{25}{7}, -\frac{1}{7}$

When, $y = \frac{25}{7}$ then, $y + 4 = \frac{25}{7} + 4 = \frac{25+28}{7} = \frac{53}{7}$

When, $y = \frac{1}{7}$ then, $y + 4 = -\frac{1}{7} + 4 = \frac{-1+28}{7} = \frac{27}{7}$

$\therefore y + 4 = \frac{53}{7}, \frac{27}{7}$ **(Ans.)**

Question ▶ 6 The quadratic equation of 'x' is $bcx^2 + cax + ab = 0$. *[Joypurhat Girls' Cadet College, Joypurhat]*

a. Write the simplified condition of the roots of the quadratic equation is equal. 2

b. Find the solution of the quardatic equation, when $a = b = c$. 4

c. Solve : $\sqrt[3]{x+6} + \sqrt{x+2} = 4$. 4

Solution to the question no. 6

a Let, $ax^2 + bx + c = 0$ be a quadratic equation. Then the discriminant of the equation is $b^2 - 4ac$. If the discriminant is equal to zero then the roots of the equation are real and equal.

b Given equation,

$bcx^2 + cax + ab = 0$

Or, $a.ax^2 + bax + a.c = 0$ [$\because a = b = c$]

Or, $a^2x^2 + abx + ac = 0$

Or, $(ax)^2 + 2(ax)\frac{b}{2} + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + ac = 0$

Or, $\left(ax + \frac{b}{2}\right)^2 = \frac{b^2}{4} - ac$

Or, $\left(ax + \frac{b}{2}\right)^2 = \frac{b^2 - 4ac}{4}$

Or, $ax + \frac{b}{2} = \pm\frac{\sqrt{b^2 - 4ac}}{2}$

Or, $ax = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4ac}}{2}$

Or, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ **(Ans.)**

c Given equation,

$\sqrt[3]{x+6} + \sqrt{x+2} = 4$

Let, $\sqrt[3]{x+6} = u$

Or, $x + 6 = u^3$

Or, $x = u^3 - 6$

$\therefore u + \sqrt{u^3 - 6 + 2} = 4$

Or, $\sqrt{u^3 - 4} = 4 - u$

Or, $u^2 - 4 = 16 - 8u + u^2$

Or, $u^3 - u^2 + 8u - 20 = 0$

Or, $u^3 - 2u^2 + u^2 - 2u + 10u - 20 = 0$

Or, $u^2(u-2) + u(u-2) + 10(u-2) = 0$

Or, $(u-2)(u^2 + u + 10) = 0$

$\therefore u - 2 = 0$ Or $u^2 + u + 10 = 0$

Or, $u = 2$ Or, $u = \frac{-1 \pm \sqrt{1 - 4.1.10}}{2}$

$= \frac{-1 \pm \sqrt{-39}}{2}$; Which is imaginary number

$\therefore u = 2$

$\therefore x = 2^3 - 6$

Or, $x = 8 - 6$

$\therefore x = 2$

Verification. L.H.S $= \sqrt[3]{2+6} + \sqrt{2+2}$

$= \sqrt[3]{8} + \sqrt{4} = 2 + 2$

$= 4$

$\therefore x = 2$ **Ans.**

Question ▶ 7 $A = \left(a + \frac{x}{3}\right)^7$ & $B = \frac{2x}{x-1}$, where $a \neq 0$. *[Pabna Cadet College, Pabna]*

a. If $a = 1$, expand A up to 4^{th} term by the help of pascal's triangle. 2

b. In the expansion of A co-efficient of a^2 is 1120, find the value of x. 4

c. If $6\sqrt{B}+\frac{5}{\sqrt{B}}-13=0$, then what is the value of x? 4

Solution to the question no. 7

a Given, $A=\left(a+\frac{x}{3}\right)^7$ and a = 1

$\therefore A=\left(1+\frac{x}{3}\right)^7$

From Pascal's triangle,

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

$\therefore A=\left(1+\frac{x}{3}\right)^7=1+7\left(\frac{x}{3}\right)+21\left(\frac{x}{3}\right)^2+35\left(\frac{x}{3}\right)^3+....$

$=1+\frac{7x}{3}+\frac{7x^2}{3}+\frac{35x^3}{27}+.........$ **(Ans.)**

b Given, $A=\left(a+\frac{x}{3}\right)^7$

Using binomial expansion we get,

$A=\left(a+\frac{x}{3}\right)^7=a^7+{}^7C_1a^6\left(\frac{x}{3}\right)+{}^7C_2a^5\left(\frac{x}{3}\right)^2+{}^7C_3a^4\left(\frac{x}{3}\right)^3$

$+{}^7C_4a^3\left(\frac{x}{3}\right)^4+{}^7C_5a^2\left(\frac{x}{3}\right)^5+..................$

$\therefore$ The co-efficient of $a^2={}^7C_5\left(\frac{x}{3}\right)^5$

$=\frac{7.6.5.4.3}{1.2.3.4.5}\frac{x^5}{3^5}=\frac{21}{243}x^5=\frac{7}{81}x^5$

According to question, $\frac{7}{81}x^5=1120$

Or, $x^5=\frac{1120\times 81}{7}$

Or, $x^5=12960$

$\therefore\quad x=\sqrt[5]{12960}$ **(Ans.)**

c Given, $6\sqrt{B}+\frac{5}{\sqrt{B}}-13=0$

Or, $6(\sqrt{B})^2-13\sqrt{B}+5=0$ [multipling by $\sqrt{B}$]

Or, $6(\sqrt{B})^2-10\sqrt{B}-3\sqrt{B}+5=0$

Or, $2\sqrt{B}(3\sqrt{B}-5)-(3\sqrt{B}-5)=0$

Or, $(3\sqrt{B}-5)(2\sqrt{B}-1)=0$

Either,

$3\sqrt{B}-5=0$

$\therefore\sqrt{B}=\frac{5}{3}$

Or, $\sqrt{\frac{2x}{x-1}}=\frac{5}{3}$

Or, $\frac{2x}{x-1}=\frac{25}{9}$ [squaring]

Or, $18x=25x-25$

Or $7x=25$

$\therefore x=\frac{25}{7}$

Or, $2\sqrt{B}-1=0$

Or, $\sqrt{B}=\frac{1}{2}$

Or, $\sqrt{\frac{2x}{x-1}}=\frac{1}{2}$

Or, $\frac{2x}{x-1}=\frac{1}{4}$ [squaring]

Or, $8x=x-1$

Or, $7x=-1$

$\therefore x=-\frac{1}{7}$

$\therefore$ Required solution $x=\frac{25}{7},-\frac{1}{7}$ **(Ans.)**

Question ▶8 Given, $f(x)=x^2-6x+15$ and $g(x)=x^2-6x+13$, $h(x)=x^2-7x-6y$, $k(x)=y^2-7y-6x$

[Bangladesh International School & College, Dhaka]

a. If f(x) = 0 find the value of x. 2

b. If $\sqrt{f(x)}-\sqrt{g(x)}=\sqrt{10}-\sqrt{8}$ solve the equation 4

c. If k(x) = 0 and h (x) = 0 find the value of (x,y) 4

Solution to the question no. 8

a Given, $f(x)=x^2-6x+15$

and $f(x)=0$

$\therefore x^2-6x+15=0$

Or, $x=\frac{-(-6)\pm\sqrt{(-6)^2-4.1.15}}{2.1}$

$=\frac{36\pm\sqrt{36-60}}{2}$

$=\frac{36\pm\sqrt{-24}}{2}$, which is not possible.

$\therefore$ There is no solution . **(Ans.)**

b Given, $f(x)=x^2-6x+15$

$g(x)=x^2-6x+13$

Now, $\sqrt{f(x)}-\sqrt{g(x)}=\sqrt{10}-\sqrt{18}$

Or, $\sqrt{x^2-6x+15}-\sqrt{x^2-6x+13}=\sqrt{10}-\sqrt{8}$

Let, $x^2-6x+13=y$

Then, $\sqrt{y+2}-\sqrt{y}=\sqrt{10}-\sqrt{8}$

Or, $\sqrt{y+2}+\sqrt{8}=\sqrt{y}+\sqrt{10}$

Or, $y+2+8+2\sqrt{8y+16}=y+10+2\sqrt{10y}$ [Squaring]

Or $\sqrt{8y+16}=\sqrt{10y}$

Or, $8y+16=10y$ [squaring]

Or, $2y=16$

Or, $y=8$

Or, $x^2-6x+13=8$

Or, $x^2-6x+5=0$

Or, $(x-1)(x-5)=0$

$\therefore x=1$, or $x=5$

Verification :

For x = 1, L.H.S $=\sqrt{10}-\sqrt{8}$ = R.H.S

For x = 5, L.H.S $=\sqrt{10}-\sqrt{8}$ = R.H.S

$\therefore$ Required solution x = 1, 5 **(Ans.)**

c Given, $h(x)=x^2-7x-6y=0$ .... (i)

$k(x)=y^2-7y-6x=0$ ......(ii)

Subtraction (ii) from (i), we have

$x^2-y^2-7x-6y+7y+6x=0$

Or, $x^2-y^2-7(x-y)+6(x-y)=0$

Or, $(x-y)(x+y)-7(x-y)+6(x-y)=0$

Or, $(x-y)(x+y-7+6)=0$

Or, $(x-y)(x+y-1)=0$

Either,

$x-y=0$ or, $x+y-1=0$

$\therefore x=y$ $\therefore x=1-y$

Putting x = y in (i), we get,

$x^2-7x-6x=0$

Or, $x^2-13x=0$

Or, $x(x-13) = 0$

$\therefore x = 0$ or, $x = 13$ but $x = y \neq 0$.

So, $x = 13$

Therefore, $y = x = 13$.

Putting $x = 1 - y$ in (i), we get,

$(1-y)^2 - 7(1 - y) - 6y = 0$

Or, $1 - 2y + y^2 - 7 + 7y - 6y = 0$

Or, $y^2 - y - 6 = 0$

Or, $(y - 3)(y + 2) = 0$

Either, $y - 3 = 0$ or, $y + 2 = 0$

$\therefore y = 3$ $\therefore y = -2$

When, $y = 3$, $x = 1 - y$
$= 1 - 3$
$= -2$

when $y = -2$, $x = 1 - y$
$= 1 - (-2)$
$= 1 + 2$
$= 3$

So that solution of $(x, y) = (13, 13), (-2, 3), (3, -2)$ **(Ans.)**

Question ▶ 9 If $x^2 + 4x = m$

[Millennium Scholastic School & College, Bogura]

a. Define: quadratic equation. 2

b. If $m = 5$, find the discrimination of the equation and the nature of root and sum of the roots 4

c. Solve : $\sqrt{x^2 - 6x + 15} - \sqrt{x^2 - 6x + 13} = \sqrt{10} - \sqrt{8}$ 4

Solution to the question no. 9

a **Quadratic equation:** A quadratic equation is any equation having the form $ax^2 + bx + c = 0$, $a \neq 0$
Where x represents an unknown and a, b and, represent known numbers.

b Given that, $x^2 + 4x = m$
If $m = 5$, then the equation will be,
$x^2 + 4x = 5$
Or, $x^2 + 4x - 5 = 0$
Comparing the equation (i) with standard quadratic equation $ax^2 + bx + c = 0$.
We get, $a = 1$, $b = 4$ and $c = -5$
$\therefore$ Discriminant, $D = b^2 - 4ac$
$= (4)^2 - 4.1.(-5)$
$= 16 + 20$
$= 36$ **(Ans.)**
Since, the discriminant $D > 0$ and $D = 36$ is a perfect square. Then the two roots of the equation are real, unequal and rational and sum of the roots $= -\frac{b}{a}$
$= -4$ **(Ans.)**

c See ch–5.2, example–5 of your text book. page-100

Question ▶ 10 Given that, $x^2 + 2x = m$

[Mainamati International School and College, Cumilla]

a. If $m = -1$ find the value of x. 2

b. If $m = 5$ find the discriminante of the equation and the nature of roots. 4

c. If $m = x^2 + 4x$ and $\sqrt{m-4} + \sqrt{m-10} = 6$, find the value of x. 4

Solution to the question no. 10

a Given, $x^2 + 2x = m$ & $m = -1$
$\therefore x^2 + 2x = -1$
Or, $x^2 + 2x + 1 = 0$
Or, $(x + 1)^2 = 0$
Or, $x + 1 = 0$
$\therefore x = -1$ **(Ans).**

b Given, $x^2 + 2x = m$ and $m = 5$,
$\therefore x^2 + 2x = 5$
$\therefore x^2 + 2x - 5 = 0$
Discriminant of the equation $= 2^2 - .4.(-5).1$
$= 4 + 20$
$= 24$ **(Ans.)**
Discriminant is positive but not a perfect square.
So, the roots of the equation are real, unequal and irrational.

c Given, $m = x^2 + 4x$
Now, $\sqrt{m-4} + \sqrt{m-10} = 6$
Or, $\sqrt{m-4} = 6 - \sqrt{m-10}$
Or, $m - 4 = 6^2 - 2.6\sqrt{m-10} + m - 10$ [Squaring both sides]
Or, $m - 4 = 36 - 12\sqrt{m-10} + m - 10$
Or, $m + 12\sqrt{m-10} - m = 36 - 10 + 4$
Or, $12\sqrt{m-10} = 30$
Or, $\sqrt{m-10} = \frac{5}{2}$
Or, $m - 10 = \frac{25}{4}$ [Again squaring]
Or, $m = 10 + \frac{25}{4}$
Or, $x^2 + 4x = \frac{65}{4}$
Or, $4x^2 + 16x - 65 = 0$
Or, $4x^2 + 26x - 10x - 65 = 0$
Or, $2x(2x + 13) - 5(2x + 13) = 0$
Or, $(2x + 13)(2x - 5) = 0$
Either, $2x + 13 = 0$ Or, $2x - 5 = 0$
$\therefore x = \frac{-13}{2}$ $\therefore x = \frac{5}{3}$
$\therefore x = \frac{-13}{2}, \frac{5}{2}$ **(Ans.)**

Question ▶ 11 (i) $x^2 - 8 = 0$
(ii) $5^x + 5^{2-x} = 26$
(iii) $3^{x-2} = 2^{2x-4}$

[Bangladesh Mahila Somitee Girls' High School & College, Chattogram]

a. Determine discriminant and nature of root of the equation $x^2 + 1 = 0$ 2

b. Show that, equation (ii) and (iii) has a common root. 4

c. Draw the graph of the equation (i) and determine roots from the graph. 4

Solution to the question no. 11

a Given equation, $x^2 + 1 = 0$
Or, $x^2 + 0.x + 1 = 0$
$\therefore$ Discriminant, $D = 0^2 - 4.1.1$
$= -4 < 0$
$\therefore$ The roots are imaginary.

b Given equation,

$$5^x + 5^{2-x} = 26 \text{ .... (ii)}$$

$$\text{and } 3^{x-2} = 2^{2x-4} \text{ .... (iii)}$$

From (ii), $5^x + 5^{2-x} = 26$

Or, $5^x + \frac{5^2}{5^x} = 26$

Or, $p + \frac{25}{p} = 26$; [Taking $5^x = p$]

Or, $p^2 + 25 = 26p$

Or, $p^2 - 26p + 25 = 0$

Or, $p^2 - 25p - p + 25 = 0$

Or, $p(p-25) - 1(p-25) = 0$

Or, $(p-25)(p-1) = 0$

Or, $p - 25 = 0$ Or, $p - 1 = 0$

Or, $p = 25$ Or, $p = 1$

Or, $5^x = 5^2$ Or, $5^x = 5^0$

$\therefore$ $x = 2$ $\therefore$ $x = 0$

From equation (iii), $3^{x-2} = 2^{2x-4}$

Or, $3^{x-2} = (2^2)^{x-2}$

Or, $\frac{3^{x-2}}{4^{x-2}} = 1$

Or, $\left(\frac{3}{4}\right)^{x-2} = \left(\frac{3}{4}\right)^0$

Or, $x - 2 = 0$

Or, $x = 2$

$\therefore$ Equation (ii) and (iii) has a common root 2 **(Shown)**

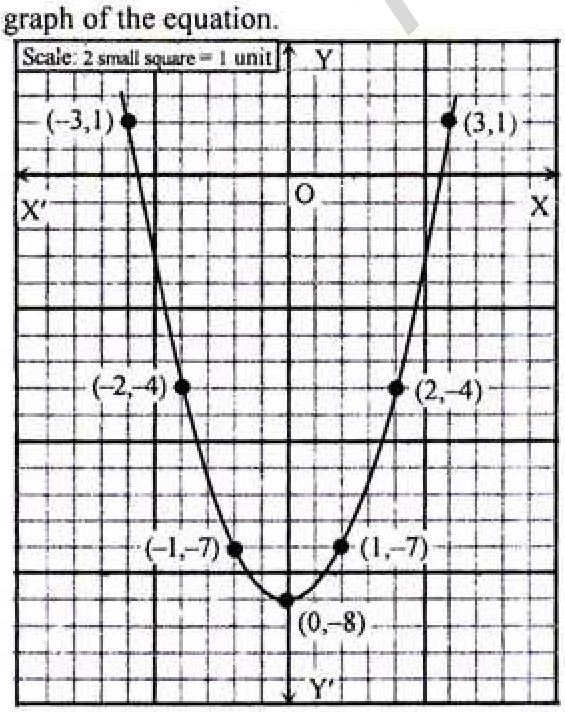
c Given equation, $x^2 = 8$ Or, $x^2 - 8 = 0$

Suppose, $y = x^2 - 8$

We find the values of y corresponding to some values of x which give the associated points for the graph:

x	0	1	2	3	–1	–2	–3
y	–8	–7	–4	1	–7	–4	1

After plotting the points obtained from the above table we draw the graph of the equation.



We can see from the figure that the graph intersects the x-axis about at (– 2.83, 0) and (2.83, 0). Since a quadratic equation has two roots, the solutions of the equation are x = – 2.83, x = 2.83.

Ans: Solution: x = –2.83 (approx.), 2.83 (approx.)

Question ▸ 12 $p = 2^{2z-1} + 3.2^z$ and $q = \left(\sqrt[3]{5}\right)^2 + \left(\sqrt[3]{5}\right)^{-2} - x^2, x > 0$ *[Secondary & Higher Secondary Education Board, Jashore]*

a. Find the solution set of $y - 6 \geq 3y + 4$ 2

b. If $p = 8$, then find the value of z. 4

c. If $q = -2$, then prove that, $5x^3 - 15x - 26 = 0$ 4

Solution to the question no. 12

a Give,

$y - 6 \geq 3y + 4$

Or, $y - 6 + 6 \geq 3y + 4 + 6$ [Adding 6 on both sides]

Or, $y \geq 3y + 10$

Or, $y - 3y \geq 3y + 10 - 3y$ [adding (– 3y) on both sides]

Or, $-2y \geq 10$

Or, $y \leq -5$

$\therefore$ The solution set, $S = \{y \in \mathbb{R} : y \leq -5\}$ **(Ans.)**

b Given,

$p = 2^{2z-1} + 3.2^z$

If $p = 8$, then

Or, $8 = 2^{2z-1} + 3.2^z$

Or, $8 = \frac{2^{2z}}{2} + 3.2^z$

Or, $8 = \frac{(2^z)^2 + 3.2^z.2}{2}$

Or, $(2^z)^2 + 6.2^z - 16 = 0$

Or, $(2^z)^2 + 8.2^z - 22^z - 16 = 0$

Or, $2^z(2^z + 8) - 2(2^z + 8) = 0$

Or, $(2^z + 8)(2^z - 2) = 0$

Either,

$2^z + 8 = 0$ Or, $2^z - 2 = 0$

Or, $2^z = -8$;   Or, $2^z = 2$

Which is not acceptable $\therefore$ z = 1 **(Ans.)**

c Given, $q = \left(\sqrt[3]{5}\right)^2 + \left(\sqrt[3]{5}\right)^{-2} - x^2, x > 0$

If $q = -2$, then

$-2 = \left(\sqrt[3]{5}\right)^2 + \left(\sqrt[3]{5}\right)^{-2} - x^2$

Or, $-2 = 5^{\frac{2}{3}} + 5^{\frac{-2}{3}} - x^2$

Or, $x^2 = 5^{\frac{2}{3}} + 5^{\frac{-2}{3}} + 2$

Or, $x^2 = \left(5^{\frac{1}{3}}\right)^2 + 2.5^{\frac{1}{3}}.5^{\frac{-1}{3}} + \left(5^{\frac{-1}{3}}\right)^2$

Or, $x^2 = \left(5^{\frac{1}{3}} + 5^{\frac{-1}{3}}\right)^2$

Or, $x = 5^{\frac{1}{3}} + 5^{\frac{-1}{3}}$

Or, $x^3 = \left(5^{\frac{1}{3}} + 5^{\frac{-1}{3}}\right)^3$

Or, $x^3 = \left(5^{\frac{1}{3}}\right)^3 + \left(5^{\frac{-1}{3}}\right)^3 + 3.5^{\frac{1}{3}}.5^{\frac{-1}{3}}\left(5^{\frac{1}{3}} + 5^{\frac{-1}{3}}\right)$

Or, $x^3 = 5 + 5^{-1} + 3x$

Or, $x^3 = 5 + \frac{1}{5} + 3x$

Or, $x^3 = \frac{25 + 1 + 15x}{5}$

Or, $5x^3 = 26 + 15x$

$\therefore$ $5x^3 - 15x - 26 = 0$ **(Proved)**