Chapter-8: Trigonometry

Question ▶ 1 $P = \tan\theta + \sec\theta$ and $Q = \cot^2\theta + \text{cosec}^2\theta$.
[All Board-18]

a. Determine the value of $\sec\theta - \tan\theta$. 2

b. Show that, $\cos\theta = \frac{2P}{P^2 + 1}$. 4

c. If $Q = 3$, then solve the given equation, where $0 < \theta < 2\pi$. 4

Solution to the question no. 1

a Given, $P = \tan\theta + \sec\theta$

Now, $\sec^2\theta - \tan^2\theta = 1$

Or, $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

Or, $P(\sec\theta - \tan\theta) = 1$

$\therefore \sec\theta - \tan\theta = \frac{1}{P}$ **(Ans.)**

b Given, $P = \tan\theta + \sec\theta$

Or, $P = \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$

Or, $P = \frac{1 + \sin\theta}{\cos\theta}$

Or, $P^2 = \frac{(1 + \sin\theta)^2}{\cos^2\theta}$ [by squaring]

Or, $\frac{(1 + \sin\theta)^2}{1 - \sin^2\theta} = P^2$

Or, $\frac{(1 + \sin\theta)^2}{(1 + \sin\theta)(1 - \sin\theta)} = P^2$

Or, $\frac{1 + \sin\theta}{1 - \sin\theta} = P^2$

Or, $\frac{1 + \sin\theta + 1 - \sin\theta}{1 + \sin\theta - 1 + \sin\theta} = \frac{P^2 + 1}{P^2 - 1}$ [by componendo and dividendo]

Or, $\frac{2}{2\sin\theta} = \frac{P^2 + 1}{P^2 - 1}$ or, $\frac{1}{\sin\theta} = \frac{p^2 + 1}{p^2 - 1}$

Or, $\sin\theta = \frac{P^2 - 1}{P^2 + 1}$

Or, $\sin^2\theta = \frac{(P^2 - 1)^2}{(P^2 + 1)^2}$

Or, $1 - \cos^2\theta = \frac{(P^2 - 1)^2}{(P^2 + 1)^2}$

Or, $1 - \frac{(P^2 - 1)^2}{(P^2 + 1)^2} = \cos^2\theta$

Or, $\frac{(P^2 + 1)^2 - (P^2 - 1)^2}{(P^2 + 1)^2} = \cos^2\theta$

Or, $\cos^2\theta = \frac{4P^2}{(P^2 + 1)^2}$

$\therefore \cos\theta = \frac{2P}{P^2 + 1}$ **(Shown)**

c Given, $Q = \cot^2\theta + \text{cosec}^2\theta$

and $Q = 3$

So, $\cot^2\theta + \text{cosec}^2\theta = 3$

Or, $\cot^2\theta + 1 + \cot^2\theta = 3$

Or, $2\cot^2\theta = 2$

Or, $\cot^2\theta = 1$

Or, $\cot\theta = \pm 1$

Taking, $\cot\theta = 1$,

$\cot\theta = \cot\frac{\pi}{4} = \cot(\pi + \frac{\pi}{4})$ [According to condition]

Or, $\cot\theta = \cot\frac{\pi}{4} = \cot\frac{5\pi}{4}$

$\therefore \theta = \frac{\pi}{4}, \frac{5\pi}{4}$ which satisfies the condition $0 < \theta < 2\pi$.

Again, taking $\cot\theta = -1$,

$\cot\theta = -\cot\frac{\pi}{4}$

Or, $\cot\theta = \cot(\pi - \frac{\pi}{4}) = \cot(2\pi - \frac{\pi}{4})$

[According to condition]

Or, $\cot\theta = \cot\frac{3\pi}{4} = \cot\frac{7\pi}{4}$

$\therefore \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$ which satisfies the condition $0 < \theta < 2\pi$.

$\therefore$ The solution in the given interval is

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ **(Ans.)**

Question ▶ 2 The wheel of a car moving from Dhaka to Khulna revolves 720 times in a minute. The radius of the wheel is 0.25 meter. [Dj.B.17]

a. Find the circumference of the wheel. 2

b. Find the speed of the car. 4

c. If the distance of Dhaka and Khulna subtends $2°$ angle at the centre of the earth, find the time required to go from Dhaka to Khulna. (The radius of the earth is 6440 km.) 4

Solution to the question no. 2

a Given, radius of the wheel, $r = 0.25$ metre

$\therefore$ Circumference of the wheel $= 2\pi r$ unit

$= 2 \times 3.1416 \times 0.25$ metre

$= 1.5708$ metre (approx.) **(Ans.)**

b From 'a' we get,

circumference of the wheel = 1.5708 metre (approx.)

We know,

after revolving one time, the wheel travel the distance which is equal to its circumference.

$\therefore$ The car travels the distance per minute

$= 720 \times 1.5708$ metre

$= 1130.976$ metre

$\therefore$ Velocity of the car $= 1130.976$ metre/minute

$= \frac{1130.976 \times 60}{1000}$ km/hour

$= 67.86$ km/hour **(Ans.)**

c Given, radius, $R = 6440$ km

The subtended angle at the centre of the earth,

$\theta = 2° = 2 \times \frac{\pi}{180}^c$

$= 0.034907^c$

$\therefore$ Distance between Dhaka = length of the arc $= r\theta$

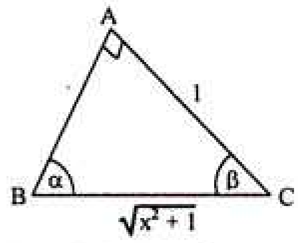
$= 6440 \times 0.034907$

$= 224.801$ km

From 'b' we get,
Velocity of the car = 67.86 km/hour

$\therefore$ The required time to go from Dhaka to Khulna
$= (224.801 \div 67.86)$ hour
$= 3.31$ hour (approx.) **(Ans.)**

Question ▶ 3



[D.B.16]

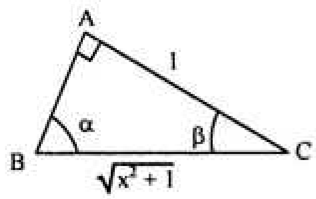
a. Find the value of $\sin(\alpha + \beta) + \cos(\alpha + \beta)$. 2

b. Considering the stem prove that, $(\sin\alpha - \cos\alpha)^2 = 1 - 2\sin\alpha.\cos\alpha$. 4

c. If $x^2 + \frac{1}{x^2} = 2$, then find the value of α. 4

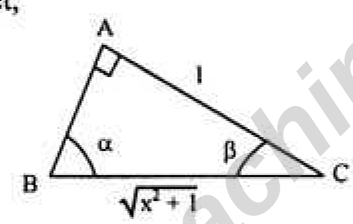
Solution to the question no. 3

a According to figure, we get in



ΔABC, $\angle A = 90°$
We know,
$\angle A + \angle B + \angle C = 180°$
Or, $\alpha + \beta = 180° - \angle A$
Or, $\alpha + \beta = 180° - 90°$
$\therefore \alpha + \beta = 90°$
Given, expression $= \sin(\alpha + \beta) + \cos(\alpha + \beta)$
$= \sin 90° + \cos 90°$
$= 1 + 0 = 1$ **(Ans.)**

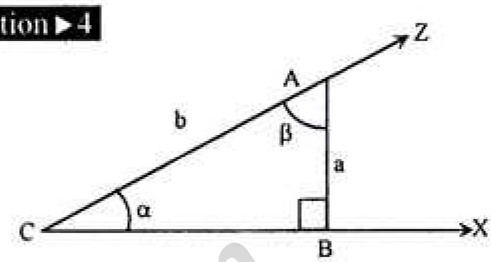
b According to figure we get,



$AC = 1$
$BC = \sqrt{x^2+1}$
since $\angle BAC = 90°$
$\therefore AB = \sqrt{BC^2 - AC^2}$
$= \sqrt{(\sqrt{x^2+1})^2 - 1^2}$
$= \sqrt{x^2+1-1} = \sqrt{x^2} = x$
Now, $\sin\alpha = \frac{AC}{BC} = \frac{1}{\sqrt{x^2+1}}$ and
$\cos\alpha = \frac{AB}{BC} = \frac{x}{\sqrt{x^2+1}}$
L. H. S. $= (\sin\alpha - \cos\alpha)^2$
$= \left(\frac{1}{\sqrt{x^2+1}} - \frac{x}{\sqrt{x^2+1}}\right)^2 = \left(\frac{1-x}{\sqrt{x^2+1}}\right)^2 = \frac{(1-x)^2}{x^2+1}$
R. H. S. $= 1 - 2\sin\alpha.\cos\alpha = 1 - 2.\frac{1}{\sqrt{x^2+1}}.\frac{x}{\sqrt{x^2+1}}$
$= 1 - \frac{2x}{x^2+1} = \frac{x^2+1-2x}{x^2+1} = \frac{1-2x+x^2}{x^2+1} = \frac{(1-x)^2}{x^2+1}$
$\therefore (\sin\alpha - \cos\alpha)^2 = 1 - 2\sin\alpha.\cos\alpha$ **(Proved)**

c From the figure,
$\tan\alpha = \frac{AC}{AB} = \frac{1}{x}$
Or, $\tan^2\alpha = \frac{1}{x^2}$
$\therefore \cot^2\alpha = x^2$
Here, $x^2 + \frac{1}{x^2} = 2$
Or, $\cot^2\alpha + \tan^2\alpha = 2$
Or, $\frac{1}{\tan^2\alpha} + \tan^2\alpha = 2$
Or, $\frac{1 + \tan^4\alpha}{\tan^2\alpha} = 2$
Or, $\tan^4\alpha + 1 = 2\tan^2\alpha$
Or, $\tan^4\alpha - 2\tan^2\alpha + 1 = 0$
Or, $(\tan^2\alpha)^2 - 2.\tan^2\alpha.1 + 1^2 = 0$
Or, $(\tan^2\alpha - 1)^2 = 0$
Or, $\tan^2\alpha - 1 = 0$
Or, $\tan^2\alpha = 1$
Or, $\tan\alpha = 1$ [$\because$ α is an acute angle, therefore taking positive value of $\tan\alpha$]
Or, $\tan\alpha = \tan 45°$
$\therefore \alpha = 45°$ **(Ans.)**

Question ▶ 4



[C.B.16]

a. Find the value of $\sec\alpha$. 2

b. If $a = 1$ and $b = 2$, then prove that, $\cos 3\beta = 4\cos^3\beta - 3\cos\beta$. 4

c. If $a + \sqrt{b^2 - a^2} = \sqrt{2}b$, then find the value of β. 4

Solution to the question no. 4

a Given,
$AB = a$, $AC = b$
In right angled triangle ABC,
$AC^2 = AB^2 + BC^2$
Or, $BC^2 = AC^2 - AB^2$
$\therefore BC = \sqrt{b^2 - a^2}$
$\therefore \sec\alpha = \frac{AC}{BC} = \frac{b}{\sqrt{b^2 - a^2}}$ **(Ans.)**

b Given, $a = 1$ and $b = 2$
In ΔABC,
$\cos\beta = \frac{AB}{AC}$ [$\because AB = a$, $AC = b$]
$= \frac{a}{b} = \frac{1}{2}$
Or, $\cos\beta = \cos\frac{\pi}{3}$ $\therefore \beta = \frac{\pi}{3}$
L. H. S. $= \cos 3\beta = \cos 3.\frac{\pi}{3} = \cos\pi = -1$
R. H. S. $= 4\cos^3\beta - 3\cos\beta$
$= 4.\left(\frac{1}{2}\right)^3 - 3.\frac{1}{2} = 4.\frac{1}{8} - \frac{3}{2} = \frac{1}{2} - \frac{3}{2} = \frac{1-3}{2} = -1$
$\therefore \cos 3\beta = 4\cos^3\beta - 3\cos\beta$ **(Proved)**

c From 'a' we get, $BC = \sqrt{b^2 - a^2}$
In, ΔABC, $\sin\beta = \frac{BC}{AC}$
$\therefore \sin\beta = \frac{\sqrt{b^2 - a^2}}{b}$ [$\therefore AC = b$]
and $\cos\beta = \frac{AB}{AC} = \frac{a}{b}$ [$\therefore AB = a$]
Given,
$a + \sqrt{b^2 - a^2} = \sqrt{2}b$

Or, $\frac{a}{b}+\frac{\sqrt{b^2-a^2}}{b}=\frac{\sqrt{2}b}{b}$ [both sides divided by b]

Or, $\cos\beta+\sin\beta=\sqrt{2}$

$\therefore \cos\beta=\sqrt{2}-\sin\beta$

Or, $\cos^2\beta=2-2\sqrt{2}\sin\beta+\sin^2\beta$

Or, $1-\sin^2\beta=2-2\sqrt{2}\sin\beta+\sin^2\beta$

Or, $2\sin^2\beta-2\sqrt{2}\sin\beta+1=0$

Or, $(\sqrt{2}\sin\beta-1)^2=0$

Or, $\sin\beta=\frac{1}{\sqrt{2}}$

Or, $\sin\beta=\sin45°$

$\therefore \beta=45°$ **(Ans.)**

Question ▶5 $P=a\cos\theta$ and $Q=b\sin\theta$. [J.B.16]

a. Find the value of $\frac{P^2}{a^2}+\frac{Q^2}{b^2}$. 2

b. If $P-Q=c$, prove that, $a\sin\theta+b\cos\theta=\pm\sqrt{a^2+b^2-c^2}$. 4

c. If $a^2=3$, $b^2=7$ and $Q^2+P^2=4$, prove that, $\tan\theta=\pm\frac{1}{\sqrt{3}}$ 4

Solution to the question no. 5

a Given, $P=a\cos\theta$, $Q=b\sin\theta$

$\therefore \frac{P^2}{a^2}+\frac{Q^2}{b^2}=\frac{(a\cos\theta)^2}{a^2}+\frac{(b\sin\theta)^2}{b^2}$

$=\frac{a^2\cos^2\theta}{a^2}+\frac{b^2\sin^2\theta}{b^2}=\cos^2\theta+\sin^2\theta=1$ **(Ans.)**

b Given, $P-Q=c$

Or, $a\cos\theta-b\sin\theta=c$

Or, $(a\cos\theta-b\sin\theta)^2=c^2$ [Squaring both sides.]

Or, $a^2\cos^2\theta-2.a\cos\theta.b\sin\theta+b^2\sin^2\theta=c^2$

Or, $a^2(1-\sin^2\theta)-2ab\cos\theta\sin\theta+b^2(1-\cos^2\theta)=c^2$

Or, $a^2-a^2\sin^2\theta-2ab\cos\theta\sin\theta+b^2-b^2\cos^2\theta=c^2$

Or, $a^2+b^2-c^2=a^2\sin^2\theta+2.a\sin\theta.b\cos\theta+b^2\cos^2\theta$

Or, $a^2+b^2-c^2=(a\sin\theta+b\cos\theta)^2$

Or, $(a\sin\theta+b\cos\theta)^2=a^2+b^2-c^2$

$\therefore a\sin\theta+b\cos\theta=\pm\sqrt{a^2+b^2-c^2}$ **(Proved)**

c Given, $a^2=3$, $b^2=7$

and $Q^2+P^2=4$

Now, $Q^2+P^2=4$

Or, $b^2\sin^2\theta+a^2\cos^2\theta=4$ [Given]

Or, $7\sin^2\theta+3\cos^2\theta=4$ [Putting the value of a^2 and b^2]

Or, $7(1-\cos^2\theta)+3\cos^2\theta=4$

Or, $7-7\cos^2\theta+3\cos^2\theta=4$

Or, $4\cos^2\theta=3$

Or, $\cos^2\theta=\frac{3}{4}$

Or, $\frac{1}{\sec^2\theta}=\frac{3}{4}$

Or, $\sec^2\theta=\frac{4}{3}$

Or, $1+\tan^2\theta=\frac{4}{3}$

Or, $\tan^2\theta=\frac{4}{3}-1$

Or, $\tan^2\theta=\frac{4-3}{3}$

Or, $\tan^2\theta=\frac{1}{3}$

$\therefore \tan\theta=\pm\frac{1}{\sqrt{3}}$ **(Proved)**

Question ▶6 $\sin A+\cos A=P$ and $Q=\sec\theta-\tan\theta$. [D.B.17]

a. Express 32′4″ in radians. 2

b. If $P=1$, then prove that, $\sin A-\cos A=\pm1$. 4

c. Find the value of θ where as $Q=(\sqrt{3})^{-1}$. (Where θ is acute angle) 4

Solution to the question no. 6

a 32′4″

$=\left(32\frac{4}{60}\right)'$ [$\because 60''=1'$]

$=\left(32\frac{1}{15}\right)'=\left(\frac{481}{15}\right)'$

$=\left(\frac{481}{15\times60}\right)^\circ$ [$\because 60'=1°$]

$=\left(\frac{481}{900}\right)^\circ$

$=\left(\frac{481}{900}\times\frac{\pi}{180}\right)^c$ $\left[\because 1°=\frac{\pi^c}{180}\right]$

$=\left(\frac{481\pi}{162000}\right)^c$

$=.0093^c$ (approx.) **(Ans.)**

b Given, $\sin A+\cos A=P$

According to question, $P=1$

Or, $\sin A+\cos A=1$

Or, $(\sin A+\cos A)^2=1^2$ [Squaring both sides]

Or, $\sin^2A+\cos^2A+2\sin A\cos A=1$

Or, $1+2\sin A\cos A=1$ [$\because \sin^2A+\cos^2A=1$]

Or, $2\sin A\cos A=1-1$

$\therefore 2\sin A\cos A=0$

Now, $(\sin A-\cos A)^2=(\sin A+\cos A)^2-4\sin A\cos A$

$=1^2-0$ [putting values]

Or, $\sin A-\cos A=\pm\sqrt{1}$

$\therefore \sin A-\cos A=\pm1$ **(Proved)**

c Given, $Q=\sec\theta-\tan\theta$

According to question, $Q=(\sqrt{3})^{-1}$

Or, $\sec\theta-\tan\theta=\frac{1}{\sqrt{3}}$

Or, $\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}=\frac{1}{\sqrt{3}}$

Or, $\frac{1-\sin\theta}{\cos\theta}=\frac{1}{\sqrt{3}}$

Or, $\sqrt{3}-\sqrt{3}\sin\theta=\cos\theta$

Or, $(\sqrt{3}-\sqrt{3}\sin\theta)^2=(\cos\theta)^2$ [Squaring both sides]

Or, $3+3\sin^2\theta-6\sin\theta=\cos^2\theta$

Or, $3+3\sin^2\theta-6\sin\theta=1-\sin^2\theta$ [$\because \sin^2\theta+\cos^2\theta=1$]

Or, $3\sin^2\theta+\sin^2\theta-6\sin\theta+3-1=0$

Or, $4\sin^2\theta-6\sin\theta+2=0$

Or, $2(2\sin^2\theta-3\sin\theta+1)=0$

Or, $2\sin^2\theta-3\sin\theta+1=0$

Or, $2\sin^2\theta-2\sin\theta-\sin\theta+1=0$

Or, $2\sin\theta(\sin\theta-1)-1(\sin\theta-1)=0$

$\therefore (\sin\theta-1)(2\sin\theta-1)=0$

That is, $\sin\theta-1=0$ or, $2\sin\theta-1=0$

Or, $\sin\theta=1$ Or, $\sin\theta=\frac{1}{2}$

Or, $\sin\theta=\sin90°$ Or, $\sin\theta=\sin30°$

$\therefore \theta=90°$ $\therefore \theta=30°$

But $\theta=90°$ is not acceptable, because θ is an acute angle.

$\therefore \theta=30°$ **(Ans.)**

Question ▶ 7 Musa Ebrahim saw that a hill subtends an angle of 7′ at a point 540 kilometre from the foot of hill and write an equation is $x = \tan\theta + \sec\theta$. [R.B.17]

a. Find the height of the hill. 2

b. From the equation find the value of $\sin\theta = \frac{x^2-1}{x^2+1}$ 4

c. From the equation if $x = 1$; find the value of θ; where $0° \le \theta < 90°$. 4

Solution to the question no. 7

a See example-9 of exercise-8.1 from your textbook. Page-153

b Given, $\tan\theta + \sec\theta = x$

Or, $\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = x$

Or, $\frac{1+\sin\theta}{\cos\theta} = x$

Or, $\frac{(1+\sin\theta)^2}{\cos^2\theta} = x^2$ [by squaring]

Or, $\frac{(1+\sin\theta)^2}{1-\sin^2\theta} = x^2$ [$\because \cos^2\theta = 1 - \sin^2\theta$]

Or, $\frac{(1+\sin\theta)(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} = x^2$

Or, $\frac{1+\sin\theta}{1-\sin\theta} = x^2$

Or, $\frac{1+\sin\theta+1-\sin\theta}{1+\sin\theta-1+\sin\theta} = \frac{x^2+1}{x^2-1}$ [by componendo and dividendo]

Or, $\frac{2}{2\sin\theta} = \frac{x^2+1}{x^2-1}$

$\therefore \sin\theta = \frac{x^2-1}{x^2+1}$ **(Shown)**

c Given, $x = 1$

From 'b' we get, $\sin\theta = \frac{x^2-1}{x^2+1}$

Or, $\sin\theta = \frac{(1)^2-1}{(1)^2+1}$ [putting the value of x]

Or, $\sin\theta = \frac{0}{2}$

Or, $\sin\theta = 0$

Or, $\sin\theta = \sin0°$ [$\because \sin0° = 0$]

$\therefore \theta = 0°$ **(Ans.)**

Question ▶ 8 $A = \sec\theta + \tan\theta$ and $B = \cos\left(-\frac{25\pi}{6}\right)$ [C.B.17]

a. Find the value of B. 2

b. If $A = x$, then show that, $\sin\theta = \frac{x^2-1}{x^2+1}$ 4

c. Find the value of θ when $A = \sqrt{3}$ and $0 < \theta < 2\pi$ 4

Solution to the question no. 8

a Given, $B = \cos\left(-\frac{25\pi}{6}\right)$

$= \cos\frac{25\pi}{6}$ [$\because \cos(-\theta) = \cos\theta$]

$= \cos\left(4\pi + \frac{\pi}{6}\right) = \cos\left(8 \cdot \frac{\pi}{2} + \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{6}$

$\therefore B = \frac{\sqrt{3}}{2}$ **(Ans.)**

b Given, $A = \sec\theta + \tan\theta$

According to question, $A = x$

Or, $\sec\theta + \tan\theta = x$

Or, $\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = x$

Or, $\frac{1+\sin\theta}{\cos\theta} = x$

Or, $\frac{(1+\sin\theta)^2}{\cos^2\theta} = x^2$ [Squaring both sides]

Or, $\frac{(1+\sin\theta)^2}{1-\sin^2\theta} = x^2$ [$\because \cos^2\theta = 1 - \sin^2\theta$]

Or, $\frac{(1+\sin\theta)(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} = x^2$

Or, $\frac{1+\sin\theta}{1-\sin\theta} = x^2$

Or, $\frac{1+\sin\theta+1-\sin\theta}{1+\sin\theta-1+\sin\theta} = \frac{x^2+1}{x^2-1}$ [by componendo and dividendo]

Or, $\frac{2}{2\sin\theta} = \frac{x^2+1}{x^2-1}$

Or, $\frac{1}{\sin\theta} = \frac{x^2+1}{x^2-1}$

$\therefore \sin\theta = \frac{x^2-1}{x^2+1}$ **(Shown)**

c Given, $A = \sec\theta + \tan\theta$

According to condition, $A = \sqrt{3}$

Or, $\sec\theta + \tan\theta = \sqrt{3}$

Or, $\sec\theta = \sqrt{3} - \tan\theta$

Or, $\sec^2\theta = (\sqrt{3} - \tan\theta)^2$ [by squaring]

Or, $1 + \tan^2\theta = 3 - 2\sqrt{3}\tan\theta + \tan^2\theta$

Or, $2\sqrt{3}\tan\theta = 3 + \tan^2\theta - 1 - \tan^2\theta$

Or, $2\sqrt{3}\tan\theta = 2$

Or, $\tan\theta = \frac{2}{2\sqrt{3}}$

Or, $\tan\theta = \frac{1}{\sqrt{3}}$

Or, $\tan\theta = \tan\frac{\pi}{6} = \tan\left(\pi + \frac{\pi}{6}\right)$ [$\because 0 < \theta < 2\pi$]

Or, $\tan\theta = \tan\frac{\pi}{6} = \tan\frac{7\pi}{6}$

$\therefore \theta = \frac{\pi}{6}, \frac{7\pi}{6}$

$\therefore$ But $\theta = \frac{7\pi}{6}$ is not acceptable because it doesn't satisfy the given equation.

$\therefore$ Required value, $\theta = \frac{\pi}{6}$

Question ▶ 9 $\cot\theta + \text{cosec}\theta = m$. [Ctg.B.17]

a. Find the value of $\text{cosec}\theta - \cot\theta$. 2

b. If $m = 2$, then show that, $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1+\sin\theta}{\cos\theta}$ 4

c. If $m = \sqrt{3}$, then find the value of θ where $0 \le \theta \le 2\pi$ 4

Solution to the question no. 9

a Given, $\text{cosec}\theta + \cot\theta = m$

We know, $\text{cosec}^2\theta - \cot^2\theta = 1$

Or, $(\text{cosec}\theta + \cot\theta)(\text{cosec}\theta - \cot\theta) = 1$

Or, $m(\text{cosec}\theta - \cot\theta) = 1$

$\therefore \text{cosec}\theta - \cot\theta = \frac{1}{m}$ **(Ans.)**

b Given, $\cot\theta + \text{cosec}\theta = m$

Or, $\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} = 2$ [$\because m = 2$]

Or, $\frac{\cos\theta + 1}{\sin\theta} = 2$

Or, $\frac{(\cos\theta + 1)^2}{\sin^2\theta} = 4$

Or, $\frac{(1 + \cos\theta)^2}{1 - \cos^2\theta} = 4$

Or, $\frac{(1 + \cos\theta)(1 + \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)} = 4$

Or, $\frac{1 + \cos\theta}{1 - \cos\theta} = 4$

Or, $\frac{1 + \cos\theta + 1 - \cos\theta}{1 + \cos\theta - 1 + \cos\theta} = \frac{4+1}{4-1}$ [by componendo and dividendo]

Or, $\frac{2}{2\cos\theta} = \frac{5}{3}$

$\therefore \cos\theta = \frac{3}{5}$

$\therefore \sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{3}{5}\right)^2}$

$= \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

L.H.S. $= \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

$= \frac{\frac{4}{5} - \frac{3}{5} + 1}{\frac{4}{5} + \frac{3}{5} - 1} = \frac{\frac{4 - 3 + 5}{5}}{\frac{4 + 3 - 5}{5}}$

$= \frac{6}{5} \times \frac{5}{2} = 3$

R.H.S. $= \frac{1 + \sin\theta}{\cos\theta} = \frac{1 + \frac{4}{5}}{\frac{3}{5}}$

$= \frac{\frac{5+4}{5}}{\frac{3}{5}} = \frac{9}{5} \times \frac{5}{3}$

$= 3$

$\therefore \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1 + \sin\theta}{\cos\theta}$ **(Shown)**

c Given, $\text{cosec}\theta + \cot\theta = m$

Or, $\text{cosec}\theta + \cot\theta = \sqrt{3}$ [$\because m = \sqrt{3}$]

Or, $\text{cosec}\theta = \sqrt{3} - \cot\theta$

Or, $\text{cosec}^2\theta = 3 - 2\sqrt{3}\cot\theta + \cot^2\theta$

Or, $1 + \cot^2\theta - 3 + 2\sqrt{3}\cot\theta - \cot^2\theta = 0$

Or, $2\sqrt{3}\cot\theta - 2 = 0$

Or, $\cot\theta = \frac{2}{2\sqrt{3}}$

Or, $\cot\theta = \frac{1}{\sqrt{3}}$

Or, $\cot\theta = \cot\frac{\pi}{3} = \cot\left(\pi + \frac{\pi}{3}\right)$

$= \cot\frac{\pi}{3} = \cot\frac{4\pi}{3}$

$\therefore \theta = \frac{\pi}{3}, \frac{4\pi}{3}$

But, for $\theta = \frac{4\pi}{3}$ the given equation is not satisfied.

$\therefore$ Required solution, $\theta = \frac{\pi}{3}$

Question ▶ 10 $f(x) = \sin x$ [S.B.17]

a. Find the length of the arc which subtends an angle 60° at the centre of a circle with radius 5 cm. 2

b. If $af(\theta) + bf\left(\frac{\pi}{2} - \theta\right) = c$, then prove that.

$af\left(\frac{\pi}{2} - \theta\right) - bf(\theta) = \pm\sqrt{a^2 + b^2 - c^2}$. 4

c. Solve : $f(x) + f\left(\frac{\pi}{2} - x\right) = \sqrt{2}$, where $0 \le x \le 2\pi$. 4

Solution to the question no. 10

a Given, radius, $r = 5$ cm

Subtend angle at the centre, $\theta = 60° = 60 \times \frac{\pi^c}{180} = \frac{\pi^c}{3}$

We know, arc length, $S = r\theta = 5 \times \frac{\pi}{3} = \frac{5 \times 3.1416}{3}$

$\therefore$ Arc length = 5.236 cm (approx.) **(Ans.)**

b Given, $f(x) = \sin x$

According to question, $af(\theta) + bf\left(\frac{\pi}{2} - \theta\right) = c$

Or, $a\sin\theta + b\sin\left(\frac{\pi}{2} - \theta\right) = c$

Or, $a\sin\theta + b\cos\theta = c$

Or, $a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = c^2$ [by squaring]

Or, $a^2(1 - \cos^2\theta) + b^2(1 - \sin^2\theta) + 2ab\sin\theta.\cos\theta = c^2$

Or, $a^2 - a^2\cos^2\theta + b^2 - b^2\sin^2\theta + 2ab\sin\theta.\cos\theta = c^2$

Or, $a^2 + b^2 - c^2 = a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta.\cos\theta$

Or, $a^2 + b^2 - c^2 = (a\cos\theta)^2 + (b\sin\theta)^2 - 2a\cos\theta.b\sin\theta$

Or, $a^2 + b^2 - c^2 = (a\cos\theta - b\sin\theta)^2$

Or, $a\cos\theta - b\sin\theta = \pm\sqrt{a^2 + b^2 - c^2}$

$\therefore af\left(\frac{\pi}{2} - \theta\right) - bf(\theta) = \pm\sqrt{a^2 + b^2 - c^2}$ **(Proved)**

c Given, $f(x) + f\left(\frac{\pi}{2} - x\right) = \sqrt{2}$, when $0 \le x \le 2\pi$

Or, $\sin x + \sin\left(\frac{\pi}{2} - x\right) = \sqrt{2}$

Or, $\sin x + \cos x = \sqrt{2}$

Or, $\cos^2x = (\sqrt{2} - \sin x)^2$ [by squaring]

Or, $\cos^2x = 2 - 2\sqrt{2}\sin x + \sin^2x$

Or, $1 - \sin^2x = 2 - 2\sqrt{2}\sin x + \sin^2x$

Or, $2\sin^2x - 2\sqrt{2}\sin x + 1 = 0$

Or, $(\sqrt{2}\sin x)^2 - 2\sqrt{2}\sin x.1 + 1^2 = 0$

Or, $(\sqrt{2}\sin x - 1)^2 = 0$

Or, $\sqrt{2}\sin x - 1 = 0$

Or, $\sin x = \frac{1}{\sqrt{2}}$

Or, $\sin x = \sin\frac{\pi}{4} = \sin\left(\pi - \frac{\pi}{4}\right)$

Or, $\sin x = \sin\frac{\pi}{4} = \sin\frac{3\pi}{4}$

$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}$

But for $x = \frac{3\pi}{4}$ the given equation is not satisfied.

$\therefore$ Required solution, $x = \frac{\pi}{4}$ **(Ans.)**

Question ▶ 11 $7\sin^2\theta + 3\cos^2\theta = P$. [J.B.17]

a. If $\theta = \frac{\pi}{4}$, find the value of P. 2

b. If P = 4, prove that, $\cot\theta = \pm\sqrt{3}$ 4

c. If P = 6 and $0 < \theta < 2\pi$, find the possible value of θ. 4

Solution to the question no. 11

a Given, $P = 7\sin^2\theta + 3\cos^2\theta$

$$= 7\left(\sin\frac{\pi}{4}\right)^2 + 3\left(\cos\frac{\pi}{4}\right)^2 \quad \left[\because \theta = \frac{\pi}{4}\right]$$

$$= 7\left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{7}{2} + \frac{3}{2} = \frac{7+3}{2}$$

$\therefore$ P = 5 **(Ans.)**

b According to question, P = 4

Or, $7\sin^2\theta + 3\cos^2\theta = 4$

Or, $7\sin^2\theta + 3(1 - \sin^2\theta) = 4$

Or, $7\sin^2\theta + 3 - 3\sin^2\theta = 4$

Or, $4\sin^2\theta = 1$

$\therefore \sin^2\theta = \frac{1}{4}$

Again, $\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$

$$\therefore \cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{\frac{3}{4}}{\frac{1}{4}} = \frac{3}{4} \times \frac{4}{1} = 3$$

$\therefore \cot\theta = \pm\sqrt{3}$ **(Proved)**

c According to question, P = 6

Or, $7\sin^2\theta + 3\cos^2\theta = 6$

Or, $7\sin^2\theta + 3(1 - \sin^2\theta) = 6$

Or, $7\sin^2\theta + 3 - 3\sin^2\theta = 6$

Or, $4\sin^2\theta = 3$

Or, $\sin^2\theta = \frac{3}{4}$

$\therefore \sin\theta = \pm\frac{\sqrt{3}}{2}$

Taking '+', $\sin\theta = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right)$

$\therefore \theta = \frac{\pi}{3}, \frac{2\pi}{3}$

Taking '–', $\sin\theta = -\frac{\sqrt{3}}{2} = \sin\left(\pi + \frac{\pi}{3}\right) = \sin\left(2\pi - \frac{\pi}{3}\right)$

$\therefore \theta = \frac{4\pi}{3}, \frac{5\pi}{3}$

$\therefore$ The possible values of θ in the given interval: $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(Ans.)

Question ▶ 12 $A = x\cos\theta$ and $B = y\sin\theta$, where $0 < \theta < 2\pi$.

[B.B.17]

a. Find the value of $\frac{A^2}{x^2} + \frac{B^2}{y^2}$ 2

b. If A + B = z, Prove that, $x\sin\theta - y\cos\theta = \pm\sqrt{x^2 + y^2 - z^2}$ 4

c. If $x^2 = 3$, $y^2 = 7$ and $A^2 + B^2 = 4$, find the value of θ. 4

Solution to the question no. 12

a Given, $A = x\cos\theta$ and $B = y\sin\theta$

Given expression $= \frac{A^2}{x^2} + \frac{B^2}{y^2}$

$$= \frac{(x\cos\theta)^2}{x^2} + \frac{(y\sin\theta)^2}{y^2} = \frac{x^2\cos^2\theta}{x^2} + \frac{y^2\sin^2\theta}{y^2}$$

$= \cos^2\theta + \sin^2\theta = 1$ **(Ans.)**

b Given, A + B = z

Or, $x\cos\theta + y\sin\theta = z$

Or, $(x\cos\theta + y\sin\theta)^2 = z^2$ [by squaring]

Or, $x^2\cos^2\theta + y^2\sin^2\theta + 2xy\sin\theta.\cos\theta = z^2$

Or, $x^2(1 - \sin^2\theta) + y^2(1 - \cos^2\theta) + 2xy\sin\theta\cos\theta = z^2$

Or, $x^2 - x^2\sin^2\theta + y^2 - y^2\cos^2\theta + 2xy\sin\theta\cos\theta = z^2$

Or, $x^2 + y^2 - z^2 = x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta$

Or, $x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta = x^2 + y^2 - z^2$

Or, $(x\sin\theta - y\cos\theta)^2 = x^2 + y^2 - z^2$

$\therefore x\sin\theta - y\cos\theta = \pm\sqrt{x^2 + y^2 - z^2}$ **(Proved)**

c Given, $A = x\cos\theta$ and $B = y\sin\theta$

Here, $A^2 + B^2 = 4$

Or, $(x\cos\theta)^2 + (y\sin\theta)^2 = 4$

Or, $x^2\cos^2\theta + y^2\sin^2\theta = 4$

Now, if $x^2 = 3$, $y^2 = 7$ then we get,

$3\cos^2\theta + 7\sin^2\theta = 4$

Or, $3(1 - \sin^2\theta) + 7\sin^2\theta = 4$

Or, $3 - 3\sin^2\theta + 7\sin^2\theta = 4$

Or, $3 + 4\sin^2\theta = 4$

Or, $4\sin^2\theta = 4 - 3$

Or, $4\sin^2\theta = 1$

Or, $\sin^2\theta = \frac{1}{4}$

$\therefore \sin\theta = \pm\frac{1}{2}$ [Taking square roots]

Taking '+', $\sin\theta = \frac{1}{2}$

Or, $\sin\theta = \sin\frac{\pi}{6}$

$= \sin\left(\pi - \frac{\pi}{6}\right)$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

taking '–', $\sin\theta = -\frac{1}{2}$

Or, $\sin\theta = -\sin\frac{\pi}{6}$

Or, $\sin\theta = \sin\left(\pi + \frac{\pi}{6}\right)$

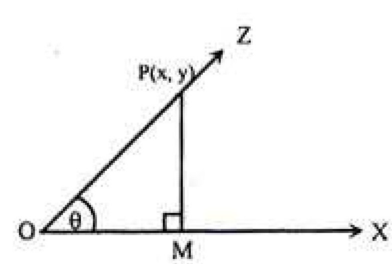
$= \sin\left(2\pi - \frac{\pi}{6}\right)$

Or, $\sin\theta = \sin\frac{7\pi}{6} = \sin\frac{11\pi}{6}$

$\therefore \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **(Ans.)**

Question ▶ 13



[R.B.16]

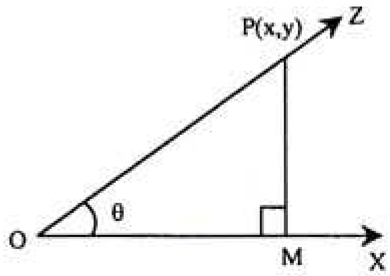
a. Find the value of $\sec\theta$. 2

b. If x = 1, $y = \sqrt{3}$, then prove that, $\sin3\theta = 3\sin\theta - 4\sin^3\theta$. 4

c. If $\sqrt{x^2 + y^2} + x = \sqrt{3}y$, then find the value of θ. 4

Solution to the question no. 13

a



From the figure we get,

OM = x and PM = y [∵ Coordinates of P is(x, y)]

ΔOPM এ ∠OMP = 90°

According to the Phythagoras theorem,

$OP^2 = OM^2 + PM^2$

Or, $OP^2 = x^2 + y^2$

$\therefore$ $OP = \sqrt{x^2 + y^2}$

$\therefore$ $\sec\theta = \frac{OP}{OM} = \frac{\sqrt{x^2 + y^2}}{x}$ **(Ans.)**

b Obtained from 'a' we get,

$\sec\theta = \frac{\sqrt{x^2 + y^2}}{x}$

Or, $\sec\theta = \frac{\sqrt{1^2 + (\sqrt{3})^2}}{1}$ [$\therefore$ $x = 1, y = \sqrt{3}$]

Or, $\sec\theta = \sqrt{4}$

Or, $\sec\theta = 2$

Or, $\sec\theta = \sec 60°$ [$\because$ $\sec 60° = 2$]

$\therefore$ $\theta = 60°$

L. H. S. $= \sin 3\theta$

$= \sin (3 \times 60°)$

$= \sin 180°$

$= 0$

R. H. S $= 3 \sin\theta - 4 \sin^3\theta$

$= 3 \sin 60° - 4 \sin^3 60°$

$= 3.\frac{\sqrt{3}}{2} - 4.\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3\sqrt{3}}{2} - 4.\frac{3\sqrt{3}}{8}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$

$= 0$

$\therefore$ $\sin 3\theta = 3\sin\theta - 4 \sin^3\theta$ **(proved)**

c Obtained from 'a' we get,

OM = x, PM = y and $OP = \sqrt{x^2 + y^2}$

We know, $\text{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{OP}{PM} = \frac{\sqrt{x^2 + y^2}}{y}$

$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{OM}{PM} = \frac{x}{y}$

Given, $\sqrt{x^2 + y^2} + x = \sqrt{3}y$

Or, $\frac{\sqrt{x^2 + y^2} + x}{y} = \frac{\sqrt{3}y}{y}$ [Dividing both sides by y]

Or, $\frac{\sqrt{x^2 + y^2}}{y} + \frac{x}{y} = \sqrt{3}$

Or, $\text{cosec}\theta + \cot\theta = \sqrt{3}$

Or, $\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \sqrt{3}$

Or, $\frac{1 + \cos\theta}{\sin\theta} = \sqrt{3}$

Or, $(1 + \cos\theta)^2 = (\sqrt{3}\sin\theta)^2$ [squaring]

Or, $1 + 2\cos\theta + \cos^2\theta = 3 \sin^2\theta$

Or, $1 + 2\cos\theta + \cos^2\theta - 3(1 - \cos^2\theta) = 0$

Or, $1 + 2\cos\theta + \cos^2\theta - 3 + 3\cos^2\theta = 0$

Or, $4\cos^2\theta + 2\cos\theta - 2 = 0$

Or, $2\cos^2\theta + \cos\theta - 1 = 0$

Or, $2\cos^2\theta + 2\cos\theta - \cos\theta - 1 = 0$

Or, $2\cos\theta(\cos\theta + 1) - 1(\cos\theta + 1) = 0$

Or, $(\cos\theta + 1)(2\cos\theta - 1) = 0$

either, $\cos\theta + 1 = 0$ Or, $2\cos\theta - 1 = 0$

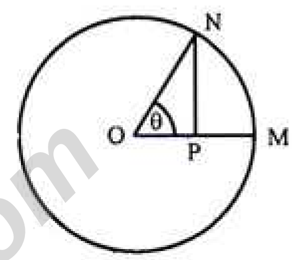
Or, $\cos \theta = -1$ Or, $\cos \theta = \frac{1}{2}$

Or, $\cos \theta = \cos 180°$ Or, $\cos \theta = \cos 60°$ $\therefore$ $\theta = 60°$

$\therefore$ $\theta = 180°$, which is not acceptable because θ is an acute angle.

The required value: $\theta = 60°$

Question ▸ 14



In the figure, O is the centre of a circle and OM = arc MN. [Dj.B.16]

a. Express θ in degree. 2

b. Prove that, θ is a constant angle. 4

c. Determine for what value of θ, $\frac{PN}{ON} + \frac{OP}{ON} = \sqrt{2}$, where $0 < \theta < 2\pi$. 4

Solution to the question no. 14

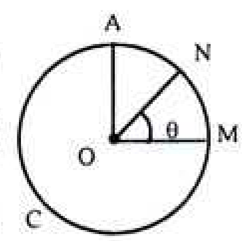
a In given figure θ is a radian angle.

We know, $\pi^c = 180°$

$1^c = \left(\frac{180}{\pi}\right)^\circ$

$\therefore$ $\theta^c = \left(\frac{180\,\theta}{\pi}\right)^\circ$ **(Ans.)**

b **Particular Enunciation:**



Suppose, in the circle AMC of raius r and centre O. ∠AOB is one radian. To prove that ∠AOB is a constant anle.

Construction: Draw perpendicular OA on OB.

Proof: OA intersect the circumsference at A

So, arc AB = one-fourth of the circumference.

$\left[\frac{1}{4} \times 2\pi r = \frac{\pi r}{2} \text{ and}\right]$

From proposition 2, $\frac{\angle POB}{\angle AOB} = \frac{\text{arc OB}}{\text{Arc AB}}$

$\therefore$ $\angle POB = \frac{\text{Arc PB}}{\text{Arc AB}} \times \angle AOB$

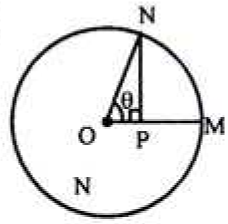
$\theta = \frac{r}{\frac{\pi r}{r}} = \frac{2}{\pi}$ = right angle and π are constant.

Since, the right angle and π are constant, therefore θ is a constant angle **(Proved)**

c In ΔOPN, $PN \perp OP$

$\therefore \angle OPN = 1$ right angle

Now, in right angled triangle OPN,



$\sin\theta = \dfrac{PN}{ON}$ and $\cos\theta = \dfrac{OP}{ON}$

Given,

$\dfrac{PN}{ON} + \dfrac{OP}{ON} = \sqrt{2}$

Or, $\sin\theta + \cos\theta = \sqrt{2}$

Or, $\sin\theta - \sqrt{2} = -\cos\theta$

Or, $(\sin\theta - \sqrt{2})^2 = (-\cos\theta)^2$

Or, $\sin^2\theta - 2\sqrt{2}\sin\theta + 2 = \cos^2\theta$

Or, $\sin^2\theta - 2\sqrt{2}\sin\theta + 2 = 1 - \sin^2\theta$

Or, $2\sin^2\theta - 2\sqrt{2}\sin\theta + 1 = 0$

Or, $(\sqrt{2}\sin\theta - 1)^2 = 0$

Or, $\sqrt{2}\sin\theta - 1 = 0$

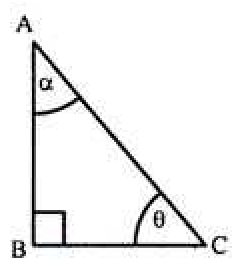
Or, $\sqrt{2}\sin\theta = 1$

Or, $\sin\theta = \dfrac{1}{\sqrt{2}}$

$\sin\theta = \sin 45°$

$\therefore \theta = 45°$ **(Ans.)**

Question ▶ 15



[Ctg.B.16]

a. Find the quadrant, in which $-700°$ lie with figure. 2

b. If $\left(\dfrac{AC}{BC}\right)^2 + \left(\dfrac{AB}{BC}\right)^2 = \dfrac{5}{3}$, then find the value of θ. 4

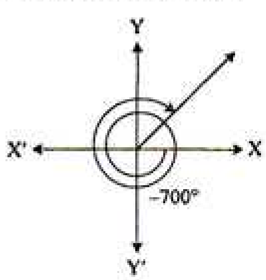
c. According to the stem,

show that, $\sin 2\alpha = 2\sin\alpha\cos\alpha = \dfrac{2\tan\alpha}{1 + \tan^2\alpha}$. 4

Solution to the question no. 15

a $-700° = -630° - 70° = -7 \times 90° - 70°$

Since, the angle $-700°$ is a negative angle. To produce $-700°$ we have revolve in the clock wise direction upto 7 right angles and more 70°



The angle $-700°$ is in 4th quadrants.**(Ans.)**

b From above information, $\sec\theta = \dfrac{AC}{BC}$

$\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{AB}{BC}$

Given, $\left(\dfrac{AC}{BC}\right)^2 + \left(\dfrac{AB}{BC}\right)^2 = \dfrac{5}{3}$

Or, $\sec^2\theta + \tan^2\theta = \dfrac{5}{3}$

Or, $1 + \tan^2\theta + \tan^2\theta = \dfrac{5}{3}$

Or, $3 + 6\tan^2\theta = 5$

Or, $6\tan^2\theta = 2$

Or, $\tan^2\theta = \dfrac{2}{6}$

Or, $\tan^2\theta = \dfrac{1}{3}$

Or, $\tan\theta = \pm\dfrac{1}{\sqrt{3}}$

Taking positive sign of $\tan\theta$

$\tan\theta = \tan 30°$ [$\because \theta$ is an acute angle]

$\therefore \theta = 30°$ **(Ans.)**

c In ΔABC,

$\angle B = 90°$ and $\angle C = \theta = 30°$ [obtained from 'b']

$\therefore \angle A + \angle B + \angle C = 180°$

Or, $\alpha + 90° + 30° = 180°$

Or, $\alpha = 180° - (90° + 30°)$

$\therefore \alpha = 60°$

$\therefore \sin 2\alpha = \sin(2 \times 60°)$

$= \sin(90° + 30°)$

$= \cos 30° = \dfrac{\sqrt{3}}{2}$

Again, $2\sin\alpha\cos\alpha = 2\sin 60°.\cos 60°$

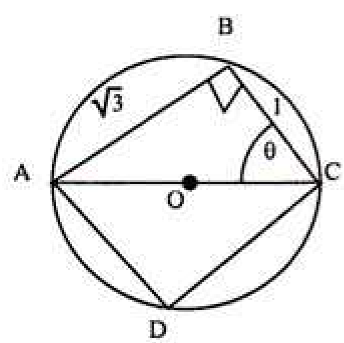
$= 2.\dfrac{\sqrt{3}}{2}.\dfrac{1}{2} = \dfrac{\sqrt{3}}{2}$

and $\dfrac{2\tan\alpha}{1 + \tan^2\alpha} = \dfrac{2\tan 60°}{1 + \tan^2 60°}$

$= \dfrac{2.\sqrt{3}}{1 + (\sqrt{3})^2} = \dfrac{2\sqrt{3}}{1 + 3} = \dfrac{2\sqrt{3}}{4} = \dfrac{\sqrt{3}}{2}$

$\therefore \sin 2\alpha = 2\sin\alpha.\cos\alpha = \dfrac{2\tan\alpha}{1 + \tan^2\alpha}$ **(Shown)**

Question ▶ 16



ABCD is a cyclic quadrilateral with centre O of the circle ABCD.

[S.B.16]

a. Find the value of θ in circular system. 2

b. In ΔABC, show that, $\cos(B + C) = \cos B\cos C - \sin B\sin C$. 4

c. What is the speed of the wheel, if ABCD is a circular wheel and it revolve ten times in a second? 4

Solution to the question no. 16

a From above figure, we get,

$\tan\angle ACB = \dfrac{AB}{BC}$

Or, $\tan\theta = \dfrac{\sqrt{3}}{1}$

Or, $\tan\theta = \sqrt{3}$

Or, $\tan\theta = \tan 60°$

$\therefore \theta = 60°$

We know, $1° = \frac{\pi}{180}$ radian

$\therefore\ 60° = \frac{\pi \times 60}{180}$ " $= \frac{\pi}{3}$ radian **(Ans.)**

b From above figure,

in ΔABC, $\angle B$ = right angle = $90°$

and $\angle C = 60°$ [obtained from 'a']

L. H. S. $= \cos(B + C)$

$= \cos(90° + 60°)$

$= -\sin 60° = -\frac{\sqrt{3}}{2}$

R.H.S $= \cos B \cos C - \sin B \sin C$

$= \cos 90°.\cos 60° - \sin 90°.\sin 60°$

$= 0.\cos 60° - 1.\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$

$\therefore \cos(B + C) = \cos B \cos C - \sin B \sin C$. **(Shown)**

c From above information, $AB = \sqrt{3}$ unit

and $BC = 1$ unit

$\therefore AC = \sqrt{AB^2 + BC^2} = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

Diameter of the circular wheel = AC = 2 unit

$\therefore$ Radious of the circular wheel, $r = \frac{2}{2}$ unit = 1 unit

$\therefore$ Circumference of the circular wheel $= 2\pi r$ unit

$= 2 \times 3.1416 \times 1 = 6.2832$ unit

we know,

in each revolution the circular wheel passes a distance which is equal to its circumference.

The wheel passes is one second 10×6.2832 units distance.

The wheel passes in one hour or is 3600 seconds $10 \times 6.2832 \times 3600$ units distance.

$\therefore$ The speed of the wheel per hour 226195.2 units **(Ans.)**

Question ▶ 17 Suppose, $P = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$ and $Q = \sec\theta + \tan\theta$. [B.B.16]

a. If $\tan 10x = \cot 5x$, find the value of x. 2

b. Show that, P = Q. 4

c. If $Q = \sqrt{3}$ and $0 < \theta < 2\pi$, find the value of θ. 4

Solution to the question no. 17

a Given, $\tan 10x = \cot 5x$

Or, $\tan 10x = \tan(90° - 5x)$

Or, $10x = 90° - 5x$

Or, $10x + 5x = 90°$

Or, $15x = 90°$

$\therefore x = 6°$ **(Ans.)**

b Given that, $P = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

$$= \frac{\cos\theta\left(\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)}{\cos\theta\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}\right)}$$

$$= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{1 - \sec\theta + \tan\theta}$$

$$= \frac{\sec\theta + \tan\theta - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{1 - \sec\theta + \tan\theta}$$

$$= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{1 - \sec\theta + \tan\theta}$$

$= \sec\theta + \tan\theta$

$= Q$ [Given that, $Q = \sec\theta + \tan\theta$]

$\therefore P = Q$ **(Shown)**

c Given that, $Q = \sec\theta + \tan\theta$

According to the condition, $Q = \sqrt{3}$

$\therefore \sec\theta + \tan\theta = \sqrt{3}$

Or, $\sec\theta = \sqrt{3} - \tan\theta$

Or, $\sec^2\theta = (\sqrt{3} - \tan\theta)^2$

Or, $1 + \tan^2\theta = 3 - 2\sqrt{3}\tan\theta + \tan^2\theta$

Or, $2\sqrt{3}\tan\theta = 3 + \tan^2\theta - 1 - \tan^2\theta$

Or, $2\sqrt{3}\tan\theta = 2$

Or, $\tan\theta = \frac{2}{2\sqrt{3}}$

Or, $\tan\theta = \frac{1}{\sqrt{3}}$

Or, $\tan\theta = \tan\frac{\pi}{6}, \tan\left(\pi + \frac{\pi}{6}\right)$ [$\because 0 < \theta < 2\pi$]

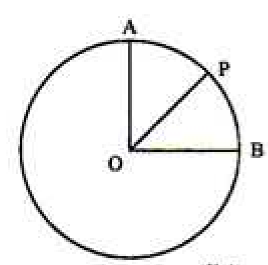
Or, $\tan\theta = \tan\frac{\pi}{6}, \tan\frac{7\pi}{6}$

$\therefore \theta = \frac{\pi}{6}, \frac{7\pi}{6}$

$\therefore$ But $\theta = \frac{7\pi}{6}$ is not acceptable. Because $\theta = \frac{7\pi}{6}$ is not satisfied the equation.

$\therefore\ \theta = \frac{\pi}{6}$

Question ▶ 18



[Mirzapur Cadet College, Tangail]

a. Find the value of $\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8}$. 2

b. From figure, prove that $\angle POB$ is a constant angle. 4

c. In the figure, if $\angle POB = 8$ and OB = 550 kilometre, find PB. 4

Solution to the question no. 18

a $\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8}$

$= \cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\left(\frac{\pi}{2} + \frac{\pi}{8}\right) + \cos^2\left(\frac{\pi}{2} + \frac{3\pi}{8}\right)$

$= \cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8}$

$= \left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right) + \left(\cos^2\frac{3\pi}{8} + \sin^2\frac{3\pi}{8}\right)$

$= 1 + 1$

$= 2$

$\therefore$ The required value = 2

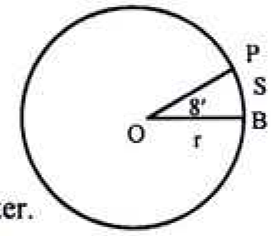
b See your text book ch-8 preposition 3.

c We know, if any arc of length S produce an angle θ at the centre of the circle of radius r, then $S = r\theta$

Here, $\theta = 8' = \left(\frac{8}{60}\right)^\circ$

$= \frac{8}{60} \times \frac{\pi}{180}$ radian

and $r = OB = 550$ kilometer.



$\therefore S = PB = \frac{8}{60} \times \frac{\pi}{180} \times 550$ kilometer.

$= 1.28$ km (appr.) **(Ans.)**

Question ▶ 19 i. $a\sec\theta + b\tan\theta = c$

ii. An unbiased coin is tossed thrice.

[Mirzapur Cadet College, Tangail]

a. If $\sin A = \frac{2}{\sqrt{5}}$, what is the value of tan A. 2

b. From (ii), find the sample space and find the probability of getting just one tail. 4

c. Solve (i), if $a = b = 1$ and $c = \sqrt{3}$ where $0° < \theta < \frac{\pi}{2}$. 4

Solution to the question no. 19

a Given that, $\sin A = \frac{2}{\sqrt{5}}$

Or, $\frac{1}{\sin^2 A} = \frac{5}{4}$

Or, $\operatorname{cosec}^2 A = \frac{5}{4}$

Or, $1 + \cot^2 A = \frac{5}{4}$

Or, $\cot^2 A = \frac{5}{4} - 1$

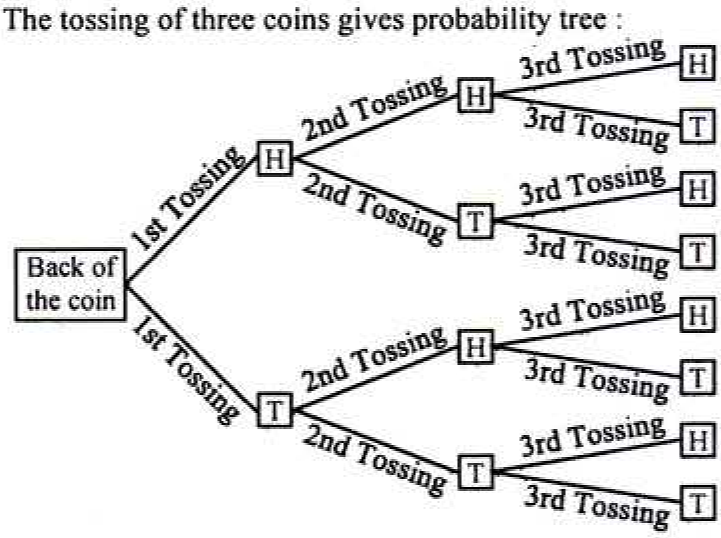
Or, $\cot^2 A = \frac{1}{4}$

Or, $\frac{1}{\tan^2 A} = \frac{1}{4}$

Or, $\tan^2 A = 4$

$\therefore \tan A = \pm 2$ **(Ans.)**

b The tossing of three coins gives probability tree :



The sample space is S = {HHT, HTH, HTT, HHH, THT, TTH, THH, TTT}

The sample point of just one tail are = {HHT, HTH, THH}

$\therefore$ Probability of just one tail is $= \frac{3}{8}$ **(Ans.)**

c Given that,

(i) $a\sec\theta + b\tan\theta = c$

If $a = b = 1$ and $c = \sqrt{3}$, then (i) will be

$\sec\theta + \tan\theta = \sqrt{3}$

Or, $\sec\theta = \sqrt{3} - \tan\theta$

Or, $\sec^2\theta = (\sqrt{3} - \tan\theta)^2$ [by squaring]

Or, $1 + \tan^2\theta = 3 - 2\sqrt{3}\tan\theta + \tan^2\theta$

Or, $2\sqrt{3}\tan\theta = 3 + \tan^2\theta - 1 - \tan^2\theta$

Or, $2\sqrt{3}\tan\theta = 2$

Or, $\tan\theta = \frac{2}{2\sqrt{3}}$

Or, $\tan\theta = \frac{1}{\sqrt{3}}$

Or, $\tan\theta = \tan\frac{\pi}{6}$ $\left[\because 0 < \theta < \frac{\pi}{2}\right]$

$\therefore \theta = \frac{\pi}{6}$

$\therefore$ Required value, $\theta = \frac{\pi}{6}$ **(Ans.)**

Question ▶ 20 Given, $A = \sec\theta - \tan\theta$

[Mymensingh Girls' Cadet College, Mymensingh]

a. If $\theta = \frac{\pi}{4}$, what is the value of $A^2 + 2A$. 2

b. Prove that $\sin\theta = \frac{1 - A^2}{1 + A^2}$ 4

c. Show that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{A}$ 4

Solution to the question no. 20

a Given, $\theta = \frac{\pi}{4}$

$A = \sec\theta - \tan\theta = \sec 45° - \tan 45°$

$= \sqrt{2} - 1$

$\therefore A^2 + 2A = (\sqrt{2} - 1)^2 + 2(\sqrt{2} - 1)$

$= 2 - 2.\sqrt{2}.1 + 1 + 2\sqrt{2} - 2$

$= 1$ **(Ans.)**

b Given, $\sec\theta - \tan\theta = A$

Or, $\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = A$

Or, $\frac{1 - \sin\theta}{\cos\theta} = A$

Or, $\frac{(1 - \sin\theta)^2}{\cos^2\theta} = A^2$

Or, $\frac{(1 - \sin\theta)^2}{1 - \sin^2\theta} = A^2$

Or, $\frac{(1 - \sin\theta)^2}{(1 - \sin\theta)(1 + \sin\theta)} = A^2$

Or, $\frac{1 - \sin\theta}{1 + \sin\theta} = A^2$

Or, $\frac{1 + \sin\theta}{1 - \sin\theta} = \frac{1}{A^2}$

Or, $\frac{1 + \sin\theta + 1 - \sin\theta}{1 + \sin\theta - 1 + \sin\theta} = \frac{1 + A^2}{1 - A^2}$

$$\text{Or, } \frac{2}{2\sin\theta} = \frac{1+A^2}{1-A^2}$$

$$\text{Or, } \frac{1}{\sin\theta} = \frac{1+A^2}{1-A^2}$$

$$\therefore \sin\theta = \frac{1-A^2}{1+A^2} \textbf{ (Proved)}$$

c $\text{L.H.S} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\cos\theta(\tan\theta - 1 + \sec\theta)}{\cos\theta(\tan\theta + 1 - \sec\theta)}$

$$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + (\sec^2\theta - \tan^2\theta)}$$

$$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$$

$$= \frac{\tan\theta + \sec\theta - 1}{-(\sec\theta - \tan\theta) + (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$$

$$= \frac{\sec\theta + \tan\theta - 1}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta - 1)} = \frac{1}{\sec\theta - \tan\theta}$$

$$= \frac{1}{A}; [\because A = \sec\theta - \tan\theta]$$

= R.H.S **(Shown)**

Question ▸21 $f(z) = \cos z$ and $g(A) = \sin A$.

[Rajshahi Cadet College, Rajshahi]

a. If $15\{g(A)\}^2 + 2f(A) = 7$, find the value of A, where A is an acute angle. 2

b. If $f(x) + g(x) = \sqrt{2}\, f(x)$, prove that, $f(x) - g(x) = \sqrt{2} g(x)$. 4

c. If $72\{f(\theta)\}^5 - 8\{f(\theta)\}^3 + 9\{g(\theta)\}^2 = 8$, find the possible value of θ. 4

Solution to the question no. 21

a Given,

$f(z) = \cos z$

$g(A) = \sin A$

Now, $f(z) = \cos z$

$\therefore f(A) = \cos A$

Here, $15\{g(A)\}^2 + 2f(A) = 7$

Or, $15\sin^2 A + 2\cos A - 7 = 0$

Or, $15(1 - \cos^2 A) + 2\cos A - 7 = 0$

Or, $15 - 15\cos^2 A + 2\cos A - 7 = 0$

Or, $15\cos^2 A - 2\cos A - 8 = 0$

Or, $15\cos^2 A - 12\cos A + 10\cos A - 8 = 0$

Or, $3\cos A(5\cos A - 4) + 2(5\cos A - 4) = 0$

Or, $(5\cos A - 4)(3\cos A + 2) = 0$

Either, $(5\cos A - 4) = 0$ or, $3\cos A + 2 = 0$

Or, $\cos A = \frac{4}{5}$ or, $\cos A = \frac{-2}{3}$

$\therefore A = 36.87°$ $\therefore A = 131.81°$

But A is an acute angle

$\therefore A = 36.87°$ **(Ans.)**

b Given,

$f(z) = \cos z$

$\therefore f(x) = \cos x$

and, $g(A) = \sin A$

$\therefore g(x) = \sin x$

$\therefore f(x) + g(x) = \sqrt{2}\, f(x)$, or, $\cos x + \sin x = \sqrt{2}\cos x$ .... (i)

Or, $\sqrt{2}\cos x + \sqrt{2}\sin x = 2\cos x$ [Multiplying both sides by $\sqrt{2}$]

Or, $\cos x + \sin x + \sqrt{2}\sin x = 2\cos x$ [from (1)]

Or, $2\cos x - \cos x - \sin x = \sqrt{2}\sin x$

Or, $\cos x - \sin x = \sqrt{2}\sin x$

$\therefore f(x) - g(x) = \sqrt{2}\, g(x)$ **(Proved.)**

c Given,

$f(z) = \cos z$

$\therefore f(\theta) = \cos\theta$

and, $g(A) = \sin A$

$\therefore g(\theta) = \sin\theta$

Now, $72\{f(\theta)\}^5 - 8\{f(\theta)\}^2 + 9\{g(\theta)\}^2 = 8$

$\therefore 72\cos^5\theta - 8\cos^3\theta + 9\sin^2\theta - 8 = 0$

Or, $72\cos^5\theta - 8\cos^3\theta + 9(1 - \cos^2\theta) - 8 = 0$

Or, $72\cos^5\theta - 8\cos^3\theta + 9 - 9\cos^2\theta - 8 = 0$

Or, $72\cos^5\theta - 8\cos^3\theta - 9\cos^2\theta + 1 = 0$

Or, $8\cos^3\theta(9\cos^2\theta - 1) - 1(9\cos^2\theta - 1) = 0$

Or, $(9\cos^2\theta - 1)(8\cos^3\theta - 1) = 0$

Either, $9\cos^2\theta - 1 = 0$

Or, $\cos^2\theta = \frac{1}{9}$

Or, $\cos\theta = \pm\frac{1}{3}$

$\therefore \theta = 70.53°, 109.47°$ **(Ans.)**

Or, $8\cos^3\theta - 1 = 0$

$\cos^3\theta = \frac{1}{8}$

Or, $\cos\theta = \frac{1}{2}$

$\therefore \theta = 60°, 300°$ **(Ans.)**

Question ▸22 We have, $\sin^2 ax + \cos^2 ax = 1$, then—

[Joypurhat Girls' Cadet College, Joypurhat]

a. What is the relation between 'sinθ' and 'tanθ'? And why $(\sin\theta)^2 = \sin^2\theta$? 2

b. Prove that, $\frac{\sin A + \cos A + 1}{\sin A - \cos A + 1} = \operatorname{cosec} A + \cot A$. 4

c. If $\sec\theta = \frac{5}{3}$ and tanθ negative, then find the value of $\frac{\operatorname{cosec}\theta - \cot\theta}{\operatorname{cosec}\theta + \cot\theta}$ 4

Solution to the question no. 22

a We know that,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\text{Or, } \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

$$\text{Or, } \tan^2\theta = \frac{\sin^2\theta}{1-\sin^2\theta}$$

$$\therefore \tan\theta = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

And $(\sin\theta)^2 = \sin\theta.\ \sin\theta$

and $\sin^2\theta = \sin\theta.\ \sin\theta$

$\therefore (\sin\theta)^2 = \sin^2\theta$

b $\text{L.H.S} = \frac{\sin A + \cos A + 1}{\sin A - \cos A + 1}$

$$= \frac{\sin A\left(\frac{\sin A}{\sin A} + \frac{\cos A}{\sin A} + \frac{1}{\sin A}\right)}{\sin A\left(\frac{\sin A}{\sin A} - \frac{\cos A}{\sin A} + \frac{1}{\sin A}\right)}$$

$= \dfrac{1 + \cot A + \operatorname{cosec} A}{1 - \cot A + \operatorname{cosec} A}$

$= \dfrac{(\operatorname{cosec}^2 A - \cot^2 A) + (\operatorname{cosec} A + \cot A)}{\operatorname{cosec} A - \cot A + 1}$

$= \dfrac{(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A) + (\operatorname{cosec} A + \cot A)}{\operatorname{cosec} A - \cot A + 1}$

$= \dfrac{(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A + 1)}{(\operatorname{cosec} A - \cot A + 1)} = \operatorname{cosec} A + \cot A$

= R.H.S

$\therefore \dfrac{\sin A + \cos A + 1}{\sin A - \cos A + 1} = \operatorname{cosec} A + \cot A$ **(Proved)**

c Given, $\sec\theta = \dfrac{5}{3}$

Or, $\sec^2\theta = \left(\dfrac{5}{3}\right)^2$ [Squaring bothsides]

Or, $1 + \tan^2\theta = \dfrac{25}{9}$

Or, $\tan^2\theta = \dfrac{25}{9} - 1$

Or, $\tan^2\theta = \dfrac{25-9}{9}$

Or, $\tan^2\theta = \dfrac{16}{9}$

Or, $\tan\theta = \pm\dfrac{4}{3}$

Or, $\tan\theta = -\dfrac{4}{3}$ [$\because$ $\tan\theta$ negative]

Or, $\dfrac{1}{\cot\theta} = -\dfrac{4}{3}$

$\therefore \cot\theta = -\dfrac{3}{4}$

Or, $\cot^2\theta = \left(-\dfrac{3}{4}\right)^2$ [Squaring both sides]

Or, $\operatorname{cosec}^2\theta - 1 = \dfrac{9}{16}$

Or, $\operatorname{cosec}^2\theta = \dfrac{9}{16} + 1$

Or, $\operatorname{cosec}^2\theta = \dfrac{9+16}{16}$

Or, $\operatorname{cosec}^2\theta = \dfrac{25}{16}$

Or, $\operatorname{cosec}^2\theta = \pm\dfrac{5}{4}$

$\therefore \operatorname{cosec}\theta = -\dfrac{5}{4}$

[Since $\sec\theta$ is positive and $\tan\theta$ is negative so the angle θ is in 4^{th} quadrant, so $\operatorname{cosec}\theta$ is also negative]

$\therefore \dfrac{\operatorname{cosec}\theta - \cot\theta}{\operatorname{cosec}\theta + \cot\theta} = \dfrac{\frac{-5}{4} - \left(-\frac{3}{4}\right)}{\frac{-5}{4} + \left(-\frac{3}{4}\right)}$

$= \dfrac{\frac{-5}{4} + \frac{3}{4}}{\frac{-5}{4} - \frac{3}{4}} = \dfrac{\frac{-5+3}{4}}{\frac{-5-3}{4}}$

$= \dfrac{-2}{4} \times \dfrac{4}{-8} = \dfrac{1}{4}$ **(Ans.)**

Question ▶ 23 $f(x) = \sin x$. *[Pabna Cadet College, Pabna]*

a. If $f(x) = 3/5$, then, $\tan x = ?$ 2

b. If $3f(\theta) + 4f\left(\dfrac{\pi}{2} - \theta\right) = c$, then prove that $3f\left(\dfrac{\pi}{2} - \theta\right) - 4f(\theta) = \pm\sqrt{25 - c^2}$. 4

c. Solve : $2f(x).f\left(\dfrac{\pi}{2} - x\right) = f(x)$, where $0 \le x \le 2\pi$. 4

Solution to the question no. 23

a Given, $f(x) = \sin x$. & $f(x) = \dfrac{3}{5}$

$\therefore \sin x = \dfrac{3}{5}$

Or, $\sin^2 x = \dfrac{9}{25}$

Or, $1 - \cos^2 x = \dfrac{9}{25}$

Or, $\cos^2 x = 1 - \dfrac{9}{25}$

$\therefore \cos x = \sqrt{\dfrac{25-9}{25}} = \sqrt{\dfrac{16}{25}} = \dfrac{4}{5}$

$\therefore \tan x = \dfrac{\sin x}{\cos x} = \dfrac{\frac{3}{5}}{\frac{4}{5}} = \dfrac{3}{4}$ **(Ans.)**

b Given, $f(x) = \sin x$

And, $3f(\theta) + 4f\left(\dfrac{\pi}{2} - \theta\right) = c$

Or, $3\sin\theta + 4\sin\left(\dfrac{\pi}{2} - \theta\right) = c$

Or, $3\sin\theta + 4\cos\theta = c$

Or, $(3\sin\theta + 4\cos\theta)^2 = c^2$

Or, $(3\sin\theta)^2 + 2.3\sin.4\cos\theta + (4\cos\theta)^2 = c^2$

Or, $9\sin^2\theta + 24\sin\theta.\cos\theta + 16\cos^2\theta = c^2$

Or, $9(1 - \cos^2\theta) + 24\sin\theta.\cos\theta + 16(1 - \sin\text{-}\theta) = c^2$

Or, $9 - 9\cos^2\theta + 24\sin\theta.\cos\theta + 16 - 16\sin^2\theta = c^2$

Or, $25 - c^2 = 9\cos^2\theta - 24\sin\theta.\cos\theta + 16\sin^2\theta$

Or, $25 - c^2 = (3\cos\theta)^2 - 2.3\cos\theta.4\sin\theta + (4\sin\theta)^2$

Or, $25 - c^2 = (3\cos\theta - 4\sin\theta)^2$

Or, $3\cos\theta - 4\sin\theta = \pm\sqrt{25 - c^2}$

Or, $3\sin\left(\dfrac{\pi}{2} - \theta\right) - 4\sin\theta = \pm\sqrt{25 - c^2}$

$\therefore 3f\left(\dfrac{\pi}{2} - \theta\right) - 4f(\theta) = \pm\sqrt{25 - c^2}$ **(Proved)**

c Given, $f(x) = \sin x$

$f\left(\dfrac{\pi}{2} - x\right) = \sin\left(\dfrac{\pi}{2} - x\right) = \cos x$

$\therefore 2f(x).\ f\left(\dfrac{\pi}{2} - x\right) = f(x)$

Or, $2\sin x \cos x = \sin x$

Or, $2\sin x \cos x - \sin x = 0$

Or, $\sin x (2\cos x - 1) = 0$

Either, $\sin x = 0$ or $2\cos x - 1 = 0$

$\therefore \cos x = \dfrac{1}{2}$

If $\sin x = 0$, then $\sin x = \sin 0, \sin\pi, \sin 2\pi$

$\therefore x = 0, \pi, 2\pi$

If $\cos x = \dfrac{1}{2}$, then $\cos x = \cos\dfrac{\pi}{3}, \cos\left(2\pi - \dfrac{\pi}{3}\right)$

Or, $\cos x = \cos\frac{\pi}{3}, \cos\frac{5\pi}{3}$

$\therefore x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\therefore$ The required solution: $\theta = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

Question ▶ 24 Consider $f(x) = \sin x$.

[Rangpur Cadet College, Rangpur]

a. Prove that "Radian is a constant angle". 2

b. Find the value of $f(\alpha) = -\frac{\sqrt{3}}{2}; \frac{\pi}{2} < \alpha < \frac{3\pi}{2}$. 4

c. Solve $\left\{f\left(\frac{\pi}{2} + x\right)\right\}^2 + f(x) = \frac{5}{4}$ where $0 < x < 2\pi$. 4

Solution to the question no. 24

a See from your text book, chapter–8.1, proposition–3, page–146

b Given,

$f(x) = \sin x$

$\therefore f(\alpha) = \sin\alpha$

Again, $f(\alpha) = -\frac{\sqrt{3}}{2}$

Or, $\sin\alpha = -\frac{\sqrt{3}}{2}$

Or, $\sin\alpha = -\sin\frac{\pi}{3}$

Or, $\sin\alpha = \sin\left(\pi + \frac{\pi}{3}\right)$ [$\because$ sin is negative in third quadrant]

Or, $\alpha = \pi + \frac{\pi}{3}$

$\therefore \alpha = \frac{4\pi}{3}$, which satisfies the condition $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

$\therefore$ The required value $= \frac{4\pi}{3}$ **(Ans.)**

c Given, $f(x) = \sin x$

$\therefore f\left(\frac{r}{2} + x\right) = \sin\left(\frac{r}{2} + x\right) = \cos x$

Now, $\left\{f\left(\frac{\pi}{2} + x\right)\right\}^2 + f(x) = \frac{5}{4}$

Or, $\cos^2 x + \sin x = \frac{5}{4}$

Or, $4(\cos^2 x + \sin x) = 5$

Or, $4(1 - \sin^2 x + \sin x) = 5$

Or, $4 - 4\sin^2 x + 4\sin x = 5$

Or, $4\sin^2 x - 4\sin x + 1 = 0$ [multiplying both sides by (–1)]

Or, $(2\sin x - 1)^2 = 0$

Or, $2\sin x - 1 = 0$ [taking square root]

Or, $\sin x = \frac{1}{2}$

Or, $\sin x = \sin\frac{\pi}{6}, \sin(\pi - \frac{\pi}{6})$ [according to the condition]

Or, $\sin x = \sin\frac{\pi}{6}, \sin\frac{5\pi}{6}$

$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}$, which satisfies the condition, $0 < x < 2\pi$

$\therefore$ The required solution: $x = \frac{\pi}{6}, \frac{5\pi}{6}$

Question ▶ 25 $x\cos A - y\sin A = z$. *[Cumilla Cadet College, Cumilla]*

a. Show that, $\tan B + \cot B = \sec B \operatorname{cosec} B$. 2

b. Show that, $y\cos A + x\sin A = \pm\sqrt{x^2 + y^2 - z^2}$ 4

c. If $x = 3$, $y = -2\sin A$ and $z = 0$, find the value of A where $0 < A < 2\pi$. 4

Solution to the question no. 25

a L.H.S $= \tan B + \cot b = \frac{\sin B}{\cos B} + \frac{\cos B}{\sin B}$

$= \frac{\sin^2 B + \cos^2 B}{\cos B \sin B}$

$= \frac{1}{\cos B.\sin B}$ [$\because \sin^2 B + \cos^2 B = 1$]

$= \sec B.\operatorname{cosec} B =$ R.H.S

$\therefore \tan B + \cot B = \sec B.\operatorname{cosec} B$ **(Shown)**

b Given, $x\cos A - y\sin A = z$

Or, $(x\cos A - y\sin A)^2 = z^2$ [squaring both sides]

Or, $x^2\cos^2 A + y^2\sin^2 A - 2xy\cos A.\sin A = z^2$

Or, $x^2(1 - \sin^2 A) + y^2(1 - \cos^2 A) - 2xy\cos A.\sin A = z^2$

Or, $x^2 - x^2\sin^2 A + y^2 - y^2\cos^2 A - 2xy\cos A.\sin A = z^2$

Or, $x^2 + y^2 - (x^2\sin^2 A + y^2\cos^2 A + 2xy\cos A.\sin A) = z^2$

Or, $-(x\sin A + y\cos A)^2 = z^2 - x^2 - y^2$

Or, $(x\sin A + y\cos A^2 = x^2 + y^2 - z^2$

$\therefore x\sin A + y\cos A = \pm\sqrt{x^2 + y^2 - z^2}$ **(Shown)**

c Given equation, $x\cos A - y\sin A = z$

If $x = 3$, $y = -2\sin A$ and $z = 0$, then the equation becomes

$3\cos A + 2\sin^2 A = 0$

Or, $3\cos A + 2(1 - \cos^2 A)\ A = 0$

Or, $3\cos A + 2 - 2\cos^2 A = 0$

Or, $2\cos^2 A - 3\cos A - 2 = 0$

Or, $2\cos^2 A - 4\cos A + \cos A - 2 = 0$

Or, $2\cos A(\cos A - 2) + 1(\cos A - 2) = 0$

Or, $(2\cos A + 1)(\cos A - 2) = 0$

$\therefore 2\cos A + 1 = 0$ [$\because \cos A - 2 \neq 0$]

Or, $\cos A = -\frac{1}{2} = -\cos\frac{\pi}{3}$

Or, $\cos A = \cos\left(\pi - \frac{\pi}{3}\right), \cos\left(\pi + \frac{\pi}{3}\right)$ [$\because 0 < \theta < 2\pi$]

Or, $\cos A = \cos\frac{2\pi}{3}, \cos\frac{4\pi}{3}$

$\therefore A = \frac{2\pi}{3}, \frac{4\pi}{3}$ **(Ans.)**

Question ▶ 26 Scenario : $f(\theta) = a\cos\theta - b\sin\theta - c$, $P(\theta) = \sin^2 17\theta + \sin^2 5\theta + \cos^2 37\theta + \cos^2 3\theta$.

[Faujdarhat Cadet College, Chattogram]

a. Solve: $\sin\theta + \cos\theta = \sqrt{2}$; $0 < \theta < \frac{\pi}{2}$ 2

b. According to scenario prove that, $a\sin\theta + b\cos\theta = \pm\sqrt{(a^2 + b^2 - c^2)}$, when $f(\theta) = 0$. 4

c. According to scenario find the value of $P\left(\frac{\pi}{10}\right)$. 4

Solution to the question no. 26

a Given, $\sin\theta + \cos\theta = \sqrt{2}$

or, $\sin\theta = \sqrt{2} - \cos\theta$

or, $\sin^2\theta = (\sqrt{2} - \cos\theta)^2$

or, $\sin^2\theta = 2 - 2\sqrt{2}\cos\theta + \cos^2\theta$

or, $\sin^2\theta - \cos^2\theta + 2\sqrt{2}\cos\theta - 2 = 0$

or, $1 - \cos^2\theta - \cos^2\theta + 2\sqrt{2}\cos\theta - 2 = 0$

or, $-2\cos^2\theta + 2\sqrt{2}\cos\theta - 1 = 0$

or, $2\cos^2\theta - 2\sqrt{2}\cos\theta + 1 = 0$

or, $(\sqrt{2}\cos\theta)^2 - 2.\sqrt{2}\cos\theta.1 + 1 = 0$

or, $(\sqrt{2}\cos\theta - 1)^2 = 0$

or, $\sqrt{2}\cos\theta - 1 = 0$

or, $\sqrt{2}\cos\theta = 1$

or, $\cos\theta = \frac{1}{\sqrt{2}}$

or, $\cos\theta = \cos 45°$

$\therefore \theta = 45°$ **(Ans.)**

b Given that, $f(\theta) = a\cos\theta - b\sin\theta - c$

When, $f(\theta) = 0$ then

$a\cos\theta - b\sin\theta - c = 0$

or, $a\cos\theta - b\sin\theta = c$

or, $(a\cos\theta - b\sin\theta)^2 = c^2$

or, $a^2\cos^2\theta - 2ab\sin\theta\cos\theta + b^2\sin^2\theta = c^2$

or, $a^2(1 - \sin^2\theta) - 2ab\sin\theta\cos\theta + b^2(1 - \cos^2\theta) = c^2$

or, $a^2 - a^2\sin^2\theta - 2ab\sin\theta\cos\theta + b^2 - b^2\cos^2\theta = c^2$

or, $a^2 + b^2 - c^2 = a^2\sin^2\theta + 2a\sin\theta\, 6\cos\theta + b^2\cos^2\theta$

or, $(a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

$\therefore a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$ **(Proved)**

c Given, $P(\theta) = \sin^2 17\theta + \sin^2 13\theta + \cos^2 37\theta + \cos^2 3\theta$

$\therefore P\left(\frac{\pi}{10}\right) = \sin^2\left(\frac{17\pi}{10}\right) + \sin^2\left(\frac{13\pi}{10}\right) + \cos^2\left(\frac{37\pi}{10}\right) + \cos^2\left(\frac{3\pi}{10}\right)$

$= \sin^2\left(\frac{17\pi}{10}\right) + \left\{\sin\left(\pi + \frac{3\pi}{10}\right)\right\}^2 + \cos^2\left(\frac{37\pi}{10}\right) + \cos^2\left(\frac{3\pi}{10}\right)$

$= \sin^2\left(\frac{17\pi}{10}\right) + \sin^2\left(\frac{3\pi}{10}\right) + \cos^2\left(\frac{37\pi}{10}\right) + \cos^2\left(\frac{3\pi}{10}\right)$

$= \sin^2\left(\frac{17\pi}{10}\right) + \sin^2\left(\frac{3\pi}{10}\right) + \left\{\cos\left(4\pi - \frac{3\pi}{10}\right)\right\}^2 + \cos^2\left(\frac{3\pi}{10}\right)$

$= \sin^2\left(\frac{17\pi}{10}\right) + \sin^2\left(\frac{3\pi}{10}\right) + \cos^2\left(\frac{3\pi}{10}\right) + \cos^2\left(\frac{3\pi}{10}\right)$

$= \sin^2\left(\frac{17\pi}{10}\right) + \sin^2\left(\frac{3\pi}{10}\right) + 2\cos^2\left(\frac{3\pi}{10}\right)$

$= \left\{\sin\left(2\pi + \frac{3\pi}{10}\right)\right\}^2 - \sin^2\left(\frac{3\pi}{10}\right) + 2\cos^2\left(\frac{3\pi}{10}\right)$

$= \sin^2\left(\frac{3\pi}{10}\right) + \sin^2\left(\frac{3\pi}{10}\right) + 2\cos^2\left(\frac{3\pi}{10}\right)$

$= 2\left(\sin^2\frac{3\pi}{10} + \cos^2\frac{3\pi}{10}\right)$

$= 2\times 1 = 2$ **(Ans.)**

Question ▶27 $a\cos\theta - b\sin\theta = c$, where a, b, c are constants.

[Sylhet Cadet College, Sylhet]

a. Find the value of $\sec\theta$ when $c = 0$. 2

b. Prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$ 4

c. If $a = 1$, $b = -1$ and $c = \sqrt{2}$, then solve the equation. 4

Solution to the question no. 27

a Given, $a\cos\theta - b\sin\theta = c$ and $c = 0$

$\therefore a\cos\theta - b\sin\theta = 0$

Or, $a\cos\theta = b\sin\theta$

Or, $\frac{a}{b} = \frac{\sin\theta}{\cos\theta}$

Or, $\frac{a}{b} = \tan\theta$

Or, $\tan^2\theta = \frac{a^2}{b^2}$

Or, $\sec^2\theta - 1 = \frac{a^2}{b^2}$

Or, $\sec^2\theta = \frac{a^2}{b^2} + 1$

Or, $\sec^2\theta = \frac{a^2 + b^2}{b^2}$

$\therefore \sec\theta = \frac{\pm\sqrt{a^2 + b^2}}{b}$ **(Ans.)**

b **Solution:** Given,

$a\cos\theta - b\sin\theta = c$

Or, $(a\cos\theta - b\sin\theta)^2 = c^2$ [squaring both sides]

Or, $a^2\cos^2\theta - 2a\cos\theta.\, b\sin\theta + b^2\sin^2\theta = c^2$

Or, $a^2(1 - \sin^2\theta) - 2a\cos\theta.b\sin\theta + b^2(1 - \cos^2\theta) = c^2$

Or, $a^2 - a^2\sin^2\theta - 2a\cos\theta.b\sin\theta + b^2 - b^2\cos^2\theta = c^2$

Or, $-(a^2\sin^2\theta + 2a\cos\theta.b\sin\theta + b^2\cos^2\theta) = -(a^2 + b^2 - c^2)$

Or, $a^2\sin^2\theta + 2a\cos\theta.b\sin\theta + b^2\cos^2\theta = a^2 + b^2 - c^2$

Or, $(a\sin\theta)^2 + 2a\sin\theta.b\cos\theta + (b\cos\theta)^2 = a^2 + b^2 - c^2$

Or, $(a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

$\therefore a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$ **(Proved)**

Alternative Solution:

Given, $a\cos\theta - b\sin\theta = c$ ... ... (i)

Let, $a\sin\theta + b\cos\theta = x$ ... ... (ii)

Squaring both sides of the equations (i) and (ii) and then adding we get,

$a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = c^2 + x^2$

Or, $a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = c^2 + x^2$

Or, $c^2 + x^2 = a^2 + b^2$

Or, $x^2 = a^2 + b^2 - c^2$

Or, $x = \pm\sqrt{a^2 + b^2 - c^2}$

$\therefore a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$ **(Proved)**

c Given, $a = 1$,

$b = -1$,

$c = \sqrt{2}$

Again, $a\cos\theta - b\sin\theta = C$

Or, $1.\cos\theta - (-1)\sin\theta = \sqrt{2}$

Or, $\sin\theta + \cos\theta = \sqrt{2}$

Or, $\sin\theta = \sqrt{2} - \cos\theta$

Or, $\sin^2\theta = 2 - 2\sqrt{2}\cos\theta + \cos^2\theta$

Or, $1 - \cos^2\theta = 2 - 2\sqrt{2}\cos\theta + \cos^2\theta$

Or, $2\cos^2\theta - 2\sqrt{2}\cos\theta + 1 = 0$

Or, $(\sqrt{2}\cos\theta - 1)^2 = 0$

Or, $\sqrt{2}\cos\theta - 1 = 0$

Or, $\cos = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$

$\therefore \theta = \frac{\pi}{4}$.

$\therefore$ The required solution is $\theta = \frac{\pi}{4}$ **(Ans.)**

Question ▶ 28 $f(x) = \cos x$ *[Jhenidah Cadet College, Jhenidah]*

a. If $f(\theta) = \frac{4}{5}$ and θ is acute angle, find the value of $f\left(\frac{\pi}{2} - \theta\right)$ 2

b. If $f(\theta) - f\left(\frac{\pi}{2} - \theta\right) = \sqrt{2}\, f\left(\frac{\pi}{2} - \theta\right)$ show that, $f(\theta) + f\left(\frac{\pi}{2} - \theta\right) = \sqrt{2}\, f(\theta)$ 4

c. If $\frac{f\left(\frac{\pi}{2} - \theta\right)}{f(\theta)} = \frac{5}{12}$ and $f(\theta)$ is negative, find the value of $\dfrac{f\left(\frac{\pi}{2} - \theta\right) + f(-\theta)}{\frac{1}{f(-\theta)} + \frac{f\left(\frac{\pi}{2} - \theta\right)}{f(\theta)}}$ 4

Solution to the question no. 28

a Given, that $f(x) = \cos x$

If $f(\theta) = \frac{4}{5}$

Or, $\cos\theta = \frac{4}{5}$

Or, $\cos^2\theta = \left(\frac{4}{5}\right)^2$

Or, $1 - \sin^2\theta = \frac{16}{25}$

Or, $\sin^2\theta = 1 - \frac{16}{25}$

Or, $\sin^2\theta = 1 - \frac{16}{25}$

Or, $\sin\theta = \sqrt{\frac{9}{25}}$

Or, $\cos\left(\frac{\pi}{2} - \theta\right) = \frac{3}{5}$ [Since θ is acute angle]

Or, $f\left(\frac{\pi}{2} - \theta\right) = \frac{3}{5}$ Ans.

b Given, $f(x) = \cos\theta$

If $f(\theta) - f\left(\frac{\pi}{2} - \theta\right) = \sqrt{2} f\left(\frac{\pi}{2} - \theta\right)$

Or, $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Or, $\cos\theta = \sqrt{2}\sin\theta + \sin\theta$

Or, $\cos\theta = (\sqrt{2} + 1)\sin\theta$

Or, $(\sqrt{2} - 1)\cos\theta = (\sqrt{2} + 1)(\sqrt{2} - 1)\sin\theta$

Or, $(\sqrt{2} - 1)\cos\theta = (2 - 1)\sin\theta$

Or, $\sqrt{2}\cos\theta - \cos\theta = \sin\theta$

Or, $\sqrt{2}\cos\theta = \sin\theta + \cos\theta$

Or, $\sqrt{2}\cos\theta = \cos\left(\frac{\pi}{2} - \theta\right) + \cos\theta$

Or, $\sqrt{2} f(\theta) = f\left(\frac{\pi}{2} - \theta\right) + f(\theta)$

$\therefore f(\theta) + f\left(\frac{\pi}{2} - \theta\right) = \sqrt{2} f(\theta)$ **(Shown)**

c $f(x) = \cos x$

Given, $\frac{f\left(\frac{\pi}{2} - \theta\right)}{f(\theta)} = \frac{5}{12}$

Or, $\frac{f\left(\frac{\pi}{2} - \theta\right)}{\cos(\theta)} = \frac{5}{12}$

Or, $\frac{\sin\theta}{\cos\theta} = \frac{5}{12}$

Or, $\tan\theta = \frac{5}{12}$

Or, $\tan^2\theta = \left(\frac{5}{12}\right)^2$

Or, $\sec^2\theta - 1 = \frac{25}{144}$

Or, $\sec^2\theta = 1 + \frac{25}{144}$

Or, $\sec^2\theta = \frac{169}{144}$

Or, $\sec\theta = \pm\frac{13}{12}$

Or, $\cos\theta = \pm\frac{12}{13}$

since, $f(\theta) = \cos\theta$ is negative

$\therefore \cos\theta = -\frac{12}{13}$

Again $\sin\theta = \pm\sqrt{1 - \cos^2\theta}$

$= \pm\sqrt{1 - \left(\frac{-12}{13}\right)^2} = \pm\sqrt{1 - \frac{144}{169}} = \pm\sqrt{\frac{25}{169}} = \pm\frac{5}{13}$

since $\tan\theta$ is positive and $\cos\theta$ is negaitve

$\therefore \sin\theta$ will be negative

$\therefore \sin\theta = -\frac{5}{13}$

Now, $\dfrac{f\left(\frac{\pi}{2} - \theta\right) + f(\theta)}{\frac{1}{f(-\theta)} + \frac{f\left(\frac{\pi}{2} - \theta\right)}{f(\theta)}}$

$= \dfrac{\cos\left(\frac{\pi}{2} - \theta\right) + \cos(\theta)}{\frac{1}{\cos(-\theta)} + \frac{\cos\left(\frac{\pi}{2} - \theta\right)}{\cos(\theta)}} = \dfrac{\sin\theta + \cos\theta}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$

$= \dfrac{-\frac{5}{13} - \frac{12}{13}}{-\frac{13}{12} + \frac{5}{12}} = \dfrac{\frac{-17}{13}}{\frac{-13 + 5}{12}} = -\frac{17}{13} \times \frac{12}{-8} = \frac{51}{26}$ **(Ans.)**

Question ▶ 29 $V = 1 - \sin x$, $K = \sec x - \tan x$ and $T = 1 + \sin x$

[Barishal Cadet College, Barishal]

a. Prove that $K = V\sec x$ 2

b. If $K = (\sqrt{3})^{-1}$, then find the value of x where $x < 90°$. 4

c. Prove that $VT^{-1} = K^2$. 4

Solution to the question no. 29

a Given, $V = 1 - \sin x$

$K = \sec x - \tan x$

$= \frac{1}{\cos x} - \frac{\sin x}{\cos x}$

$= \frac{1 - \sin x}{\cos x}$

$= (1 - \sin x).\frac{1}{\cos x}$

$= V.\sec x$ [putting value]

$\therefore K = V.\sec x$ **(Shown)**

b Given,

$K = (\sqrt{3})^{-1}$

Or, $\sec x - \tan x = \frac{1}{\sqrt{3}}$ [$\because K = \sec x - \tan x$]

Or, $\frac{1}{\cos x} - \frac{\sin x}{\cos x} = \frac{1}{\sqrt{3}}$

Or, $\frac{1-\sin x}{\cos x} = \frac{1}{\sqrt{3}}$

Or, $\sqrt{3}(1 - \sin x) = \cos x$

Or, $3(1 - 2\sin x + \sin^2 x) = \cos^2 x$ [squaring both sides]

Or, $3 - 6\sin x + 3\sin^2 x = 1 - \sin^2 x$

Or, $4\sin^2 x - 6\sin x + 2 = 0$

Or, $2\sin^2 x - 3\sin x + 1 = 0$

Or, $2\sin^2 x - 2\sin x - \sin x + 1 = 0$

Or, $2\sin x(\sin x - 1) - 1(\sin x - 1) = 0$

Or, $(\sin x - 1)(2\sin x - 1) = 0$

Either, $\sin x = 1 = \sin\frac{\pi}{2}$

$\therefore x = \frac{\pi}{2}$ [not acceptale since x is acute]

or, $2\sin x - 1 = 0$

or, $\sin x = \frac{1}{2} = \sin\frac{\pi}{3}$

$\therefore x = \frac{\pi}{3}$

$\therefore$ The required solution, $x = \frac{\pi}{3}$ **(Ans.)**

c L.H.S $= VT^{-1}$

$= \frac{V}{T}$

$= \frac{1-\sin x}{1+\sin x}$ [$\because V = 1 - \sin x$ and $T = 1 + \sin x$]

$= \frac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)}$

$= \frac{(1-\sin x)^2}{1-\sin^2 x} = \frac{(1-\sin x)^2}{\cos^2 x}$

$= \left(\frac{1-\sin x}{\cos x}\right)^2 = \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2$

$= (\sec x - \tan x)^2$

$= K^2$ [$\because K = \sec x - \tan x$]

$=$ R.H.S.

$\therefore VT^{-1} = K^2$ **(Proved)**

Question ▶30 $(x^2 + 3)\sin^2\theta + (x^2 - 1)\cos^2\theta = x + 2$

[RAJUK Uttara Model College, Dhaka]

a. If $\theta = \frac{\pi}{2}$, express in term of x. 2

b. If $x = 2$, prove that, $\tan\theta = \pm\frac{1}{\sqrt{3}}$ 4

c. If $x = 0$ and $0 < \theta < 2\pi$, find the possible value of x. 4

Solution to the question no. 30

a Given, $(x^2 + 3)\sin^2\theta + (x^2 - 1)\cos^2\theta = x + 2$

If $\theta = \frac{\pi}{2}$ then

$(x^2 + 3)\left(\sin\frac{\pi}{2}\right)^2 + (x^2 - 1)\left(\cos\frac{\pi}{2}\right)^2 = x + 2$

Or, $(x^2 + 3)\cdot 1 + (x^2 - 1)\cdot 0 = x + 2$

Or, $x^2 + 3 = x + 2$

Or, $x^2 - x + 1 = 0$ **(Ans.)**

b Given, $(x^2 + 3)\sin^2\theta + (x^2 - 1)\cos^2\theta = x + 2$

If $x = 2$ then

$(2^2 + 3)\sin^2\theta + (2^2 - 1)\cos^2\theta = 2 + 2$

Or, $7\sin^2\theta + 3\cos^2\theta = 4$

Or, $7(1 - \cos^2\theta) + 3\cos^2\theta = 4$

Or, $7 - 7\cos^2\theta + 3\cos^2\theta = 4$

Or, $-4\cos^2\theta = 4 - 7$

Or, $4\cos^2\theta = 3$

Or, $\cos^2\theta = \frac{3}{4}$

Or, $\sec^2\theta = \frac{4}{3}$

Or, $\sec^2\theta - 1 = \frac{4}{3} - 1$

Or, $\tan^2\theta = \frac{1}{3}$

$\therefore \tan\theta = \pm\frac{1}{\sqrt{3}}$ **(proved)**

c Given,

$(x^2 + 3)\sin^2\theta + (x^2 - 1)\cos^2\theta = x + 2$

If $x = 0$ then

$(0 + 3)\sin^2\theta + (0 - 1)\cos^2\theta = 0 + 2$

Or, $3\sin^2\theta - \cos^2\theta = 2$

Or, $3\sin^2\theta - (1 - \sin^2\theta) = 2$

Or, $3\sin^2\theta - 1 + \sin^2\theta = 2$

Or, $4\sin^2\theta = 2 + 1$

Or, $4\sin^2\theta = 3$

Or, $\sin^2\theta = \frac{3}{4}$

$\therefore \sin\theta = \pm\frac{\sqrt{3}}{2}$

For (+): $\sin\theta = \frac{\sqrt{3}}{2}$

$= \sin\frac{\pi}{3}, \sin\left(\pi - \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{3}, \sin\frac{2\pi}{3}$

$\therefore \theta = \frac{\pi}{3}, \frac{2\pi}{3}$

For (–): $\sin\theta = -\frac{\sqrt{3}}{2}$

$= -\sin\frac{\pi}{3}$

$= \sin\left(\pi + \frac{\pi}{3}\right), \sin\left(2\pi - \frac{\pi}{3}\right)$

$= \sin\frac{4\pi}{3}, \sin\frac{5\pi}{3}$

$\therefore \theta = \frac{4\pi}{3}, \frac{5\pi}{3}$

So, the value of $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **(Ans.)**

Question ▶31 The radius of the earth is 6440 k.m. The two places A and B on the surface of the earth which subtend an angle of 2° at the centre of the earth. Sumaiya takes 't' hours to reach from A to B. The wheel of the car revolves 880 times in a minute. The radius of the wheel is 25 cm.

[Viqarunnisa Noon School & College, Dhaka]

a. What distance (in metre) does the wheel of the car cover by revolving 5 times. 2

b. What is the speed of the car? Determine it. 4

c. Find the value of 't'. 4

Solution to the question no. 31

a Given, radius of the wheel, $r = 25$ cm $= 0.25$m

$\therefore$ Circumference of the wheel $= 2\pi r$

$= 2 \times 3.1416 \times 0.25$

$= 1.5708$ m.

So, the distance when the wheel of the car cover by revolving 5 times $= 5 \times 1.5708$ m

$= 7.854$ m **(Ans.)**

b From 'a' we get,

Circumference of the wheel = 1.5708 m.

We know that, after revolving one time, the wheel travel the distance which is equal to its circumference.

$\therefore$ The car travels the distance per minute $= 880 \times 1.5708$ m

$= 1382.04$ m

$\therefore$ Velocity of the car = 1382.04m/minute

$= \dfrac{1382.04 \times 60}{1000}$ km/hour

$= 82.9224$ km/hour **(Ans.)**

c Given, radius, $r = 6440$ km

The subtended angle at the center of the earth, $\theta = 2° = 2 \times \dfrac{\pi}{180}$

$= 0.034907^c$

$\therefore$ Distance between A and B = length of the arc

$= r\theta$

$= 6440 \times 0.034907$

$= 224.801$ km

From 'b' we get,

Velocity of the car = 82.9224 km/hour

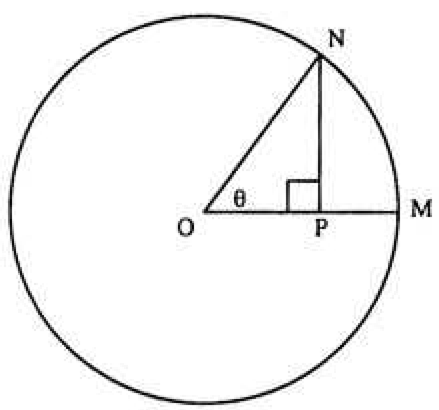
$\therefore$ The required time to go from A to B

$= (224.801 \div 82.9924)$ hour

$= 2.708$

$\cong 2.71$ **(Ans.)**

Question ▶ 32 In the figure, O is the centre of a circle and OM = arc MN



[Dhaka Residential Model School and College, Dhaka]

a. Express the angle θ in degree. 2

b. Prove that, θ is a constant angle. 4

c. Determine for what value of θ

$\dfrac{PN}{ON} + \dfrac{OP}{ON} = \sqrt{2}$ where, $0 < \theta < 2\pi$. 4

Solution to the question no. 32

a The know that,

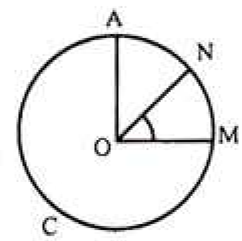
$1^c = \dfrac{180°}{\pi}$

Now, θ^c

$= \theta \times \dfrac{180}{\pi}$ degree

$= \dfrac{180\theta}{\pi}$ degree **(Ans.)**

b Let, in the circle AMC with center O, $\angle MON = \theta$ is a radian angle. It is to be proved that θ is constant angle.



Here, OA is perpendicular on the line segment OM.

Proof : OA intersects the circumference at A.

So, arc AM = one - fourth of the circumference

$= \dfrac{1}{4} \times 2\pi r = \dfrac{\pi r}{2}$

and the arc MN = radius = r

Now, we know that, the centered angled produced by any arc of a circle is proportional to its arc.

So, $\dfrac{\angle MON}{\angle AOM} = \dfrac{\text{arc } MN}{\text{arc } AM}$

Or, $\angle MON = \dfrac{\text{arc } MN}{\text{arc } AM} \times \angle AOM$

Or, $\theta = \dfrac{r}{\frac{\pi r}{2}} \times 1$ right angle

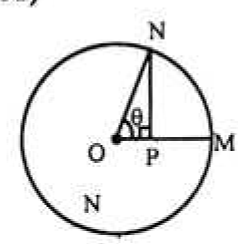
[Since, OA is the radius and perpendicular to OM]

$\therefore \theta = \dfrac{2}{\pi} \times 1$ right angle.

Since, the right angle and π are constant

So, $\theta = \angle MON$ is a constant angle. **(Proved)**

c In ΔOPN, $PN \perp OP$



$\therefore \angle OPN = 1$ right angle

Now, in right angled triangle OPN,

$\sin\theta = \dfrac{PN}{ON}$ and $\cos\theta = \dfrac{OP}{ON}$

Given,

$\dfrac{PN}{ON} + \dfrac{OP}{ON} = \sqrt{2}$

Or, $\sin\theta + \cos\theta = \sqrt{2}$

Or, $\sin\theta - \sqrt{2} = -\cos\theta$

Or, $(\sin\theta - \sqrt{2})^2 = (-\cos\theta)^2$

Or, $\sin^2\theta - 2\sqrt{2}\sin\theta + 2 = \cos^2\theta$

Or, $\sin^2\theta - 2\sqrt{2}\sin\theta + 2 = 1 - \sin^2\theta$

Or, $2\sin^2\theta - 2\sqrt{2}\sin\theta + 1 = 0$

Or, $(\sqrt{2}\sin\theta - 1)^2 = 0$

Or, $\sqrt{2}\sin\theta - 1 = 0$

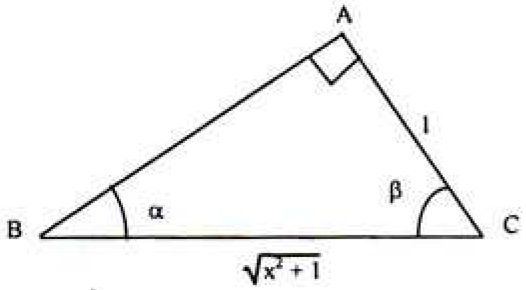
Or, $\sqrt{2}\sin\theta = 1$

Or, $\sin\theta = \dfrac{1}{\sqrt{2}}$

Or, $\sin\theta = \sin 45°$

$\therefore \theta = 45°$ **(Ans.)**

Question ▶ 33 *[Milestone College, Dhaka]*



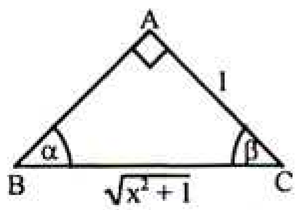
a. What is the value of $\tan\alpha$ and $\tan\beta$? 2

b. If $x = 1$, then prove that $\cos 3\beta = 4\cos^3\beta - 3\cos\beta$. 4

c. If $x^2 + \frac{1}{x^2} = 2$, then find the value of α. 4

Solution to the question no. 33

a



In right angled ΔABC,

$AB^2 + AC^2 = BC^2$

Or, $AB^2 = BC^2 - AC^2$

$= \left(\sqrt{x^2+1}\right)^2 - 1^2$

$= x^2 + 1 - 1$

$= x^2$

Here, $BC = \sqrt{x^2+1}$

$AC = 1$

$\therefore AB = x$

$\therefore \tan\alpha = \frac{AC}{AB} = \frac{1}{x}$

and $\tan\beta = \frac{AB}{AC} = \frac{x}{1} = x$ **(Ans.)**

b From 'a' we get,

$\tan\beta = 1 = \tan\frac{\pi}{4}$ [$\because x = 1$]

$\therefore \beta = \frac{\pi}{4}$

L.H.S $= \cos 3\beta$

$= \cos 3.\frac{\pi}{4}$

$= \cos\frac{3\pi}{4}$

$= -\frac{1}{\sqrt{2}}$

R.H.S $= 4\cos^3\beta - 3\cos\beta$

$= 4.\left(\cos\frac{\pi}{4}\right)^3 - 3\cos\frac{\pi}{4}$

$= 4.\left(\frac{1}{\sqrt{2}}\right)^3 - 3.\frac{1}{\sqrt{2}}$

$= 4.\frac{1}{2\sqrt{2}} - \frac{3}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} - \frac{3}{\sqrt{2}}$

$= -\frac{1}{\sqrt{2}}$

$\therefore \cos 3\beta = 4\cos^3\beta - 3\cos\beta$ **(Proved)**

c From the figure,

$\tan\alpha = \frac{AC}{AB} = \frac{1}{x}$

Or, $\tan^2\alpha = \frac{1}{x^2}$

$\therefore \cot^2\alpha = x^2$

Here, $x^2 + \frac{1}{x^2} = 2$

Or, $\cot^2\alpha + \tan^2\alpha = 2$

Or, $\frac{1}{\tan^2\alpha} + \tan^2\alpha = 2$

Or, $\frac{1 + \tan^4\alpha}{\tan^2\alpha} = 2$

Or, $\tan^4\alpha + 1 = 2\tan^2\alpha$

Or, $\tan^4\alpha - 2\tan^2\alpha + 1 = 0$

Or, $(\tan^2\alpha)^2 - 2.\tan^2\alpha.1 + 1^2 = 0$

Or, $(\tan^2\alpha - 1)^2 = 0$

Or, $\tan^2\alpha - 1 = 0$

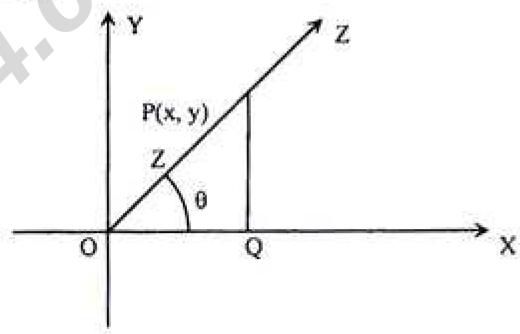
Or, $\tan^2\alpha = 1$

Or, $\tan\alpha = 1$ [$\because$ α is an acute angle, therefore taking positive value of $\tan\alpha$]

Or, $\tan\alpha = \tan 45°$

$\therefore \alpha = 45°$ **(Ans.)**

Question ▶ 34



In above diagram, $\angle POQ = 90°$, $OP = z$

[Saint Joseph Higher Secondary School, Dhaka]

a. Find the value of $\sec\theta$. 2

b. If $x = \sqrt{3}$ and $y = 1$, then prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. 4

c. If $y = 1$ and $z = 2$ and $y + \sqrt{z^2 - y^2} = \sqrt{2}z$ then find $\angle OPQ$. 4

Solution to the question no. 34

a Given, $P(x, y)$ is a point of OZ and $OP = z$ and $OQ = x$

$\therefore \sec\theta = \frac{OP}{OQ}$

$= \frac{z}{x}$ **(Ans.)**

b If $x = \sqrt{3}$ and $y = \perp$

then $z = \sqrt{3+1} = 2$

$\cos\theta = \frac{OQ}{OP} = \frac{\sqrt{3}}{2}$

$\therefore \theta = \frac{\pi}{6}$

$\therefore$ L.H.S $= \cos 3\theta$

$= \cos\left(3 \times \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{2}$

$= 0$

and R.H.S $= 4\cos^3\theta - 3\cos\theta$

$$= 4.\left(\cos\frac{\pi}{3}\right)^3 - 3.\cos\frac{\pi}{6}$$

$$= 4.\left(\frac{\sqrt{3}}{2}\right)^3 - 2.\frac{\sqrt{3}}{2}$$

$$= 4.\frac{3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$$= 0$$

$\therefore \cos 3\theta = 4\cos^3\theta - 3\cos\theta$ **(Proved)**

c In ΔOPQ, let $\angle OPQ = \alpha$

Then, $\sin\alpha = \frac{OQ}{OP}$

$$= \frac{x}{z} = \frac{\sqrt{z^2 - y^2}}{z}$$

and $\cos\alpha = \frac{PQ}{OP}$

$$= \frac{y}{z}$$

Now, $y + \sqrt{z^2 - y^2}$

Or, $\frac{y}{z} + \frac{\sqrt{z^2 - y^2}}{z} = \sqrt{2}$

Or, $\cos\alpha + \sin\alpha = \sqrt{2}$

$\therefore \cos\alpha = \sqrt{2} - \sin\alpha$

Or, $\cos^2\alpha = (\sqrt{2} - \sin\alpha)^2$

Or, $1 - \sin^2\alpha = 2 - 2\sqrt{2}\sin\alpha + \sin^2\alpha$

Or, $0 = 2 - 1 + \sin^2\alpha - 2\sqrt{2}\sin\alpha + \sin^2\alpha$

Or, $0 = 1 - 2\sqrt{2}\sin\alpha + 2\sin^2\alpha$

Or, $0 = (1 - \sqrt{2}\sin\alpha)^2$

Or, $1 - \sqrt{2}\sin\alpha = 0$

Or, $\sin\alpha = \frac{1}{\sqrt{2}}$

Or, $\sin\alpha = \sin\frac{\pi}{4}$

Or, $\alpha = \frac{\pi}{4}$

$\therefore \angle OPQ = \frac{\pi}{4}$ **(Ans.)**

Question ▶35 $A = x\cos\theta$ and $B = y\sin\theta$, where $0 < \theta < 2\pi$.

[BAF Shaheen College, Tejgaon, Dhaka]

a. Find the value of $\frac{A^2}{x^2} + \frac{B^2}{y^2}$. 2

b. If $A + B = z$, prove that, $x\sin\theta - y\cos\theta = \pm\sqrt{x^2 + y^2 - z^2}$ 4

c. If $x^2 = 3$, $y^2 = 7$ and $A^2 + B^2 = 4$, then find the value of θ. 4

Solution to the question no. 35

a Given, $A = x\cos\theta$ and $B = y\sin\theta$

Given expression $= \frac{A^2}{x^2} + \frac{B^2}{y^2}$

$$= \frac{(x\cos\theta)^2}{x^2} + \frac{(y\sin\theta)^2}{y^2}$$

$$= \frac{x^2\cos^2\theta}{x^2} + \frac{y^2\sin^2\theta}{y^2}$$

$$= \cos^2\theta + \sin^2\theta$$

$= 1$ **(Ans.)**

b Given, $A + B = z$

Or, $x\cos\theta + y\sin\theta = z$

Or, $(x\cos\theta + y\sin\theta)^2 = z^2$ [by squaring]

Or, $x^2\cos^2\theta + y^2\sin^2\theta + 2xy\sin\theta.\cos\theta = z^2$

Or, $x^2(1 - \sin^2\theta) + y^2(1 - \cos^2\theta) + 2xy\sin\theta\cos\theta = z^2$

Or, $x^2 - x^2\sin^2\theta + y^2 - y^2\cos^2\theta + 2xy\sin\theta\cos\theta = z^2$

Or, $x^2 + y^2 - z^2 = x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta$

Or, $x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta = x^2 + y^2 - z^2$

Or, $(x\sin\theta - y\cos\theta)^2 = x^2 + y^2 - z^2$

$\therefore x\sin\theta - y\cos\theta = \pm\sqrt{x^2 + y^2 - z^2}$ **(Proved)**

c Given, $A = x\cos\theta$ and $B = y\sin\theta$

Here, $A^2 + B^2 = 4$

Or, $(x\cos\theta)^2 + (y\sin\theta)^2 = 4$

Or, $x^2\cos^2\theta + y^2\sin^2\theta = 4$

Now, if $x^2 = 3$, $y^2 = 7$ then we get,

$3\cos^2\theta + 7\sin^2\theta = 4$

Or, $3(1 - \sin^2\theta) + 7\sin^2\theta = 4$

Or, $3 - 3\sin^2\theta + 7\sin^2\theta = 4$

Or, $3 + 4\sin^2\theta = 4$

Or, $4\sin^2\theta = 4 - 3$

Or, $4\sin^2\theta = 1$

Or, $\sin^2\theta = \frac{1}{4}$

$\therefore \sin\theta = \pm\frac{1}{2}$ [Taking square roots]

Taking '+' , $\sin\theta = \frac{1}{2}$

Or, $\sin\theta = \sin\frac{\pi}{6}$

$= \sin\left(\pi - \frac{\pi}{6}\right)$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

taking '–' , $\sin\theta = -\frac{1}{2}$

Or, $\sin\theta = -\sin\frac{\pi}{6}$

Or, $\sin\theta = \sin\left(\pi + \frac{\pi}{6}\right)$

$= \sin\left(2\pi - \frac{\pi}{6}\right)$

Or, $\sin\theta = \sin\frac{7\pi}{6} = \sin\frac{11\pi}{6}$

$\therefore \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **(Ans.)**

Question ▶36 $p = \frac{\sin\theta + \cos(-\theta)}{\sec(-\theta) + \tan\theta}$, $q = \sec^2\theta + \tan^2\theta$, $\tan\theta = \frac{5}{12}$ and $\cos\theta$ is negative. *[BAF Shaheen College, Kurmitola, Dhaka]*

a. Find the value of $\cos\theta$. 2

b. Prove that, $p = \frac{51}{26}$ 4

c. If $q = \frac{5}{3}$ and $0 < \theta < 2\pi$ then find all possible values of θ. 4

Solution to the question no. 36

a See chapter-8.3, page-180, example-16 of text book.

b See chapter-8.3, page-180, example-16 of text book.

c Given,

$q = \sec^2\theta + \tan^2\theta$

and $q = \frac{5}{3}$

$\therefore \sec^2\theta + \tan^2\theta = \frac{5}{3}$

Or, $3(1 + \tan^2\theta + \tan^2\theta) = 5$

Or, $3 + 6\tan^2\theta - 5 = 0$

Or, $6\tan^2\theta = 2$

Or, $\tan^2\theta = \frac{1}{3}$

$\therefore \tan\theta = \pm\frac{1}{\sqrt{3}}$

Now, by taking $\tan\theta = \frac{1}{\sqrt{3}}$ we get,

$\tan\theta = \tan\frac{\pi}{6}, \tan(\pi + \frac{\pi}{6})$ [according to the condition]

Or, $\tan\theta = \tan\frac{\pi}{6},\ \tan\frac{7\pi}{6}$

$\therefore \theta = \frac{\pi}{6}, \frac{7\pi}{6}$

Again, by taking $\tan\theta = -\frac{1}{\sqrt{3}}$ we get,

Or, $\tan\theta = -\tan\frac{\pi}{6}$

Or, $\tan\theta = \tan(\pi - \frac{\pi}{6}),\ \tan(2\pi - \frac{\pi}{6})$

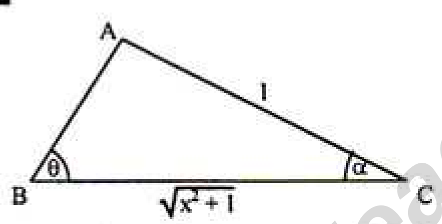
[according to the condition]

Or, $\tan\theta = \tan\frac{5\pi}{6},\ \tan\frac{11\pi}{6}$

$\therefore \theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

$\therefore$ The required solution: $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Question ▶ 37



[Bangladesh International School & College, Dhaka]

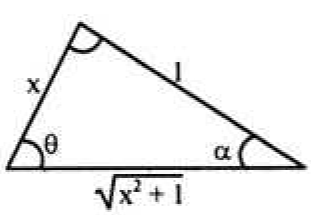
a. Find the value of $\sin(\theta + \alpha)$. 2

b. Based on the stem, show that, $(\sin\alpha + \cos\alpha)^2 = 1 + 2\sin\alpha.\cos\alpha$. 4

c. If $x + \sqrt{x^2+1} = \sqrt{3}$, what is the value of θ? 4

Solution to the question no. 37

a



$\therefore \sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$

$$= \frac{1}{\sqrt{1+x^2}} \times \frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} \times \frac{x}{\sqrt{1+x^2}}$$

$$= \frac{1}{1+x^2} + \frac{x^2}{1+x^2}$$

$$= \frac{1+x^2}{1+x^2}$$

$= 1$ **(Ans.)**

b L.H.S $= (\sin\alpha + \cos\alpha)^2$

$$= \left(\frac{x}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right)^2$$

$$= \left(\frac{1+x}{\sqrt{1+x^2}}\right)^2$$

$$= \frac{(1+x)^2}{1+x^2}$$

R.H.S $= 1 + 2\sin\alpha\cos\alpha$

$$= 1 + 2.\frac{x}{\sqrt{1+x^2}}.\frac{1}{\sqrt{1+x^2}}$$

$$= 1 + \frac{2x}{1+x^2}$$

$$= \frac{1+x^2+2x}{1+x^2} = \frac{(1+x)^2}{1+x^2}$$

$\therefore$ L.H.S. = R.H.S. **(Shown)**

c $x + \sqrt{x^2+1} = \sqrt{3}$

Or, $(\sqrt{x^2+1})^2 = (\sqrt{3} - x)^2$

Or, $x^2 + 1 = 3 + x^2 - 2\sqrt{3}x$

Or, $1 = 3 - 2\sqrt{3}x$

Or, $2\sqrt{3}x = 2$

Or, $x = \frac{1}{\sqrt{3}}$

$$\therefore \sin\theta = \frac{1}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1+\frac{1}{3}}} = \frac{1}{\sqrt{\frac{4}{3}}} = \frac{\sqrt{3}}{2}$$

$\theta = 60°$ **(Ans.)**

Question ▶ 38 $\cot\alpha = \frac{m}{n}$, $\tan^2B + \cot^2B = 2$.

[Mirpur Girls' Ideal Laboratory Institute, Dhaka]

a. Find $\cot\left(-\frac{11\pi}{6}\right)$ 2

b. From 1st equation find the value of $\sin\alpha$ and $\cos\alpha$ also show that $\sin^2\alpha + \cos^2\alpha = 1$ 4

c. Solve the 2nd equation where $0<B<2\pi$ 4

Solution to the question no. 38

a $\cot\left(\frac{-11\pi}{6}\right)$

$= -\cot\left(\frac{11\pi}{6}\right)$

$= -\cot\left(2\pi - \frac{\pi}{6}\right)$

$= -\cot\left(\frac{-\pi}{6}\right)$

$= \cot\left(\frac{\pi}{6}\right)$

$= \sqrt{3}$ **(Ans.)**

b Given, $\cot\alpha = \frac{m}{n}$

or, $\cot^2\alpha = \frac{m^2}{n^2}$

or, $\operatorname{cosec}^2\alpha - 1 = \frac{m^2}{n^2}$

or, $\operatorname{cosec}^2\alpha = 1 + \frac{m^2}{n^2}$

or, $\operatorname{cosec}^2\alpha = \frac{m^2+n^2}{n^2}$

$\therefore \operatorname{cosec}\alpha = \pm\frac{\sqrt{m^2+n^2}}{n}$

$\therefore \sin\alpha = \pm\frac{n}{\sqrt{m^2+n^2}}$ **(Ans.)**

And $\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$

$= \pm\sqrt{1-\frac{n^2}{m^2+n^2}}$

$= \pm\sqrt{\frac{m^2}{m^2+n^2}}$

$= \pm\frac{m}{\sqrt{m^2+n^2}}$ **(Ans.)**

$\therefore \sin^2\alpha + \cos^2\alpha = \left(\pm\frac{n}{\sqrt{m^2+n^2}}\right)^2 + \left(\pm\frac{m}{\sqrt{m^2+n^2}}\right)$

$= \frac{n^2+m^2}{m^2+n^2}$

$= 1$ **(Shown)**

c $\tan^2B + \cot^2B = 2$

or, $\tan^2B + \frac{1}{\tan^2B} = 2$

or, $\tan^4B + 1 = 2\tan^2B$ [multiplying both sides by $\tan^2\theta$]

or, $\tan^4B - 2\tan^2B + 1 = 0$

or, $(\tan^2B - 1)^2 = 0$

or, $\tan^2B - 1 = 0$

or, $\tan^2B = 1$

or, $\tan B = \pm 1$

Now, taking $\tan B = 1$ we get,

$\tan B = \tan\frac{\pi}{4}, \tan(\pi + \frac{\pi}{4})$ [according to the condition]

Or, $\tan B = \tan\frac{\pi}{4}, \tan\frac{5\pi}{4}$

$\therefore B = \frac{\pi}{4}, \frac{5\pi}{4}$

Again by taking $\tan B = -1$ we get,

$\tan B = -\tan\frac{\pi}{4}$

Or, $\tan B = \tan(\pi - \frac{\pi}{4}), \tan(2\pi - \frac{\pi}{4})$

[according to the condition]

Or, $\tan B = \tan\frac{3\pi}{4}, \tan\frac{7\pi}{4}$

$\therefore B = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\therefore$ The required solution: $B = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Question ▶ 39 Given $m = \cot\theta + \operatorname{cosec}\theta$

[Baridhara Scholars' Institution (BSI), Dhaka]

a. Find the value of $\operatorname{cosec}\theta - \cot\theta$ 2

b. If $m = 2$, then show that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1 + \sin\theta}{\cos\theta}$ 4

c. If $m = \sqrt{3}$, then find the value of θ, where $0 \le \theta \le 2\pi$ 4

Solution to the question no. 39

a Given, $\operatorname{cosec}\theta + \cot\theta = m$

We know, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Or, $(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

Or, $m(\operatorname{cosec}\theta - \cot\theta) = 1$

$\therefore \operatorname{cosec}\theta - \cot\theta = \frac{1}{m}$ **(Ans.)**

b Given, $\cot\theta + \operatorname{cosec}\theta = m$

Or, $\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} = 2$; [$\because m = 2$]

Or, $\frac{\cos\theta + 1}{\sin\theta} = 2$

Or, $\frac{(\cos\theta + 1)^2}{\sin^2\theta} = 4$

Or, $\frac{(1 + \cos\theta)^2}{1 - \cos^2\theta} = 4$

Or, $\frac{(1 + \cos\theta)(1 + \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)} = 4$

Or, $\frac{1 + \cos\theta}{1 - \cos\theta} = 4$

Or, $\frac{1 + \cos\theta + 1 - \cos\theta}{1 + \cos\theta - 1 + \cos\theta} = \frac{4+1}{4-1}$, [by componendo and dividendo]

Or, $\frac{2}{2\cos\theta} = \frac{5}{3}$

$\therefore \cos\theta = \frac{3}{5}$

$\therefore \sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{3}{5}\right)^2}$

$= \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

L.H.S. $= \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

$= \frac{\frac{4}{5} - \frac{3}{5} + 1}{\frac{4}{5} + \frac{3}{5} - 1} = \frac{\frac{4-3+5}{5}}{\frac{4+3-5}{5}}$

$= \frac{6}{5} \times \frac{5}{2} = 3$

R.H.S. $= \frac{1 + \sin\theta}{\cos\theta} = \frac{1 + \frac{4}{5}}{\frac{3}{5}}$

$= \frac{\frac{5+4}{5}}{\frac{3}{5}} = \frac{9}{5} \times \frac{5}{3}$

$= 3$

$\therefore \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1 + \sin\theta}{\cos\theta}$ **(Shown)**

c Given, $\operatorname{cosec}\theta + \cot\theta = m$

Or, $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$; [$\because m = \sqrt{3}$]

Or, $\operatorname{cosec}\theta = \sqrt{3} - \cot\theta$

Or, $\operatorname{cosec}^2\theta = 3 - 2\sqrt{3}\cot\theta + \cot^2\theta$

Or, $1 + \cot^2\theta - 3 + 2\sqrt{3}\cot\theta - \cot^2\theta = 0$

Or, $2\sqrt{3}\cot\theta - 2 = 0$

Or, $\cot\theta = \frac{2}{2\sqrt{3}}$

Or, $\cot\theta = \frac{1}{\sqrt{3}}$

Or, $\cot\theta = \cot\frac{\pi}{3} = \cot\left(\pi + \frac{\pi}{3}\right)$

$= \cot\frac{\pi}{3} = \cot\frac{4\pi}{3}$

$\therefore \theta = \frac{\pi}{3}, \frac{4\pi}{3}$

But, for $\theta = \frac{4\pi}{3}$ the given equation is not satisfied.

$\therefore$ Required solution, $\theta = \frac{\pi}{3}$ **(Ans.)**

Question ▶ 40 $R = 2\sin\theta\cos\theta$

[Chetona Model Academy (CMA), Dhaka]

a. Find the value of $\sec\left(\frac{-31\pi}{6}\right)$ 2

b. Show that, $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1 + \sin\theta}{\cos\theta}$ 4

c. If $R = \sin\theta$, find the value of θ. where $0 \le \theta \le 2\pi$ 4

Solution to the question no. 40

a $\sec\left(-\frac{31\pi}{6}\right)$

$= \sec\frac{31\pi}{6}$; [$\because \sec(-\theta) = \sec\theta$]

$= \sec\left(5\pi + \frac{\pi}{6}\right)$

$= \sec\left(10 \times \frac{\pi}{2} + \frac{\pi}{6}\right)$

$= -\sec\frac{\pi}{6}$; [n = 10, angle is in 2nd quadrant]

$= -\frac{2}{\sqrt{3}}$ **(Ans.)**

b $L.S = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

$= \frac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$; [$\sec^2A - \tan^2A = 1$]

$= \frac{(\sec\theta + \tan\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$

$= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{(1 - \sec\theta + \tan\theta)}$

$= \sec\theta + \tan\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$

$= \frac{1 + \sin\theta}{\cos\theta}$

$= R.S$

$\therefore$ L.S = R.S **(Shown)**

c $2\sin\theta\cos\theta = \sin\theta$

Or, $2\sin\theta\cos\theta - \sin\theta = 0$

Or, $\sin\theta(2\cos\theta - 1) = 0$

Either, $\sin\theta = 0$ or $2\cos\theta - 1 = 0$

$\therefore \cos\theta = \frac{1}{2}$

If $\sin\theta = 0$, then $\sin\theta = \sin 0, \sin\pi, \sin 2\pi$

$\therefore \theta = 0, \pi, 2\pi$

If $\cos\theta = \frac{1}{2}$, then $\cos\theta = \cos\frac{\pi}{3}, \cos\left(2\pi - \frac{\pi}{3}\right)$

Or, $\cos\theta = \cos\frac{\pi}{3}, \cos\frac{5\pi}{3}$

$\therefore \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$\therefore$ The required solution: $\theta = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

Question ▶ 41 $P = 7\sin^2\theta + 3\cos^2\theta$

[Rajshahi Cantonment Public School & College, Rajshahi]

a. Find the value of P when $\theta = \frac{\pi}{3}$. 2

b. If P = 4 prove that $\frac{\cos\theta}{\sin\theta} = \pm\sqrt{3}$. 4

c. If P = 6 and $0 < \theta < 2\pi$, find the possible value of θ. 4

Solution to the question no. 41

a Given that,

$P = 7\sin^2\theta + 3\cos^2\theta$

$= 7\left(\cos\frac{\pi}{3}\right)^2 + 3\left(\cos\frac{\pi}{3}\right)^2 \left[\because \theta = \frac{\pi}{3}\right]$

$= 7\left(\frac{\sqrt{3}}{2}\right)^2 + 3\left(\frac{1}{2}\right)^2$

$= 7.\frac{3}{4} + 3.\frac{1}{4}$

$= \frac{21}{4} + \frac{3}{4}$

$= \frac{21 + 3}{4}$

$= \frac{24}{4}$

$= 6$ **(Ans.)**

b Given that,

$P = 7\sin^2\theta + 3\cos^2\theta$

and, P = 4

$\therefore 7\sin^2\theta + 3\cos^2\theta = 4$

Or, $7\sin^2\theta + 3(1 - \sin^2\theta) = 4$

Or, $7\sin^2\theta + 3 - 3\sin^2\theta = 4$

Or, $4\sin^2\theta = 1$

$\therefore \sin^2\theta = \frac{1}{4}$

Again, $\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore \frac{\cos^2\theta}{\sin^2\theta} = \frac{\frac{3}{4}}{\frac{1}{4}}$

Or, $\left(\frac{\cos\theta}{\sin\theta}\right)^2 = \frac{3}{4} \times \frac{4}{1}$

Or, $\left(\frac{\cos\theta}{\sin\theta}\right)^2 = 3$

$\therefore \frac{\cos\theta}{\sin\theta} = \pm\sqrt{3}$. **(Proved)**

c According to question, P = 6

Or, $7\sin^2\theta + 3\cos^2\theta = 6$

Or, $7\sin^2\theta + 3(1 - \sin^2\theta) = 6$

Or, $7\sin^2\theta + 3 - 3\sin^2\theta = 6$

Or, $4\sin^2\theta = 3$

Or, $\sin^2\theta = \frac{3}{4}$

$\therefore \sin\theta = \pm\frac{\sqrt{3}}{2}$

Taking '+', $\sin\theta = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right)$

$\therefore \theta = \frac{\pi}{3}, \frac{2\pi}{3}$

Taking '–', $\sin\theta = -\frac{\sqrt{3}}{2} = \sin\left(\pi + \frac{\pi}{3}\right) = \sin\left(2\pi - \frac{\pi}{3}\right)$

$\therefore \theta = \frac{4\pi}{3}, \frac{5\pi}{3}$

$\therefore$ The possible values of θ in the given interval: $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(Ans.)

Question ▶ 42 Riding a bi-cycle Karim traverses an arc in 10 seconds which makes an angle 28° at the centre of a circular region. The wheel of the bi-cycle makes four revolutions per second and its diameter is 0.84 m.

[Millennium Scholastic School & College, Bogura]

a. If $\sin\theta + \cos\theta = \sqrt{2}$ then find the value of θ, where $0° < \theta < 90°$. 2
b. Determine the speed of Karim in km/hour. 4
c. Determine the area of the circular region. 4

Solution to the question no. 42

a See chapter-8.3, example-17 of your textbook. Page-181

b Given, diameter of the wheel = 0.84 m.

$\therefore$ The radius of the wheel, $r = \frac{0.84}{2}$ m. = 0.42 m.

$\therefore$ The circumference of the wheel $= 2\pi r$
$= 2 \times 3.1416 \times 0.42$ m.
$= 2.6389$ m.

$\therefore$ The wheel covers the distance 2.6389 m. in 1 revolution.
Again, it makes 4 revolutions in 1 second.
So, the distance covers in 1 second is 2.6389×4 m.
$\therefore$ The distance covers in 1 hour
$= 2.6389 \times 4 \times 60 \times 60$ m.
$= 38000.16$ m.
$= \frac{38000.16}{1000}$ km.
$= 38.00016$ km.

$\therefore$ The speed of Karim is 38 km/hour. (approx.) **(Ans.)**

c From 'b' the distance covers in 1 second is 2.6389×4m
$\therefore$ covers in 10 seconds is $2.6389 \times 4 \times 10$ m
$= 105.556$ m
$\therefore$ Arc length = 105.556 m

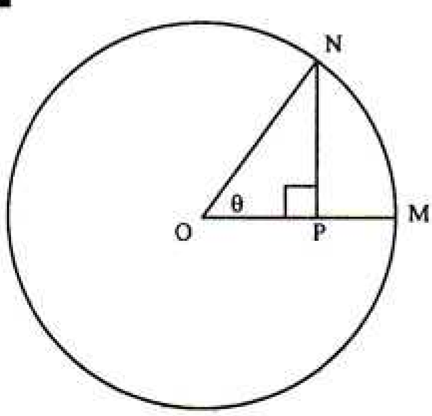
If the arc makes angle θ in the centre then $s = \frac{\pi r\theta°}{180°}$

According to the question $105.556 = \frac{\pi.r.28}{180}$

or, $r = 216$m (appr.)

$\therefore$ Area of the circular region $= \frac{\theta}{360}\pi r^2$
$= \frac{28}{360}.\pi.(216)^2$
$= 11399.79\text{m}^2$ **(Ans.)**

Question ▶ 43



In the figure, O is the centre of a circle and OM = arc MN.

[Dinajpur Laboratory School & College, Dinajpur]

a. Express θ in degree. 2
b. Prove that, θ is a constant angle. 4
c. Determine for what value of θ, $\frac{PN}{ON} + \frac{OP}{ON} = \sqrt{2}$ 4

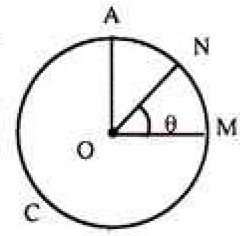
Solution to the question no. 43

a In given figure θ is a radian angle.
We know, $\pi^c = 180°$

$$1^c = \left(\frac{180}{\pi}\right)°$$

$\therefore \theta^c = \left(\frac{180\,\theta}{\pi}\right)°$ **(Ans.)**

b **Particular Enunciation:**



Suppose, in the circle AMC of raius r and centre O. ∠AOM is one radian. We have to prove that ∠NOM is a constant angle.

Proof: OA intersect the circumsference at A
So, arc AM = one-fourth of the circumference.

$\left[\frac{1}{4} \times 2\pi r = \frac{\pi r}{2} \text{ and}\right]$

From proposition 2, $\frac{\angle NOM}{\angle AOM} = \frac{\text{arc NM}}{\text{Arc AM}}$

$\therefore \angle NOM = \frac{\text{Arc NM}}{\text{Arc AM}} \times \angle AOM$

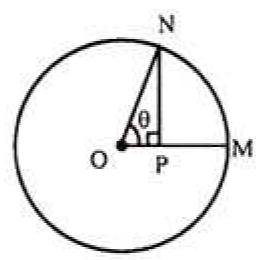
$\theta = \frac{r}{\frac{\pi r}{2}} \times 1 \text{ right angle} = \frac{2}{\pi} \times 1 \text{ right angle}$

Since, the right angle and π are constant, therefore θ is a constant angle **(proved)**

c In ΔOPN, PN ⊥ OP
$\therefore$ ∠OPN = 1 right angle
Now, in right angled triangle OPN,

$\sin\theta = \frac{PN}{ON}$ and $\cos\theta = \frac{OP}{ON}$

Given,



$\frac{PN}{ON} + \frac{OP}{ON} = \sqrt{2}$

Or, $\sin\theta + \cos\theta = \sqrt{2}$
Or, $\sin\theta - \sqrt{2} = -\cos\theta$
Or, $(\sin\theta - \sqrt{2})^2 = (-\cos\theta)^2$
Or, $\sin^2\theta - 2\sqrt{2}\sin\theta + 2 = \cos^2\theta$
Or, $\sin^2\theta - 2\sqrt{2}\sin\theta + 2 = 1 - \sin^2\theta$
Or, $2\sin^2\theta - 2\sqrt{2}\sin\theta + 1 = 0$
Or, $(\sqrt{2}\sin\theta - 1)^2 = 0$
Or, $\sqrt{2}\sin\theta - 1 = 0$
Or, $\sqrt{2}\sin\theta = 1$
Or, $\sin\theta = \frac{1}{\sqrt{2}}$
Or, $\sin\theta = \sin 45°$
$\therefore \theta = 45°$ **(Ans.)**

Question ▶ 44 $p(\theta) = a\cos\theta - b\sin\theta$

[Cantonment Public School & College, Saidpur]

a. Express in radian : 55°52′53″ 2

b. If a = b and $p(\theta) = 0$, then solve it when $0 \le \theta \le 2\pi$. 4

c. If $p(\theta) = c$ then show that, $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$ 4

Solution to the question no. 44

a $55°52'53'' = 55° + 52' + 53''$

$= 55° + 52' + \left(\frac{53}{60}\right)'$ $[\because 1'' = \left(\frac{1}{60}\right)']$

$= 55° + \left(52 + \frac{53}{60}\right)'$

$= 55° + \left(\frac{3173}{60}\right)'$

$= 55° + \left(\frac{3173}{60\times60}\right)°$ $[\because 1' = \left(\frac{1}{60}\right)°]$

$= \left(55 + \frac{3173}{3600}\right)°$

$= \left(\frac{201173}{3600}\right)°$

$= \frac{201173 \times \pi^c}{3600 \times 180}$ $[\because 1° = \frac{\pi^c}{180}]$

$= 0.31045 \times 3.1416$ radian $[\because \pi = 3.1416]$

$= 0.9753$ radian (approx.) **(Ans.)**

b Given, $p(\theta) = a\cos\theta - b\sin\theta$

Or, $0 = a\cos\theta - a\sin\theta$ $[\because a = b$ and $p(\theta) = 0]$

Or, $a\sin\theta = a\cos\theta$

Or, $\frac{a\sin\theta}{a\cos\theta} = 1$

Or, $\tan\theta = 1$

In first quadrant $\tan\theta$ is + ve

$\therefore \tan\theta = \tan\frac{\pi}{4}$

$\therefore \theta = \frac{\pi}{4}$

In 3rd quadrant $\tan\theta$ is + ve

$\therefore \tan\theta = \tan\left(\pi + \frac{\pi}{4}\right)$

$\therefore \theta = \frac{5\pi}{4}$

$\therefore$ The required value of θ is $= \frac{\pi}{4}, \frac{5\pi}{4}$ **(Ans.)**

c Here, $p(\theta) = c$

Or, $a\cos\theta - b\sin\theta = c$

Or, $(a\cos\theta - b\sin\theta)^2 = c^2$ [squaring both sides]

Or, $a^2\cos^2\theta - 2a\cos\theta.\, b\sin\theta + b^2\sin^2\theta = c^2$

Or, $a^2(1 - \sin^2\theta) - 2a\cos\theta.b\sin\theta + b^2(1 - \cos^2\theta) = c^2$

Or, $a^2 - a^2\sin^2\theta - 2a\cos\theta.b\sin\theta + b^2 - b^2\cos^2\theta = c^2$

Or, $-(a^2\sin^2\theta + 2a\cos\theta.b\sin\theta + b^2\cos^2\theta) = -(a^2 + b^2 - c^2)$

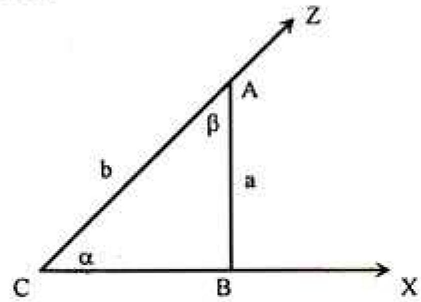
Or, $a^2\sin^2\theta + 2a\cos\theta.b\sin\theta + b^2\cos^2\theta = a^2 + b^2 - c^2$

Or, $(a\sin\theta)^2 + 2a\sin\theta.b\cos\theta + (b\cos\theta)^2 = a^2 + b^2 - c^2$

Or, $(a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

$\therefore a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$ **(Proved)**

Question ▶ 45



[Mainamati International School and College, Cumilla]

a. Determine the value of $\sec\alpha$. 2

b. If a = 1 and b = 2, prove that, $\cos3\beta = 4\cos^3\beta - 3\cos\beta$. 4

c. If $a + \sqrt{b^2 - a^2} = \sqrt{2}b$ then, find the value of β. 4

Solution to the question no. 45

a Given,
AB = a, AC = b
In right angle triangle ABC,
$AC^2 = AB^2 + BC^2$
Or, $BC^2 = AC^2 - AB^2$
$\therefore BC = \sqrt{b^2 - a^2}$
$\therefore \sec\alpha = \frac{AC}{BC} = \frac{b}{\sqrt{b^2 - a^2}}$ **(Ans.)**

b Given, a = 1 and b = 2
In ΔABC,
$\cos\beta = \frac{AB}{AC}$ $[\because AB = a, AC = b]$
$= \frac{a}{b} = \frac{1}{2}$
Or, $\cos\beta = \cos\frac{\pi}{3}$
$\therefore \beta = \frac{\pi}{3}$

L. H. S. $= \cos3\beta = \cos3.\frac{\pi}{3} = \cos\pi = -1$

R. H. S. $= 4\cos^3\beta - 3\cos\beta$

$= 4.\left(\frac{1}{2}\right)^3 - 3.\frac{1}{2} = 4.\frac{1}{8} - \frac{3}{2} = \frac{1}{2} - \frac{3}{2} = \frac{1-3}{2} = -1$

$\therefore \cos3\beta = 4\cos^3\beta - 3\cos\beta$ **(Proved)**

c From 'a' we get, $BC = \sqrt{b^2 - a^2}$
In, ΔABC, $\sin\beta = \frac{BC}{AC}$
$\therefore \sin\beta = \frac{\sqrt{b^2 - a^2}}{b}$ $[\therefore AC = b]$
and $\cos\beta = \frac{AB}{AC} = \frac{a}{b}$ $[\therefore AB = a]$
Given,
$a + \sqrt{b^2 - a^2} = \sqrt{2}b$
Or, $\frac{a}{b} + \frac{\sqrt{b^2 - a^2}}{b} = \frac{\sqrt{2}b}{b}$ [Both sides divided by b]
Or, $\cos\beta + \sin\beta = \sqrt{2}$
$\therefore \cos\beta = \sqrt{2} - \sin\beta$
Or, $\cos^2\beta = 2 - 2\sqrt{2}\sin\beta + \sin^2\beta$ [Squaring both sides]
Or, $1 - \sin^2\beta = 2 - 2\sqrt{2}\sin\beta + \sin^2\beta$
Or, $2\sin^2\beta - 2\sqrt{2}\sin\beta + 1 = 0$
Or, $(\sqrt{2}\sin\beta - 1)^2 = 0$
Or, $\sin\beta = \frac{1}{\sqrt{2}}$
Or, $\sin\beta = \sin45°$
$\therefore \beta = 45°$**(Ans.)**

Question ▶ 46 $A = 7\sin^2\theta + 3\cos^2\theta$ and $\tan P = \frac{b}{a}$ where P is an acute angle. *[Cantonment English School & College, Chattogram]*

a. Find $\tan\left(-\frac{11\pi}{6}\right)$. 2

b. Find the value of cos P and sin P. Also show that $\sin^2P + \cos^2P = 1$. 4

c. If A = 6, then find the value of θ (where $0 < \theta < 2\pi$). 4

Solution to the question no. 46

a $\tan\left(\frac{-11\pi}{6}\right) = -\tan\left(\frac{11\pi}{6}\right)$

$= -\tan\left(2\pi - \frac{\pi}{6}\right)$

$= \tan\frac{\pi}{6}$

$= \frac{1}{\sqrt{3}}$ **(Ans.)**

b Given, $\tan P = \frac{b}{a}$

Or, $\tan^2P = \frac{b^2}{a^2}$ [Squaring both sides]

Or, $\sec^2P - 1 = \frac{b^2}{a^2}$ [$\because \tan^2\theta = \sec^2\theta - 1$]

Or, $\sec^2P = \frac{b^2}{a^2} + 1$

Or, $\frac{1}{\cos^2P} = \frac{b^2 + a^2}{a^2}$

Or, $\cos^2P = \frac{a^2}{a^2 + b^2}$

$\therefore$ $\cos P = \frac{\pm a}{\sqrt{a^2 + b^2}}$ **(Ans.)**

Again, $\cos^2P = \frac{a^2}{a^2 + b^2}$

Or, $1 - \sin^2P = \frac{a^2}{a^2 + b^2}$

Or, $\sin^2P = 1 - \frac{a^2}{a^2 + b^2}$

Or, $\sin^2P = \frac{a^2 + b^2 - a^2}{a^2 + b^2}$

$\therefore$ $\sin P = \frac{\pm b}{\sqrt{a^2 + b^2}}$ **(Ans.)**

2nd part :

L.H.S $= \sin^2P + \cos^2P$

$= \frac{b^2}{a^2 + b^2} + \frac{a^2}{a^2 + b^2}$

$= \frac{b^2 + a^2}{a^2 + b^2}$

$= 1$

= R. H. S

$\therefore \sin^2P + \cos^2P = 1$ **(Shown)**

c Given, $A = 7\sin^2\theta + 3\cos^2\theta$

Or, $7\sin^2\theta + 3\cos^2\theta = 6$ [$\because$ A = 6]

Or, $7\sin^2\theta + 3(1 - \sin^2\theta) = 6$

Or, $7\sin^2\theta + 3 - 3\sin^2\theta = 6$

Or, $4\sin^2\theta = 6 - 3$

Or, $\sin^2\theta = \frac{3}{4}$

Or, $\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\therefore$ $\sin\theta = \frac{\sqrt{3}}{2}$

Or, $\sin\theta = \sin\frac{\pi}{3}, \sin\left(\pi + \frac{\pi}{3}\right)$

$\therefore$ $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

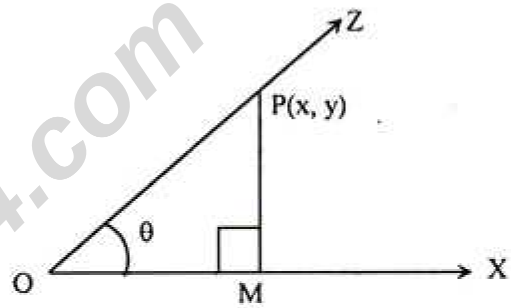
Or, $\sin\theta = -\frac{\sqrt{3}}{2}$

Or, $\sin\theta = \sin\left(\pi + \frac{\pi}{3}\right), \sin\left(2\pi - \frac{\pi}{3}\right)$

$\therefore$ $\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$

$\therefore$ $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **(Ans.)**

Question ▶ 47 Observe the following figure.



[Bangladesh Mahila Somitee Girls' High School & College, Chattogram]

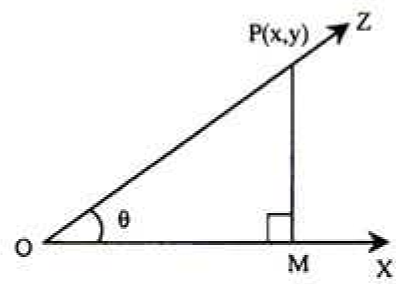
a. Find the value of secθ. 2

b. If x = 1, $y = \sqrt{3}$, then prove that, $\sin^3\theta = 3\sin\theta - 4\sin^3\theta$. 4

c. If $\sqrt{x^2 + y^2} + x = \sqrt{3}y$, then find the value of θ. 4

Solution to the question no. 47

a



From the figure, we get,

OM = x and PM = y [$\because$ Coordinates of P is(x, y)]

From ΔOPM ∠OMP = 90°

According to the Pythagoras Theorem,

$OP^2 = OM^2 + PM^2$

Or, $OP^2 = x^2 + y^2$

$\therefore$ $OP = \sqrt{x^2 + y^2}$

$\therefore$ $\sec\theta = \frac{OP}{OM} = \frac{\sqrt{x^2 + y^2}}{x}$ **(Ans.)**

b Obtained from 'a' we get,

$\sec\theta = \frac{\sqrt{x^2 + y^2}}{x}$

Or, $\sec\theta = \frac{\sqrt{1^2 + (\sqrt{3})^2}}{1}$; [$\therefore$ x = 1, $y = \sqrt{3}$]

Or, $\sec\theta = \sqrt{4}$

Or, $\sec\theta = 2$

Or, $\sec\theta = \sec 60°$ [$\because \sec 60° = 2$]

$\therefore \theta = 60°$

L. H. S. $= \sin 3\theta$

$= \sin(3 \times 60°)$

$= \sin 180°$

$= 0$

R. H. S $= 3\sin\theta - 4\sin^3\theta$

$= 3\sin 60° - 4\sin^3 60°$

$= 3.\frac{\sqrt{3}}{2} - 4.\left(\frac{\sqrt{3}}{2}\right)^3$

$= \frac{3\sqrt{3}}{2} - 4.\frac{3\sqrt{3}}{8}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$

$= 0$

$\therefore \sin 3\theta = 3\sin\theta - 4\sin^3\theta$ **(proved)**

c Obtained from 'a' we get,

OM = x, PM = y and $OP = \sqrt{x^2 + y^2}$

We know, $\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{OP}{PM} = \frac{\sqrt{x^2+y^2}}{y}$

$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{OM}{PM} = \frac{x}{y}$

Given, $\sqrt{x^2+y^2} + x = \sqrt{3}y$

Or, $\frac{\sqrt{x^2+y^2}+x}{y} = \frac{\sqrt{3}y}{y}$; [Dividing both sides by y]

Or, $\frac{\sqrt{x^2+y^2}}{y} + \frac{x}{y} = \sqrt{3}$

Or, $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$

Or, $\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \sqrt{3}$

Or, $\frac{1+\cos\theta}{\sin\theta} = \sqrt{3}$

Or, $(1+\cos\theta)^2 = (\sqrt{3}\sin\theta)^2$; [squaring]

Or, $1 + 2\cos\theta + \cos^2\theta = 3\sin^2\theta$

Or, $1 + 2\cos\theta + \cos^2\theta - 3(1 - \cos^2\theta) = 0$

Or, $1 + 2\cos\theta + \cos^2\theta - 3 + 3\cos^2\theta = 0$

Or, $4\cos^2\theta + 2\cos\theta - 2 = 0$

Or, $2\cos^2\theta + \cos\theta - 1 = 0$

Or, $2\cos^2\theta + 2\cos\theta - \cos\theta - 1 = 0$

Or, $2\cos\theta(\cos\theta + 1) - 1(\cos\theta + 1) = 0$

Or, $(\cos\theta + 1)(2\cos\theta - 1) = 0$

either, $\cos\theta + 1 = 0$ Or, $2\cos\theta - 1 = 0$

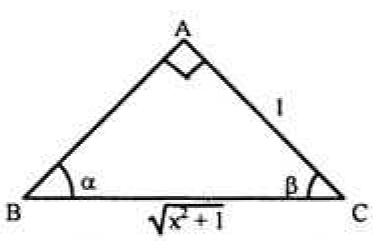
Or, $\cos\theta = -1$ Or, $\cos\theta = \frac{1}{2}$

Or, $\cos\theta = \cos 180°$ Or, $\cos\theta = \cos 60°$ $\therefore \theta = 60°$

$\therefore \theta = 180°$, which is not acceptable because θ is an acute angle.

The required value: $\theta = 60°$ **(Ans.)**

Question ▶ 48



[Navy Anchorage School and College, Chattogram]

a. Find the value of $\sin(\alpha + \beta) + \cos(\alpha + \beta)$. 2

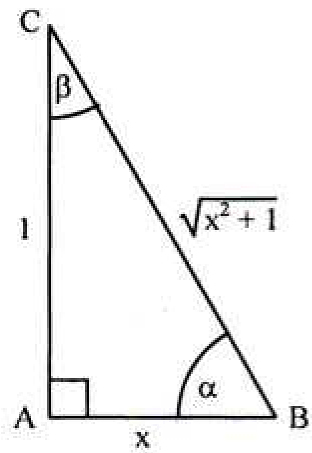
b. considering the stem prove that $(\sin\alpha - \cos\alpha)^2 = 1\text{-}2\sin\alpha.\cos\alpha$. 4

c. If $x^2 + \frac{1}{x^2} = 2$ then find the value of α. 4

Solution to the question no. 48

a From, the right angled triangle ABC, we get $\alpha + \beta = 90°$

$\therefore \sin(\alpha + \beta) + \cos(\alpha + \beta)$



$= \sin 90° + \cos 90°$

$= 1 + 0$

$= 1$ **(Ans.)**

b From the stem, we get

$\sin\alpha = \frac{1}{\sqrt{x^2+1}}$

and $\cos\alpha = \frac{x}{\sqrt{x^2+1}}$

Now, L.H.S $= (\sin\alpha - \cos\alpha)^2$

$= \left(\frac{1}{\sqrt{x^2+1}} - \frac{x}{\sqrt{x^2+1}}\right)^2$

$= \left(\frac{1-x}{\sqrt{x^2+1}}\right)^2$

$= \frac{(1-x)^2}{x^2+1}$

and, R.H.S $= 1 - 2\sin\alpha.\cos\alpha$

$= 1 - 2.\frac{1}{\sqrt{x^2+1}}.\frac{x}{\sqrt{x^2+1}}$

$= 1 - \frac{2x}{x^2+1}$

$= \frac{x^2+1-2x}{x^2+1}$

$= \frac{(1-x)^2}{x^2+1}$

$\therefore (\sin\alpha - \cos\alpha)^2 = 1 - 2\sin\alpha.\cos\alpha$ **(Proved)**

c Given that, $x^2 + \frac{1}{x^2} = 2$

Or, $x^2 + \frac{1}{x^2} - 2 = 0$

Or, $\left(x - \frac{1}{x}\right)^2 = 0$

Or, $\left(x - \frac{1}{x}\right) = 0$

Or, $x^2 - 1 = 0$

Or, $x^2 = 1$

From, ΔABC,

we get, $\sin\alpha = \frac{AC}{BC} = \frac{1}{\sqrt{x^2+1}}$

$= \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$

Or, $\sin\alpha = \sin 45°$

$\therefore \alpha = 45°$ **(Ans.)**

Question ▶ 49 Let, $m = \tan\theta + \sec\theta$ and $n = 5\text{cosec}^2\alpha - 7\text{cosec}\alpha.\cot\alpha$ *[SCHOLARSHOME, Sylhet]*

a. If $\theta = \frac{\pi}{3}$, then prove that, $\sin3\theta = 3\sin\theta - 4\sin^3\theta$. 2

b. Prove that, $\sin\theta = \frac{m^2-1}{m^2+1}$. 4

c. If $n = 2$, then find the value of α, where $0 < \alpha < 2\pi$. 4

Solution to the question no. 49

a Given, $\theta = \frac{\pi}{3}$

$\therefore \sin3\theta = \sin 3.\frac{\pi}{3} = \sin\pi = 0$

And, $3\sin\theta - 4\sin^3\theta = 3\sin\frac{\pi}{3} - 4.\left(\sin\frac{\pi}{3}\right)^3$

$= 3.\frac{\sqrt{3}}{2} - 4.\left(\frac{\sqrt{3}}{2}\right)^3$

$= \frac{3\sqrt{3}}{2} - 4.\frac{3\sqrt{3}}{8}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$

$\therefore \sin3\theta = 3\sin\theta - 4\sin^3\theta$. **(Proved)**

b Given,

$\tan\theta + \sec\theta = m$

Or, $\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = m$ [$\because \tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$]

Or, $\frac{1+\sin\theta}{\cos\theta} = m$

Or, $\frac{(1+\sin\theta)^2}{\cos^2\theta} = m^2$ [squaring both sides]

Or, $\frac{(1+\sin\theta)^2}{1-\sin^2\theta} = m^2$ [$\because \cos^2\theta = 1 - \sin^2\theta$]

Or, $\frac{(1+\sin\theta)(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} = m^2$

Or, $\frac{1+\sin\theta}{1-\sin\theta} = m^2$

Or, $\frac{1+\sin\theta-1+\sin\theta}{1+\sin\theta+1-\sin\theta} = \frac{m^2-1}{m^2+1}$ [by compodendo-dividendo]

Or, $\frac{2\sin\theta}{2} = \frac{m^2-1}{m^2+1}$

$\therefore \sin\theta = \frac{m^2-1}{m^2+1}$

c $5\text{cosec}^2\alpha - 7\text{cosec}\alpha = 2$, when $n = 2$

Or, $5\text{cosec}^2\alpha - 7\cot\alpha\,\text{cosec}\alpha - 2 = 0$

Or, $\frac{5}{\sin^2\alpha} - \frac{7\cos\alpha}{\sin^2\alpha} - 2 = 0$

Or, $5 - 7\cos\alpha - 2\sin^2\alpha = 0$

Or, $5 - 7\cos\alpha - 2(1-\cos^2\alpha) = 0$

Or, $5 - 7\cos\alpha - 2 + 2\cos^2\alpha = 0$

Or, $2\cos^2\alpha - 7\cos\alpha + 3 = 0$

Or, $2\cos^2\alpha - 6\cos\alpha - \cos\alpha + 3 = 0$

Or, $2\cos\alpha(\cos\alpha - 3) - 1(\cos\alpha - 3) = 0$

Or, $(2\cos\alpha - 1)(\cos\alpha - 3) = 0$

Either, $2\cos\alpha - 1 = 0$ or, $\cos\alpha - 3 = 0$

$\therefore \cos\alpha = \frac{1}{2}$ $\qquad \therefore \cos\alpha = 3$

But, the value of $\cos\alpha$ can never be greater than 1.

$\therefore \cos\alpha = \frac{1}{2}$

$\cos\alpha = \cos\frac{\pi}{3}, \cos(2\pi - \frac{\pi}{3})$ [according to the condition]

$\therefore \alpha = \frac{\pi}{3}, \frac{5\pi}{3}$, which lie in the given limit, $0 < \alpha < 2\pi$

$\therefore$ The required solution : $\alpha = \frac{\pi}{3}, \frac{5\pi}{3}$

Question ▶ 50 $\cot\theta + \text{cosec}\theta = m$ is a trigonometric equation. *[Jalalabad Cantonment Public School & College, Sylhet]*

a. Find the value of $\cot\theta - \text{cosec}\theta$ if $m = \frac{3}{2}$. 2

b. If $m = 2$ then show that, $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1+\sin\theta}{\cos\theta}$ 4

c. Determine the value of θ if $m = \sqrt{3}$ [where $0° \le \theta \le 2\pi$] 4

Solution to the question no. 50

a Given, $\text{cosec}\theta + \cot\theta = m$

We know, $\text{cosec}^2\theta - \cot^2\theta = 1$

Or, $(\text{cosec}\theta + \cot\theta)(\text{cosec}\theta - \cot\theta) = 1$

Or, $m(\text{cosec}\theta - \cot\theta) = 1$

$\therefore \text{cosec}\theta - \cot\theta = \frac{1}{m}$

$\cot\theta - \text{cosec}\theta = \frac{1}{\frac{3}{2}}$; $\qquad \left[\text{if } m = \frac{3}{2}\right]$

$= \frac{2}{3}$ **(Ans.)**

b Given, $\cot\theta + \text{cosec}\theta = m$

Or, $\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} = 2$; [$\because m = 2$]

Or, $\frac{\cos\theta + 1}{\sin\theta} = 2$

Or, $\frac{(\cos\theta+1)^2}{\sin^2\theta} = 4$

Or, $\frac{(1+\cos\theta)^2}{1-\cos^2\theta} = 4$

Or, $\frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} = 4$

Or, $\frac{1+\cos\theta}{1-\cos\theta} = 4$

Or, $\frac{1+\cos\theta+1-\cos\theta}{1+\cos\theta-1+\cos\theta} = \frac{4+1}{4-1}$; [by componendo and dividendo]

Or, $\frac{2}{2\cos\theta} = \frac{5}{3}$

$\therefore \cos\theta = \frac{3}{5}$

$\therefore \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{3}{5}\right)^2}$

$= \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

L.H.S. $= \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

$= \frac{\frac{4}{5} - \frac{3}{5} + 1}{\frac{4}{5} + \frac{3}{5} - 1} = \frac{\frac{4-3+5}{5}}{\frac{4+3-5}{5}}$

$= \frac{6}{5} \times \frac{5}{2} = 3$

$$\text{R.H.S.} = \frac{1+\sin\theta}{\cos\theta} = \frac{1+\frac{4}{5}}{\frac{3}{5}}$$

$$= \frac{\frac{5+4}{5}}{\frac{3}{5}} = \frac{9}{5} \times \frac{5}{3}$$

$$= 3$$

$$\therefore \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1+\sin\theta}{\cos\theta} \textbf{ (Shown)}$$

c Given, $\operatorname{cosec}\theta + \cot\theta = m$

Or, $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$ $\quad [\because m = \sqrt{3}]$

Or, $\operatorname{cosec}\theta = \sqrt{3} - \cot\theta$

Or, $\operatorname{cosec}^2\theta = 3 - 2\sqrt{3}\cot\theta + \cot^2\theta$

Or, $1 + \cot^2\theta - 3 + 2\sqrt{3}\cot\theta - \cot^2\theta = 0$

Or, $2\sqrt{3}\cot\theta - 2 = 0$

Or, $\cot\theta = \frac{2}{2\sqrt{3}}$

Or, $\cot\theta = \frac{1}{\sqrt{3}}$

Or, $\cot\theta = \cot\frac{\pi}{3} = \cot\left(\pi + \frac{\pi}{3}\right)$

$= \cot\frac{\pi}{3} = \cot\frac{4\pi}{3}$

$\therefore \theta = \frac{\pi}{3}, \frac{4\pi}{3}$

But, for $\theta = \frac{4\pi}{3}$ the given equation is not satisfied.

$\therefore$ Required solution, $\theta = \frac{\pi}{3}$. **(Ans.)**

Question ▶51 $\tan\theta = \frac{3}{4}$ and $\cos\theta$ is negative.

[The Sylhet Khajanchibari International School & College, Sylhet]

a. What is the value of $\sec\theta$? 2

b. Find the value of $(\cot\theta - \operatorname{cosec}\theta)^{\frac{1}{2}}$? 4

c. Prove that, $\frac{\sin\theta + \cos(-\theta)}{\sec(-\theta) + \tan\theta} = \frac{14}{5}$ 4

Solution to the question no. 51

a Given, $\tan\theta = \frac{3}{4}$ and $\cos\theta$ is negative.

Or, $\tan^2\theta = \frac{9}{16}$; [Squaring]

Or, $\sec^2\theta - 1 = \frac{9}{16}$

Or, $\sec^2\theta = \frac{9}{16} + 1$

Or, $\sec^2\theta = \frac{9+16}{16}$

Or, $\sec\theta = -\frac{5}{4}$; $\quad [\because \cos\theta - \text{ve}]$

; $[\therefore \sec\theta$ also $-$ ve] **(Ans.)**

b From 'a',

$\sec\theta = \frac{-5}{4}$

$\therefore \cos\theta = \frac{-4}{5}$

$\therefore \sin^2\theta = 1 - \cos^2\theta$

$= 1 - \frac{16}{25}$

$= \frac{25-16}{25}$

$\therefore \sin\theta = \frac{-3}{5}$

$[\because \tan\theta$ is (+) ve. and $\cos\theta$ and $\sin\theta$ both are (−) ve]

$\therefore \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{-3}{5}} = \frac{-5}{3}$

Again, $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

$\therefore (\cot\theta - \operatorname{cosec}\theta)^{\frac{1}{2}} = \sqrt{\cot\theta - \operatorname{cosec}\theta}$

$= \sqrt{\frac{4}{3} - \left(-\frac{5}{3}\right)}$

$=$

$= \sqrt{\frac{4+5}{3}}$

$= \sqrt{\frac{9}{3}}$

$= \sqrt{3}$ **(Ans.)**

c From 'a' and 'b',

$\cos\theta = -\frac{4}{5}$, $\sec\theta = -\frac{5}{4}$, $\sin\theta = -\frac{3}{5}$ and $\tan\theta = \frac{3}{4}$

$\therefore \frac{\sin\theta + \cos(-\theta)}{\sec(-\theta) + \tan\theta}$

$= \frac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta}$; $\quad [\because \cos(-\theta) = \cos\theta, \sec(-\theta) = \sec\theta]$

$= \frac{-\frac{3}{5} - \frac{4}{5}}{-\frac{5}{4} + \frac{3}{4}} = \frac{\frac{-3-4}{5}}{\frac{-5+3}{4}}$

$= \frac{\frac{-7}{5}}{\frac{-2}{4}} = \frac{7}{5} \times \frac{4}{2} = \frac{14}{5}$

$\therefore \frac{\sin\theta + \cos(-\theta)}{\sec(-\theta) + \tan\theta} = \frac{14}{5}$ **(Proved)**

Question ▶52 $\cot A + \operatorname{cosec} A = m$, $3\cos^2\theta - 2\sin^2\theta = n$ when $0 < \theta < 2\pi$. *[Secondary & Higher Secondary Education Board, Jashore]*

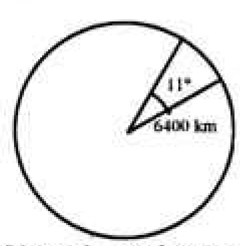
a. Find the distance between the two points which makes an angle 11′ at the centre of the earth. And where the radius of the earth is 6400 km. 2

b. Prove that $\sin A = \frac{2m}{m^2 + 1}$ 4

c. If $n = \frac{1}{2}$, $\theta = ?$ 4

Solution to the question no. 52

a



we know, if any arc of length produces an angle θ at the centre of the circle of radius then $S = r\theta$

there, $\theta = 11' = \frac{11^\circ}{60} = \frac{11 \times \pi}{60 \times 180}$ radian

and $r = 6400$km

$\therefore \quad S = r\theta = 6400 \times \frac{11 \times \pi}{60 \times 180}$

$= \frac{704 \times 3.1416}{108}$

$= 20.476$ km (approx.) **(Ans.)**

b Given, $\cot A + \operatorname{cosec} A = m$

we have to prove that $\sin A = \frac{2m}{m^2 + 1}$

$$\text{R.H.S} = \frac{2m}{m^2+1}$$

$$= \frac{2(\cot A + \operatorname{cosec} A)}{(\cot A + \operatorname{cosec} A)^2 + 1}$$

$$= \frac{2\left(\frac{\cos A}{\sin A} + \frac{1}{\sin A}\right)}{\left(\frac{\cos A}{\sin A} + \frac{1}{\sin A}\right)^2 + 1}$$

$$= \frac{2\,\frac{\cos A + 1}{\sin A}}{\left(\frac{\cos A + 1}{\sin A}\right)^2 + 1}$$

$$= \frac{2\,\frac{(\cos A + 1)}{\sin A}}{\frac{(\cos A + 1)^2}{\sin^2 A} + 1}$$

$$= \frac{2\,\frac{(\cos A + 1)}{\sin A}}{\frac{\cos^2 A + 2\cos A + 1 + \sin^2 A}{\sin^2 A}}$$

$$= \frac{2\,\frac{(\cos A + 1)}{\sin A}}{\frac{1 + 1 + 2\cos A}{\sin^2 A}}$$

$$= \frac{2\,\frac{(\cos A + 1)}{\sin A}}{\frac{2(\cos A + 1)}{\sin^2 A}} = \sin A$$

$= \text{L.H.S}$

$\therefore$ L.H.S. = R.H.S **(Proved)**

c Given,

$3\cos^2\theta - 2\sin^2\theta = n$

if $\quad n = \frac{1}{2}$ then

$3\cos^2\theta - 2\sin^2\theta = \frac{1}{2}$

Or, $\quad 3\cos^2\theta - 2(1 - \cos^2\theta) = \frac{1}{2}$

Or, $\quad 3\cos^2\theta - 2 + 2\cos^2\theta = \frac{1}{2}$

Or, $\quad 5\cos^2\theta = \frac{1}{2} + 2$

Or, $\quad 5\cos^2\theta = \frac{5}{2}$

Or, $\quad \cos^2\theta = \frac{1}{2}$

Or, $\quad \cos\theta = \pm\frac{1}{\sqrt{2}}$

taking positive value, $\cos\theta = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4} = \cos\left(2\pi - \frac{7\pi}{4}\right)$

$= \cos\frac{7\pi}{4}$

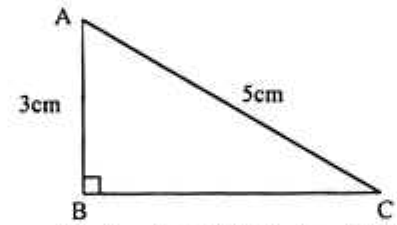
taking negative values, $\cos\theta = -\frac{1}{\sqrt{2}} = \cos\frac{3\pi}{4}$

$= \cos\left(2\pi - \frac{5\pi}{4}\right) = \cos\frac{5\pi}{4}$

$\therefore$ The possible values of θ in the given interval is $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(Ans.)

Question ▶ 53 Observe the following figure:



[Jashore English School and College (JESC), Jashore]

a. Determine the value of $\tan\theta$ and $\cos\theta$. 2

b. Solve : $2\sin^2\theta - 3\cos\theta = 0$ (when $0 < \theta < \frac{\pi}{2}$) 4

c. Find the value of $\sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14}$. 4

Solution to the question no. 53

a For ΔABC, $AB^2 + BC^2 = AC^2$

Or, $3^2 + BC^2 = 5^2$

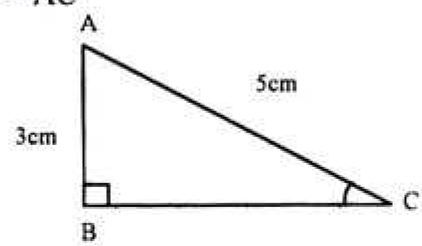
Or, $BC^2 = 25 - 9$

Or, $BC^2 = 16$

$\therefore BC = 4$

$\therefore \tan\theta = \frac{AB}{BC} = \frac{3}{4}$

and $\cos\theta = \frac{BC}{AC} = \frac{4}{5}$ **(Ans.)**



b Given,

$2\sin^2\theta - 3\cos\theta = 0$

Or, $2(1 - \cos^2\theta) - 3\cos\theta = 0$

Or, $2 - 2\cos^2\theta - 3\cos\theta = 0$

Or, $2\cos^2\theta + 3\cos\theta - 2 = 0$

Or, $2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

Or, $2\cos\theta(\cos\theta + 2) - 1(\cos + 2) = 0$

Or, $(\cos\theta + 2)(2\cos\theta - 1) = 0$

Either, $2\cos - 1 = 0$ Or, $\cos\theta + 2 = 0$

Or, $2\cos\theta = 1$ $\cos = -2$ [Not accepted]

Or, $\cos\theta = \frac{1}{2}$

Or, $\cos\theta = \cos\frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$

$\therefore$ Required solution $= \frac{\pi}{3}$ **(Ans.)**

c According to the question,

$= \sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14}$

$= \sin^2\frac{\pi}{7} + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{7}\right) + \sin^2\left(\pi + \frac{\pi}{7}\right) + \sin^2\left(\frac{\pi}{2} + \frac{\pi}{7}\right)$

$= \sin^2\frac{\pi}{7} + \left\{\sin\left(\frac{\pi}{2} - \frac{\pi}{7}\right)\right\}^2 + \left\{\sin\left(\pi + \frac{\pi}{7}\right)\right\}^2 + \left\{\sin\left(\frac{\pi}{2} + \frac{\pi}{7}\right)\right\}^2$

$= \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} + \left(-\sin\frac{\pi}{7}\right)^2 + \cos^2\frac{\pi}{7}$

$= \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} + \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7}$

$= 1 + 1 \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$= 2$ **(Shown)**